Micro-mechanical consideration of poroplasticity: Applications to porous geomaterial

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Hong Kong, China
14-16 December 2016

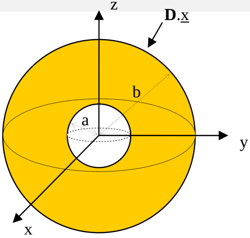
Main issues

- Plastic criteria of porous materials : effect of porosity
- Effective stress concept : effect of pore pressure
- Experimental validation

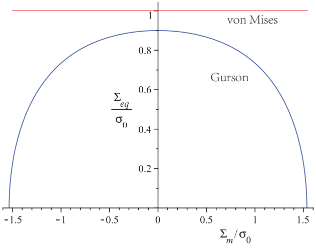
Plan

1. Non-linear homogenization methods
    - Analytical methods
        - Limit analysis
        - Modified secant method
        - Stress Variational Homogenization
    - Semi-analytical methods and numerical methods
        - Incremental approach of Hill
        - FFT based numerical homogenization
2. Porous geomaterials with a Mises-Schleicher matrix
3. Effective stress concept
4. Double porous medium saturated by two different pore pressures

Reminder on the Gurson criterion (1977)



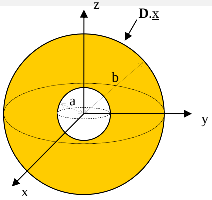
Hollow sphere model



Criterion of the solid matrix

- Limit analysis of a hollow sphere with a von Mises plastic matrix :
  $\Phi(\boldsymbol{\sigma}) = \sigma_{eq} - \sigma_0 \leq 0, \quad \sigma_{eq} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'}$
- Porosity : $\phi = a^3/b^3$
- Velocity field : $\underline{v}(x) = \mathbf{D}'.\underline{x} + \frac{b^3}{r^2} D_m \underline{e}_r$
- Determination of the macroscopic dissipation $\Pi(\boldsymbol{D})$ and the surface of plasticity
- The limit stress states : $\boldsymbol{\Sigma} = \frac{\partial \Pi(\boldsymbol{D})}{\partial \boldsymbol{D}}$
- Criterion of Gurson (1977) :
  $$\frac{\Sigma_{eq}^2}{\sigma_0^2} + 2\phi \cosh\left(\frac{3}{2}\frac{\Sigma_m}{\sigma_0}\right) - 1 - \phi^2 = 0$$

Extension of Gurson's criterion to a general elliptic matrix



- Hypothesis on the criterion of the matrix :

$$\Phi = \beta\sigma_{eq}^2 + \frac{9\alpha}{2}\sigma_m^2 - L\sigma_m - \sigma_0^2 \leq 0$$

- Support function :

$$\pi(\boldsymbol{d}) = \boldsymbol{\sigma} : \boldsymbol{d} = \frac{L}{3\alpha}d_m + \sqrt{\sigma_0^2 + \frac{L^2}{18\alpha}}\sqrt{\frac{2d_m^2}{\alpha} + \frac{d_{eq}^2}{\beta}}$$

- Velocity field chosen (Eshelby-like velocity field) :

$$\underline{v} = A\underline{x} + \boldsymbol{B}.\underline{x} + \underline{v}^E \quad \text{with} \quad \underline{v}^E = \frac{a^5}{5r^4}\left[5d_m^*\boldsymbol{1} + 2\bar{\boldsymbol{d}}^*\right].\underline{e}_r + \frac{a^3}{r^2}\left[1 - \frac{a^2}{r^2}\right]d_{rr}^*\underline{e}_r$$

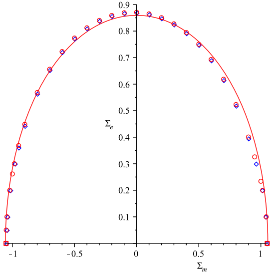
- Macroscopic dissipation : $\Pi(\boldsymbol{D}) = \min_A\left[\widetilde{\Pi}(\boldsymbol{D}, A)\right]$

- Determination of the macroscopic criterion : $\boldsymbol{\Sigma} = \frac{\partial\Pi(\boldsymbol{D},A)}{\partial\boldsymbol{D}}, \ \frac{\partial\widetilde{\Pi}(\boldsymbol{D},A)}{\partial A} = 0$

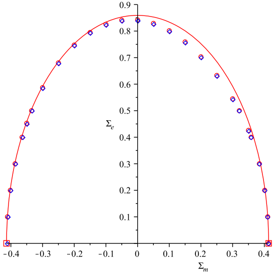
- Macroscopic yield criterion :

$$\beta\frac{\Sigma_{eq}^2}{\Sigma_0^2} + \frac{9\alpha}{2}\left[\frac{\Sigma_m - \frac{L}{9\alpha}(1-\phi)}{\Sigma_0}\right]^2 + 2\phi\cosh\left(\sqrt{\frac{9\beta}{4}\frac{\Sigma_m^2}{\Sigma_0^2} + \frac{2\beta}{3\bar{\Gamma}(\phi)}\frac{\Sigma_{eq}^2}{\Sigma_0^2}}\right) - 1 - \phi^2 = 0$$

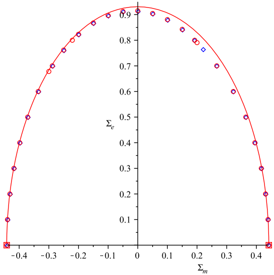
Validation by means of numerical Limit analysis bounds



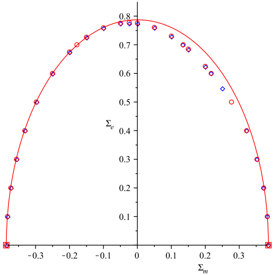
$\alpha = 0.1, \beta = 1, f = 0.1, \sigma_0 = 1$



$\alpha = 1, \beta = 1, f = 0.1, \sigma_0 = 1$

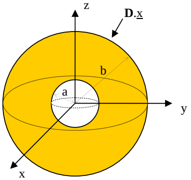


$\alpha = 1, \beta = 1, f = 0.05, \sigma_0 = 1$



$\alpha = 1, \beta = 1, f = 0.15, \sigma_0 = 1$

Plastic behavior of porous geomaterial

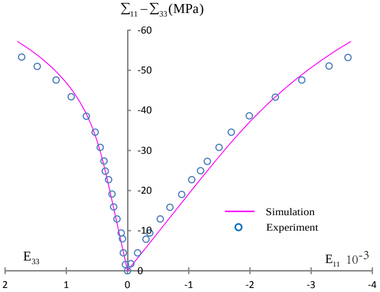


- Dependency on mean stress
- Dissymmetry between compression - tension

- Mises-Schleicher type matrix :
  $f(\boldsymbol{\sigma}) = \sigma_{eq}^2 + 3\alpha\sigma_0\sigma_m - \sigma_0^2 \leq 0, \qquad \sigma_0 = \sqrt{CT}, \alpha = \frac{C-T}{\sqrt{CT}}$
- Macroscopic yield criterion :
  $$F = \frac{\frac{\Sigma_{eq}^2}{\sigma_0^2}}{\left(\frac{1-f}{1-\Gamma}\right)^2 - 3\alpha\frac{1-f}{(1-\Gamma)^2}\frac{\Sigma_m}{\sigma_0}} + 2\Gamma\cosh\left(A\ln\left(1 - 3\alpha\frac{\Sigma_m}{\sigma_0}\right)\right) - 1 - \Gamma^2 = 0$$
- An associated plastic flow rule : $\boldsymbol{D}^p = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\Sigma}}$
- Plastic hardening law : $\bar{\sigma} = \sigma_0 + H(\varepsilon_{eq}^p)^m$
- Evolution of porosity : $\dot{f} = (1-f)tr\boldsymbol{D}^p - \frac{1}{\Omega}\int_{\Omega_m} tr\boldsymbol{d}^p d\Omega$

Experimental verification on dry materials

- An associated plastic model :

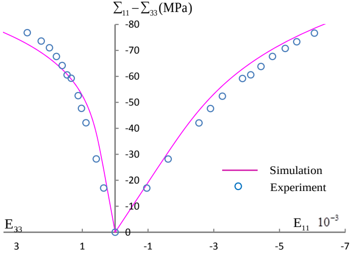


Triaxial compression test, Pc=5MPa

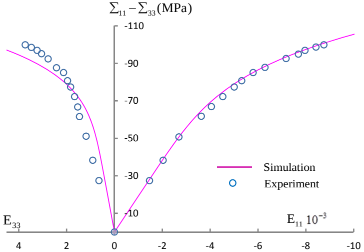
Elastic	$E_s = 30\ GPa,\ \nu_s = 0.3$
Plastic	$\alpha = 5,\ \sigma_0 = 2\ MPa,$ $H = 50, m = 0.27$
Porosity	$f = 0.22$

- The local plastic parameters are determined by an optimization procedure from experimental data
- Verification for different confining pressures

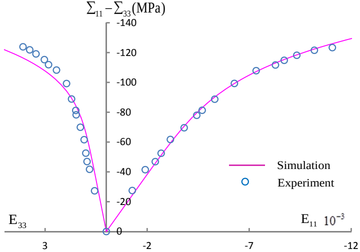
Experimental verification on dry materials :



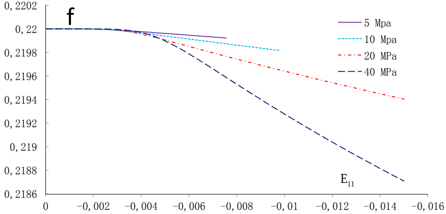
Triaxial compression test, Pc=10MPa



Triaxial compression test, Pc=20MPa



Triaxial compression test, Pc=40MPa



Evolution of porosity f for different confining pressures.

Effective stress concept in plasticity

Macroscopic consideration :

- Plastic flow rule :

$$\dot{\mathbf{E}}^p = \dot{\lambda}\frac{\partial Q(\Sigma,p,A_\gamma)}{\partial \Sigma}\ ;\ \dot{\phi}^p = \dot{\lambda}\frac{\partial Q(\Sigma,p,A_\gamma)}{\partial p}\ ;\ \dot{\gamma}_p = \dot{\lambda} h(\Sigma,p,A_\gamma)$$

- Intrinsic dissipation :

$$\Sigma : \dot{\mathbf{E}}^p + p\dot{\phi}^p + A_\gamma \dot{\gamma}_p \geq 0$$

- Kinematic assumption :

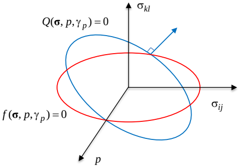
$$\dot{\phi}^p = \beta \dot{E_k k}^p \Rightarrow (\Sigma + \beta p \delta) : \dot{\mathbf{E}}^p + A_\gamma \dot{\gamma}_p \geq 0$$

- Plastic potential with effective stress :

$$Q(\Sigma,p,A_\gamma) \equiv Q(\Sigma^{ep},A_\gamma) \Rightarrow \text{with } \Sigma^{ep} = \Sigma + \beta p \delta$$

- Plastic yield function with effective stress to be verified :

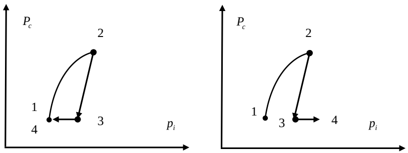
$$F(\Sigma,p,A_\gamma) \equiv F(\Sigma^{ep},A_\gamma)\ ?$$



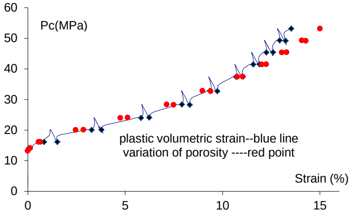
Micro-mechanical consideration :

$$F(\Sigma,p,A_\gamma) \equiv F(\Sigma^{ep},A_\gamma) \Rightarrow \text{with } \Sigma^{ep} = \frac{\Sigma + \beta p \delta}{1 + (p \tan \varphi)/c}$$

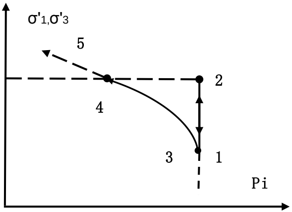
Experimental verification of effective stress :



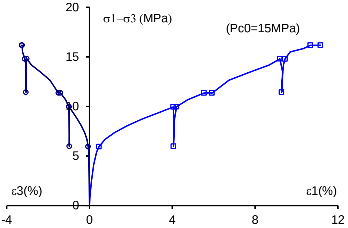
Hydrostatic Yamada's test



Plastic porosity versus volumetric strain

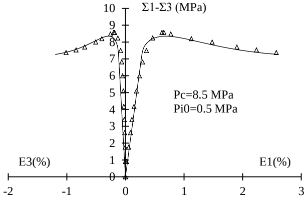


Triaxial Yamada's test

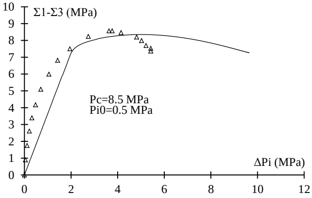


Stress-strain curve with effective stress cycles

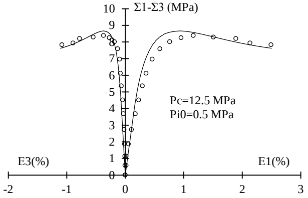
Experimental verification on saturated materials :



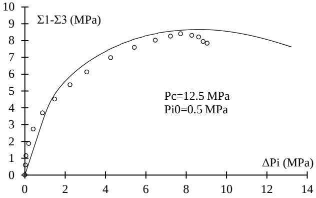
Triaxial compression test, Pc=8.5MPa



Pore pressure evolution, Pc=8.5MPa

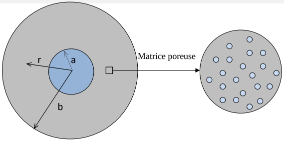


Triaxial compression test, Pc=12.5MPa



Pore pressure evolution, Pc=12.5MPa

Double porous material saturated by 2 differ. pore pressures



- Saturated double porous materials :
  - Micropores f, mesopores ϕ
  - The solid phase obeys to Drucker-Prager criterion
  - Pores are saturated by a fluid p at mesoscale and q at microscale
- Construction of the macroscopic criterion :

$$\beta \frac{\Sigma_{eq}^2}{(1+q/h)^2\Sigma_0^2} + \frac{9\alpha}{2}\left[\frac{\frac{\Sigma_m+p}{(1+q/h)} - \left(\frac{L}{9\alpha} + \frac{p-q}{(1+q/h)}\right)(1-\phi)}{\Sigma_0}\right]^2 + 2\phi\cosh(\Theta) - 1 - \phi^2 = 0$$

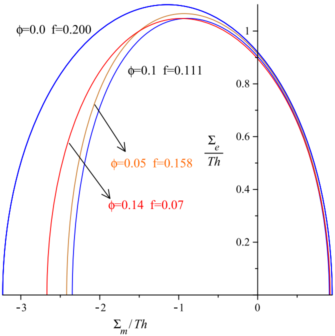
with $\beta = \frac{2}{3}\frac{1+2f/3}{T^2}, \quad \frac{9\alpha}{2} = \frac{3f}{2T^2} - 1, \quad L = -2(1-f)h, \quad \sigma_0 = (1-f)h$

- The triplet of loading parameters $(\boldsymbol{\Sigma}, q, p) \Rightarrow (\boldsymbol{\Sigma}^e, 0, \frac{p-q}{1+q/h})$, with $\boldsymbol{\Sigma}^e = \frac{\boldsymbol{\Sigma}+q\boldsymbol{1}}{1+q/h}$.
  - when $p = q$, the macroscopic criterion :

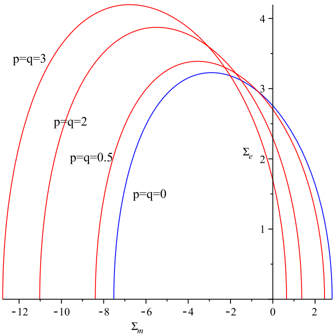
$$\beta\left(\frac{\Sigma_{eq}^e}{\Sigma_0}\right)^2 + \frac{9\alpha}{2}\left[\frac{\Sigma_m^e - \frac{L}{9\alpha}(1-\phi)}{\Sigma_0}\right]^2 + 2\phi\cosh\left(\sqrt{\frac{9\beta}{4}\frac{\Sigma_m^{e2}}{\Sigma_0^2} + \frac{2\beta}{3\bar{\Gamma}(\phi)}\frac{\Sigma_{eq}^{e2}}{\Sigma_0^2}}\right) - 1 - \phi^2 = 0$$

  - when $p \neq q$, there is no effective stress

Influences of pore pressures on the strength of the material



$T = 0.3, p = 0, q = 0$



$f = 0.1, \phi = 0.1, T = 0.3, h = 10$

- The proportions of f, ϕ have effect on the yield surface
- Pore pressures p, q have a great influence on the strength
- The yield surface has a tension-compression asymmetry.

Conclusions and Perspectives

Conclusions

- Analytical macroscopic criteria obtained from non-linear homogenization methods
- Porous geomaterials with different behaviors of solid matrix
- Application to the materials having two populations of voids
- Porous geomaterials with different shapes of pores
- Effective stress : still an open issue

Perspectives

- Engineering application of micro-macro models
- Extend to partially saturated materials
- Further studies on the effect of fluid pressure.