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Constitutive Modeling of Chalk with Considering Effect of Intergranular Physicochemical forces

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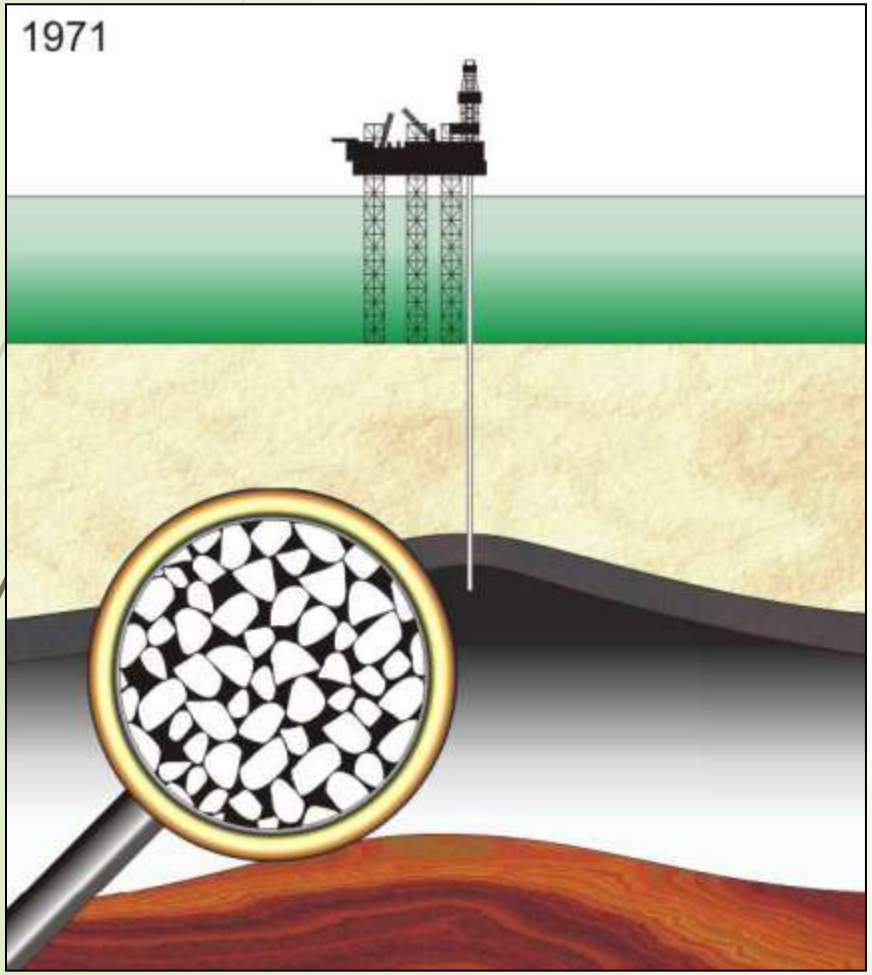
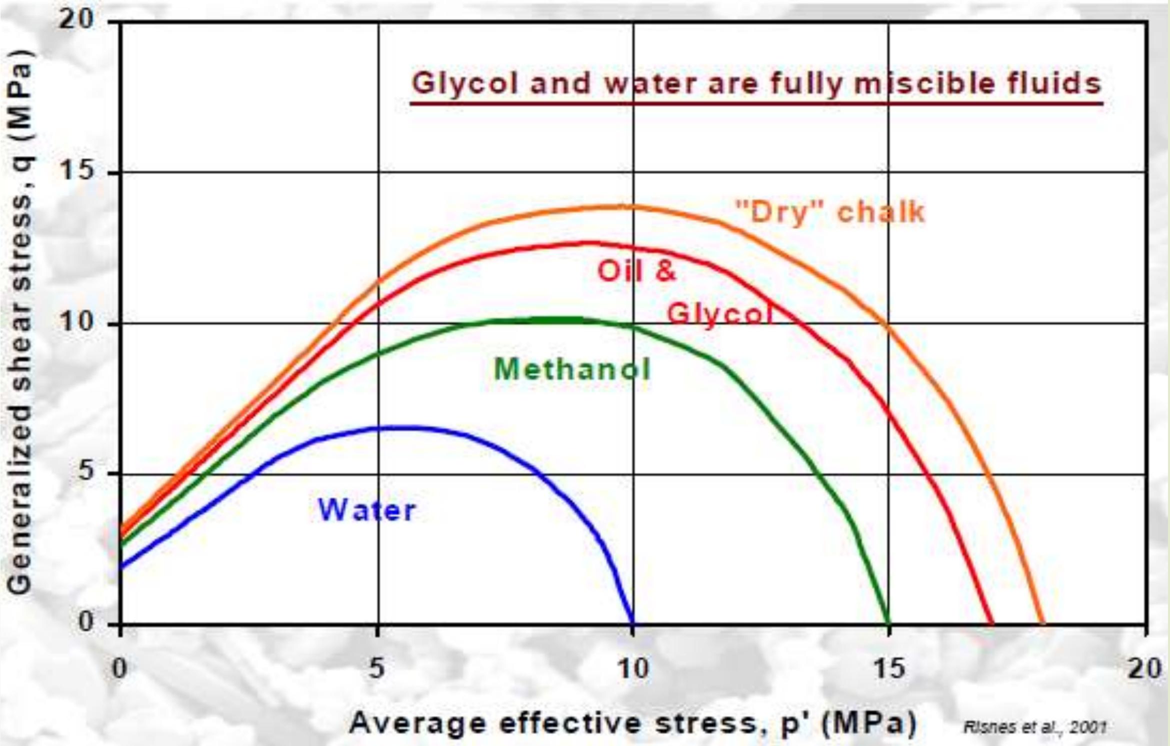
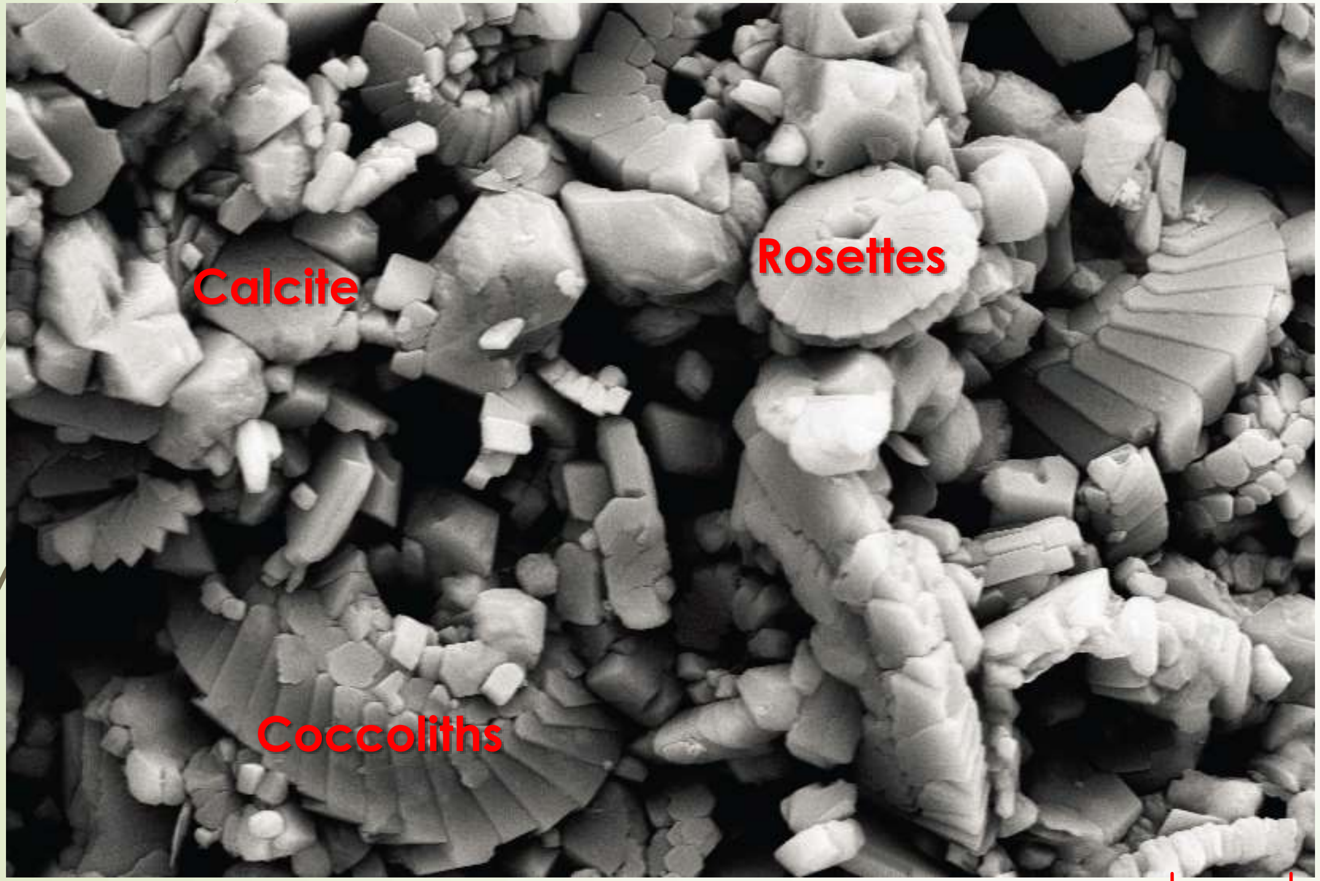
A notorious example: Subsidence of Ekofisk oil field



Water-weakening effect of chalk



Microstructure and features of Chalk Material



SEM image

1μm

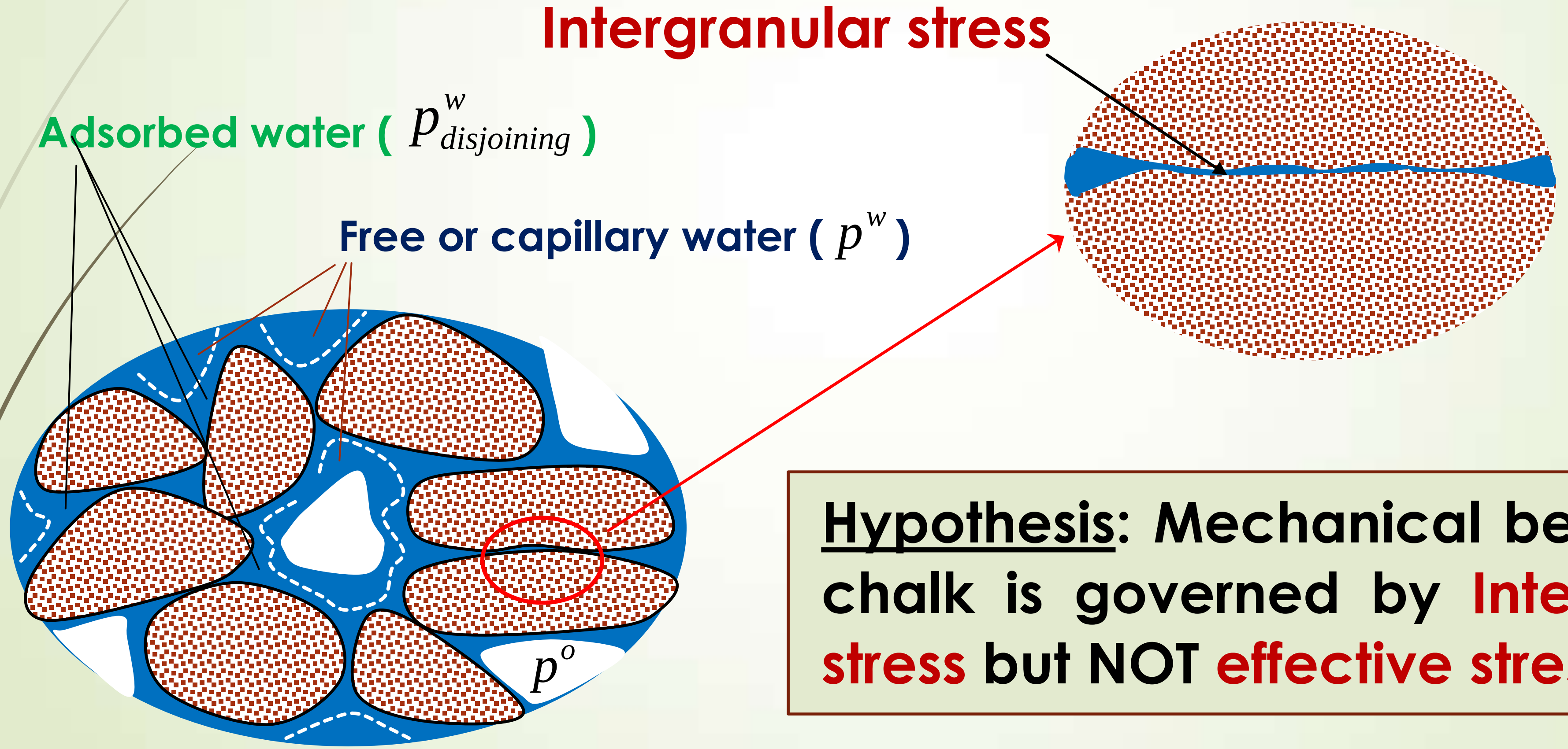
- "grains" originated from the skeleton of algae organism
- High porosity (40% - 50%)
- Large surface area (~2.0 m^2/g)
- Chemically active

Zeta potential data

Equilibrium water: - 20 mV
Methanol: +10 mV
Oil: 0 mV

Our working hypothesis

❖ **What is the relevance of these features**



<u>Hypothesis</u>: Mechanical behavior of chalk is governed by Intergranular stress but NOT effective stress!

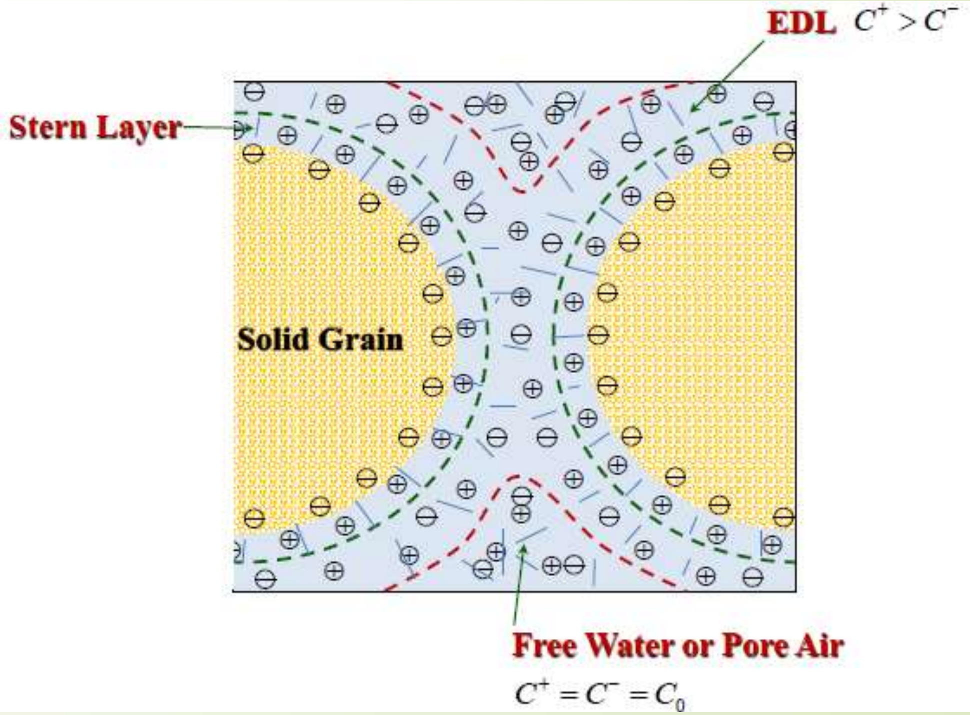
Outline

1. Surface forces & surface potential
2. Matric potential & Chemical potential
3. Pore water pressure & effective stresses
4. Constitutive modeling of chalk
5. Concluding remarks

1. Surface forces and surface potential

❖ **Surface forces** in the soil pores saturated with water solution:

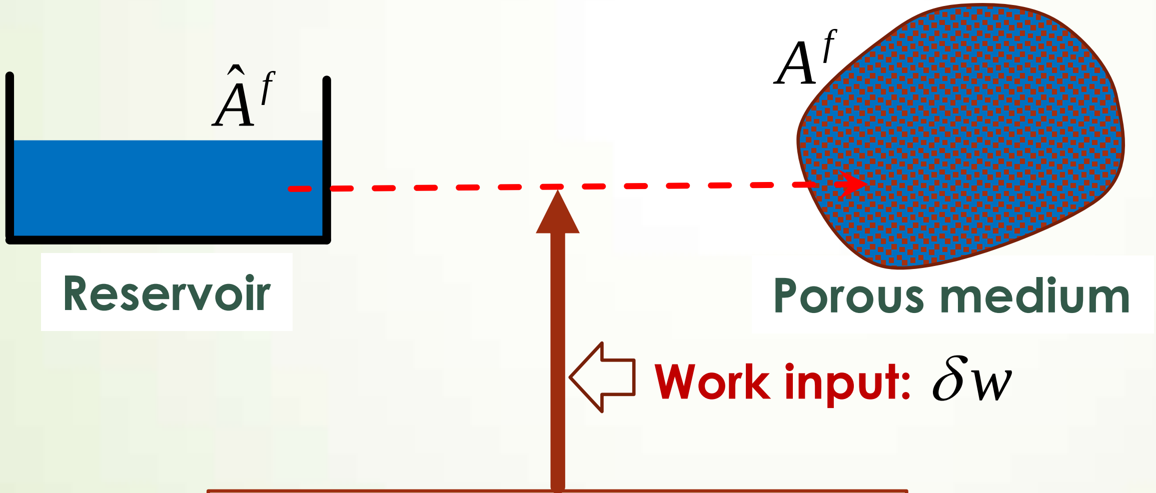
- capillary forces (in case of unsaturation)
- interactions between water dipoles and charged solid surfaces
- electrostatic forces (Columbic force)
- van der Waals forces
- hydrogen bonding forces
- diffuse double layer repulsion
- others …



1. Surface forces and surface potential

❖ **How to characterize these surface forces?**

Energy conserved: $\hat{A}^f + \delta w = A^f$



Solution (water) is moved slowly from the reservoir to the porous medium

【Definition】

Surface potential

$$\Omega^f = \delta w$$

$$A^f = \hat{A}^f + \Omega^f$$

2. Matric and chemical potentials

❖ **Chemical potential of a species in the pore fluid**

Definition:

$$\mu^{f_k} = \left.\frac{\partial(Jn^f \rho^f A^f)}{\partial(Jn^f \rho^{f_k})}\right|_{T,\rho^f,Jn^f\rho^{f_m},m\neq k}$$

Surface potential

$$\mu^{f_k}(T,p^f,C^{f_j},n^f,\zeta_0) = \mu_{\oplus}^{f_k}(T,p^f) + \frac{RT}{\mathrm{M}_k}\ln a^{f_k}(T,p^f,C^{f_j}) + \Omega^f(T,n^f,\zeta_0) + \frac{\mathcal{F}\xi\nu_k}{\mathrm{M}_k}$$

Classical chemical potential

2. Matric and chemical potentials

❖ **Surface potential**

Matric suction → s_M

Donnan osmotic pressure → Π_D

$$\Pi_D = \frac{RT\rho_{\oplus}^{f_{H_2O}}}{M_{H_2O}} \ln\left(\frac{a_W^{f_{H_2O}}}{a^{f_{H_2O}}}\right)$$

$$S_r \rho^f \Omega^f = \rho^f \Omega_0^f + \int_{S_r}^{1} \left(s_M - \Pi_D\right) dS_r$$

Equilibrium conditions:

1. Between two bulk phase:

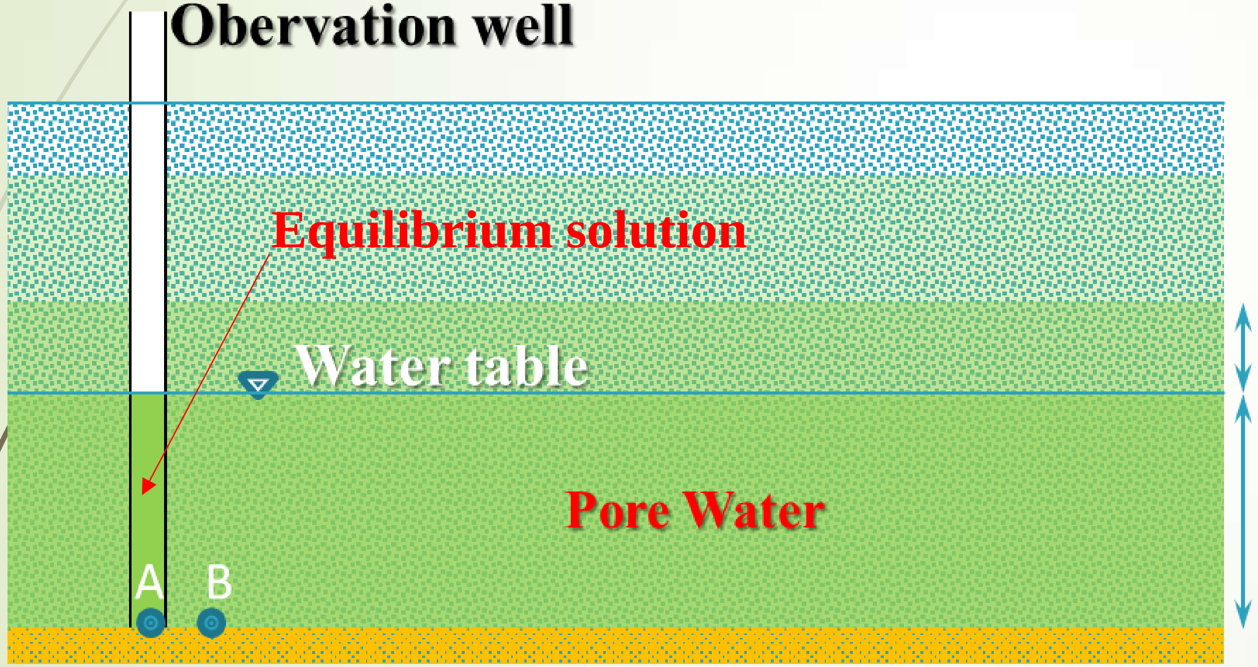
$$\mu^{f_k}(T, p^f, C^{f_j}, n^f, \zeta_0) = \mu^{g_k}(T, p^g, C^{g_j}, n^g, \zeta_0)$$

2. In an individual phase:

$$\mu^{f_k}(T, p^f, C^{f_j}, n^f, \zeta_0) + \mathbf{g}\cdot(\mathbf{Z} - \mathbf{Z}_0) = const$$

3. Pore water pressure & effective stresses

What is the pore water pressure?



$$p_A^f \neq p_B^f \qquad \mu_A^{f_k} = \mu_B^{f_k} \quad \Rightarrow \quad p_B^f = p_A^f + \Pi$$

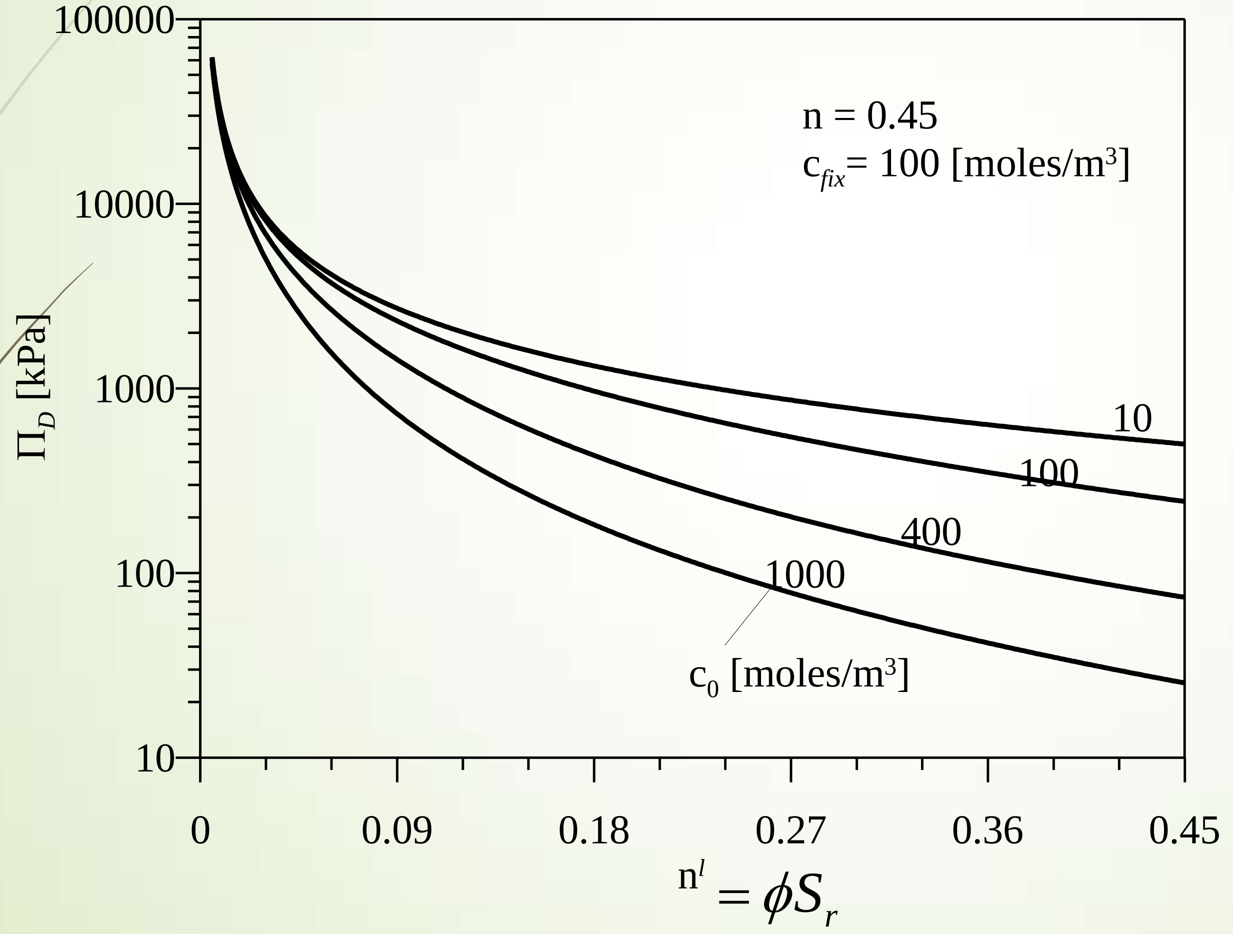
Generalized osmotic pressure

$$\Pi = \Pi_D - \rho_{\oplus}^{f_{H_2O}} \Omega^f$$

$$\Pi_D = \frac{RT\rho_{\oplus}^{f_{H_2O}}}{M_{H_2O}} \ln\left(\frac{a_W^{f_{H_2O}}}{a^{f_{H_2O}}}\right)$$

$$S_r \rho^f \Omega^f = \rho^f \Omega_0^f + \int_{S_r}^{1} \left(s_M - \Pi_D\right) dS_r$$

3. Pore water pressure & effective stresses



$$\Pi = \Pi_D - \rho^l \Omega_0^l \geq \Pi_D$$

$$\Pi_D = \frac{RT\rho_{\oplus}^{l_{H_2O}}}{M_{H_2O}} \ln\left(\frac{a_W^{l_{H_2O}}}{a_B^{l_{H_2O}}}\right)$$

3. Pore water pressure & effective stresses

❖ Effective stresses of unsaturated soils are given by

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p^o\mathbf{1} - p_{\mathrm{Int}}\mathbf{1}$$

$$p_{\mathrm{Int}} = \phi S_r\left(s_M - \Pi\right)$$

@ full saturation

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p^w\mathbf{1} + \Pi\mathbf{1}$$

Matric suction: $s_M = p^o - p^w$

Generalized osmotic pressure: $\Pi = \Pi_D - \rho^w\Omega^w$

Donnan osmotic pressure: $\Pi_D = \dfrac{RT\rho^w}{\mathrm{M}_w}\ln\left(\dfrac{a^w_{eq}}{a^w_{pore}}\right)$

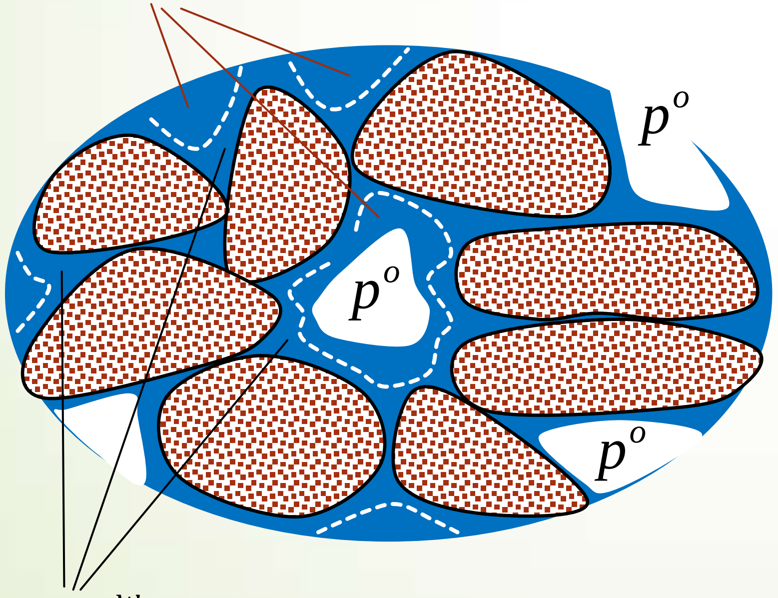
Surface potential: $S_r\rho^w\Omega^w = \rho^w\Omega^w_0 + \int_{S_r}^{1}\left(S_M - \Pi_D\right)dS_r$

3. Pore water pressure & effective stresses

❖ Physical significance of average **intergranular stress**

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p_{\text{neutral}} \mathbf{1}$$

Free or capillary water (p^w)



Adsorbed water ($p^w_{disjoining}$)

❖ For oil- AND water- saturated:

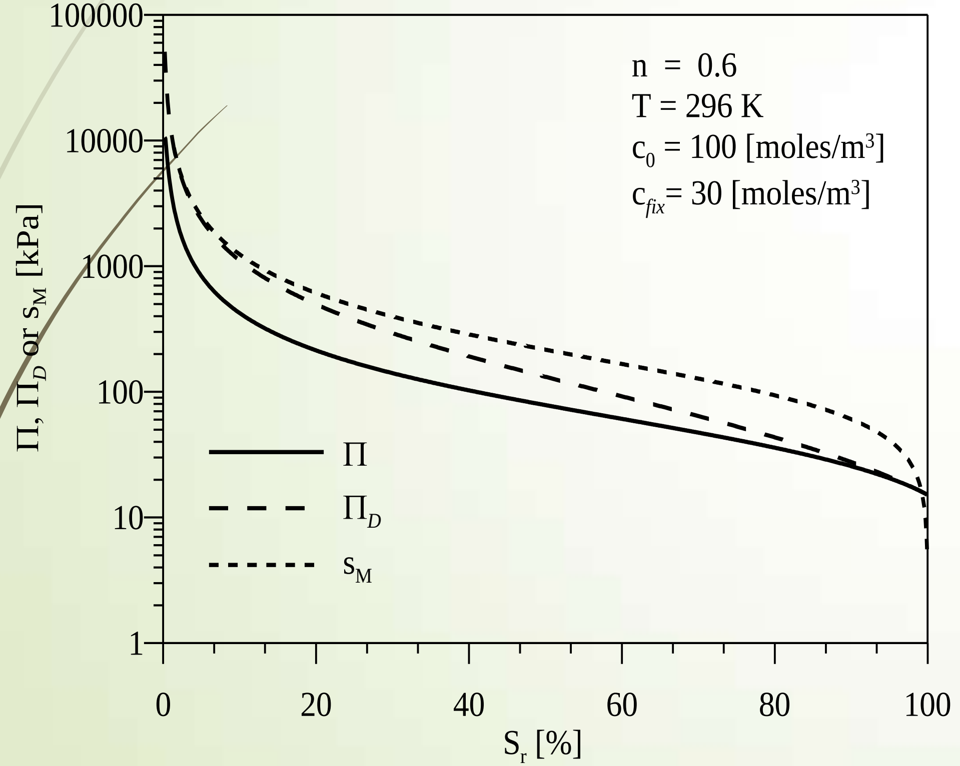
$$p_{neutral} = (1 - S_r) p^o + S_r p^w + \left[(1 - \phi S_r) s_M + \phi S_r \Pi \right]$$

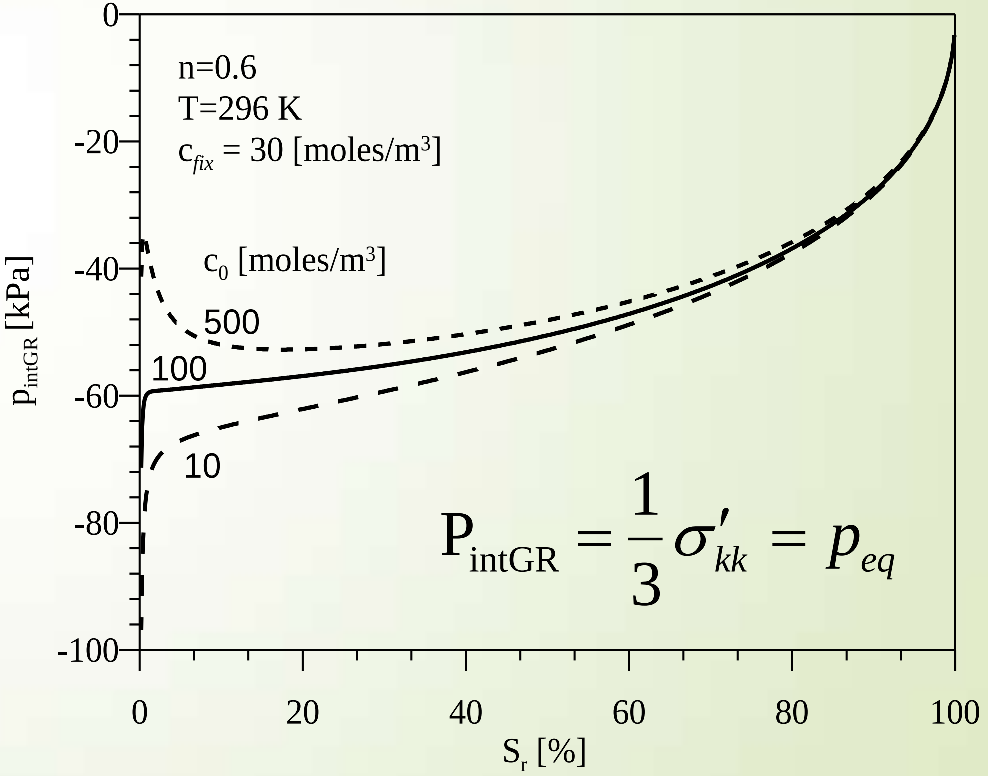
❖ For water-saturated

$$p_{neutral} = p^w + \Pi$$

3. Pore water pressure & effective stresses

The average intergranular stress (P_{intGR}) in a clay during drying

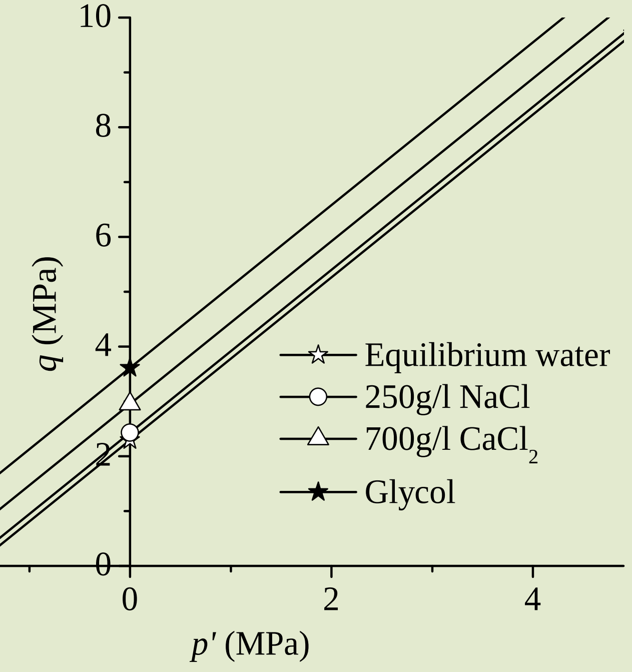


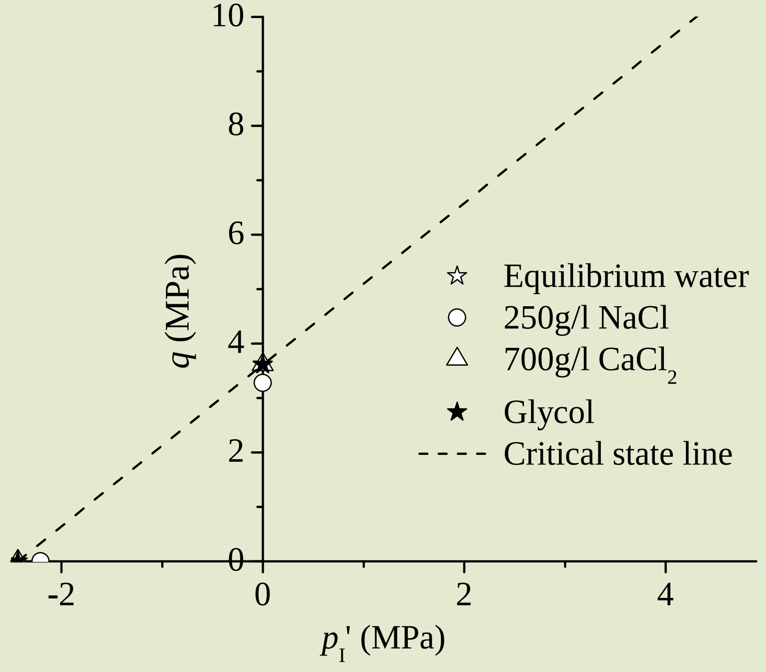


$$P_{intGR} = \frac{1}{3}\sigma'_{kk} = p_{eq}$$

3. Pore water pressure & effective stresses

Failure lines for the chalks saturated by various fluids



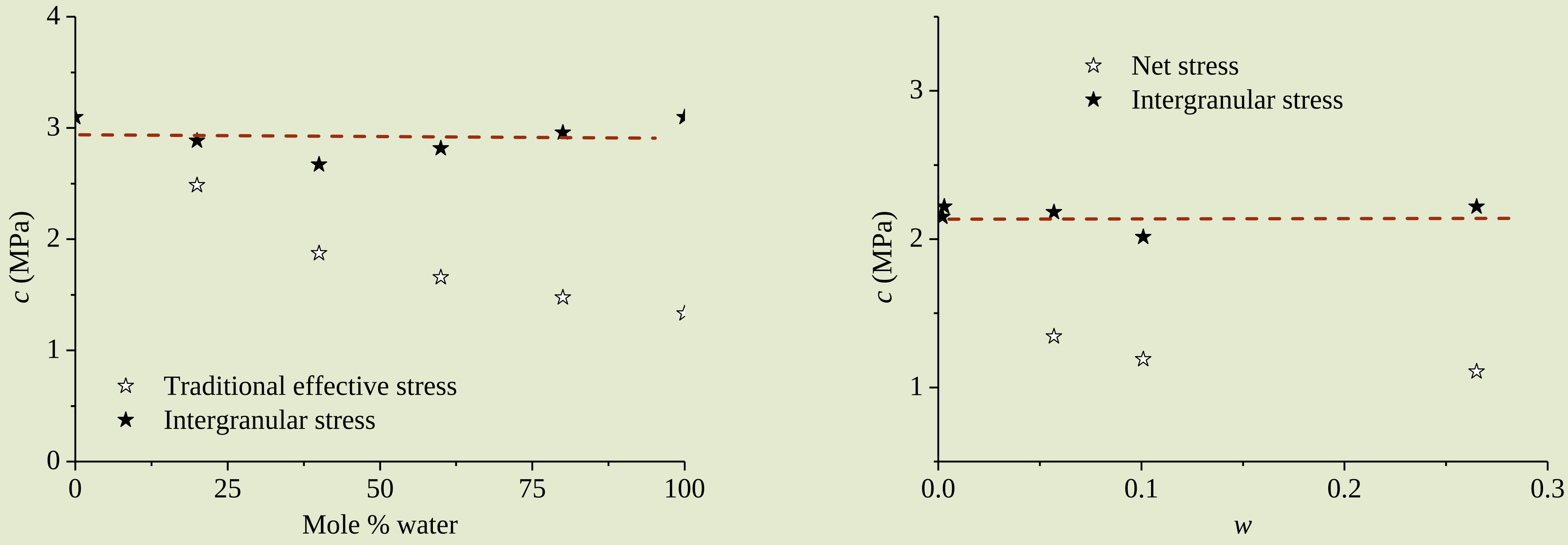


Terzaghi: $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p^w \mathbf{1}$

Current: $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p^w \mathbf{1} + \left(\Pi_D - \rho^w \Omega^w\right) \mathbf{1}$

3. Pore water pressure & effective stresses

Apparent cohesive pressure of various chalks



High-porosity outcrop chalk (Risness et al. 2005)

Lewes chalk (Taibi et al. 2009)

4. Constitutive modeling of chalk

Constitutive framework: The Modified Cam Clay model

Intergranular PhysicoChemical pressure

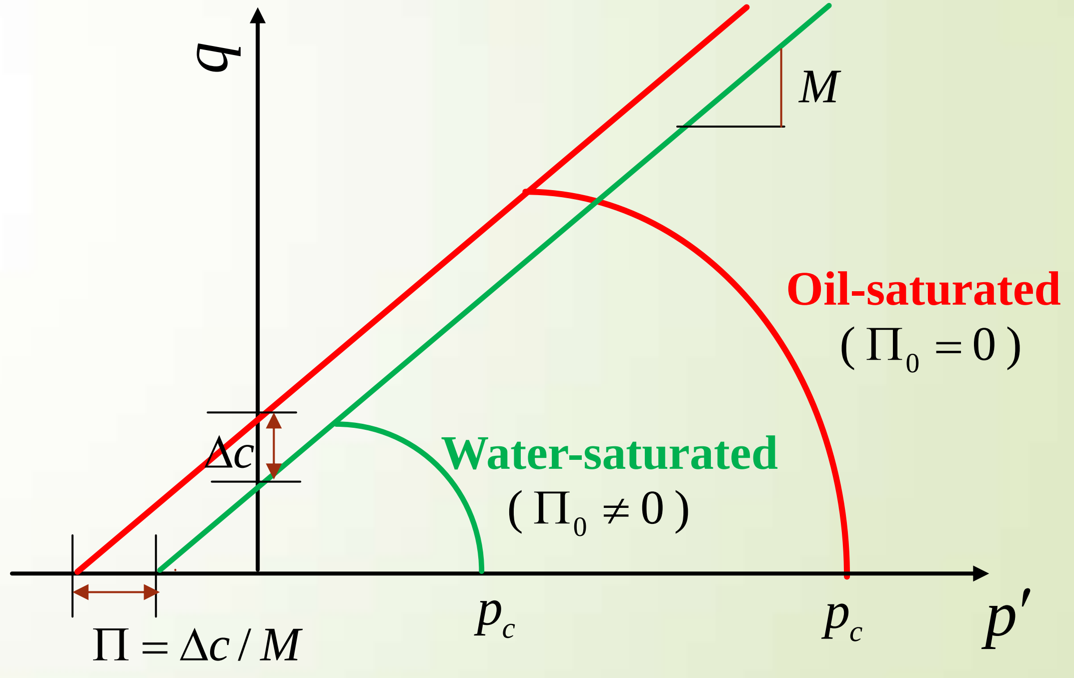
$$p_c = p_{c0}\left(\varepsilon_v^p, \Pi_0\right) h\left(p_{\text{Int}}\right)$$

$$p_{c0}\left(\varepsilon_v^p, \Pi_0\right) = p_{c0}^* \exp\left(\frac{\upsilon \varepsilon_v^p}{\lambda - \kappa}\right) \exp\left(-\beta \Pi_0\right)$$

$$h\left(p_{\text{Int}}\right) = \exp\left(\beta p_{\text{Int}}\right)$$

$$p_{\text{Int}} = \phi S_r \left(s_{\text{M}} - \Pi\right)$$

$$@ S_r = 100\% : \quad p_{\text{Int}} = 0, \quad h = 1.0$$



4. Constitutive modeling of chalk

Material parameters

Conventional: **(as in the Modified CamClay model)**	$E, \nu, \lambda, \kappa, M, c, p_{c0}^{*}$
Physicochemical Effect:	c_{fix}, Π_0, β
Water Retention Characteristics:	α, n

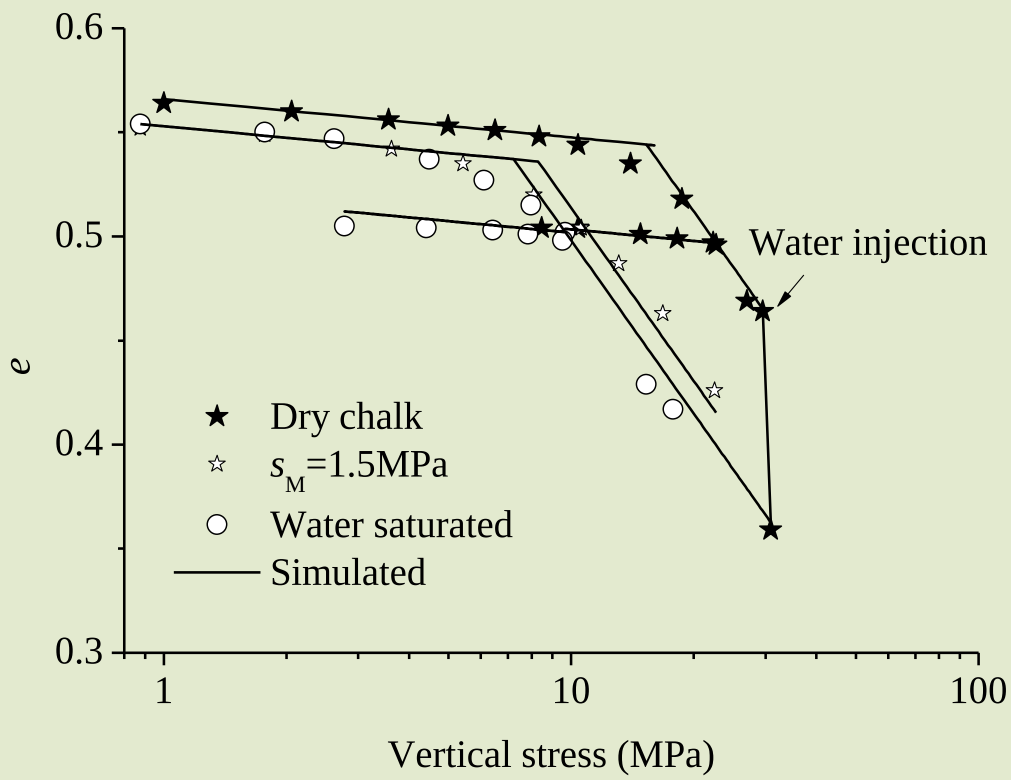
For oil-saturated, $\Pi_0 = 0$

c_{fix} can be estimated using CEC value

$$S_r = \left[\frac{1}{1 + \left(s_M / \alpha \right)^n} \right]^{1-\frac{1}{n}}$$

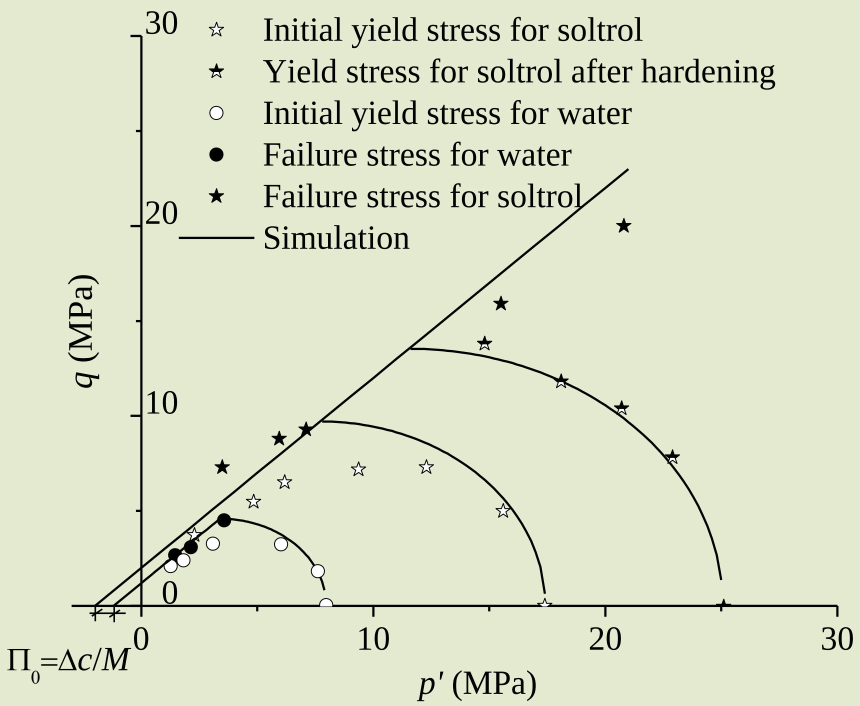
4. Constitutive modeling of chalk

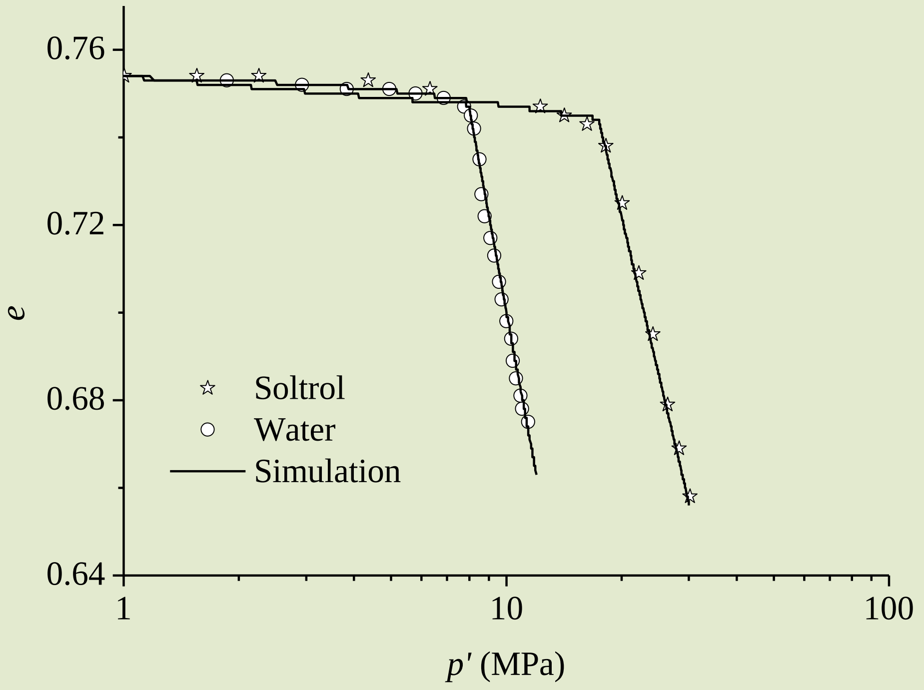
Compression of Estreux chalk (De Gennaro et al. 2005)



4. Constitutive modeling of chalk

Mechanical behavior of water- or saltrol-saturated chalk
(Homand and Shao 2000a)





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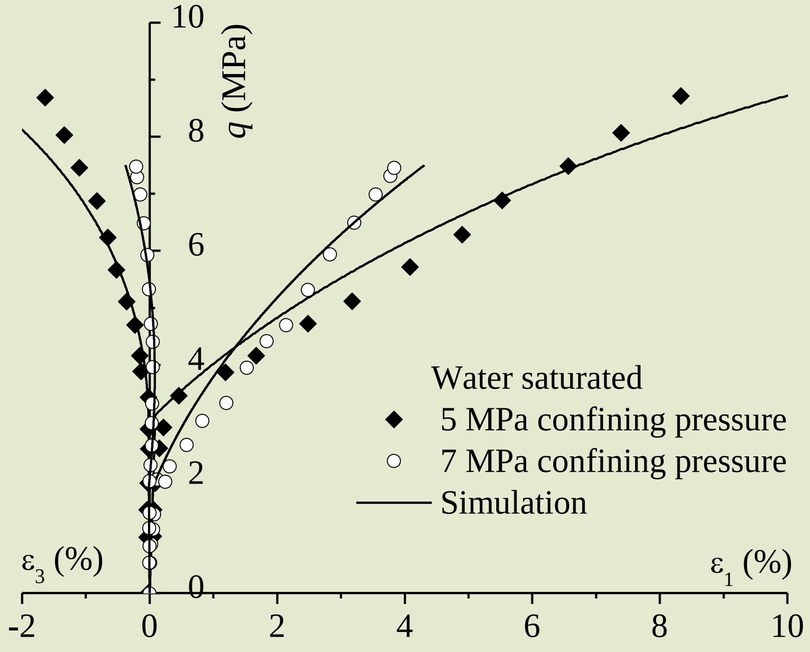

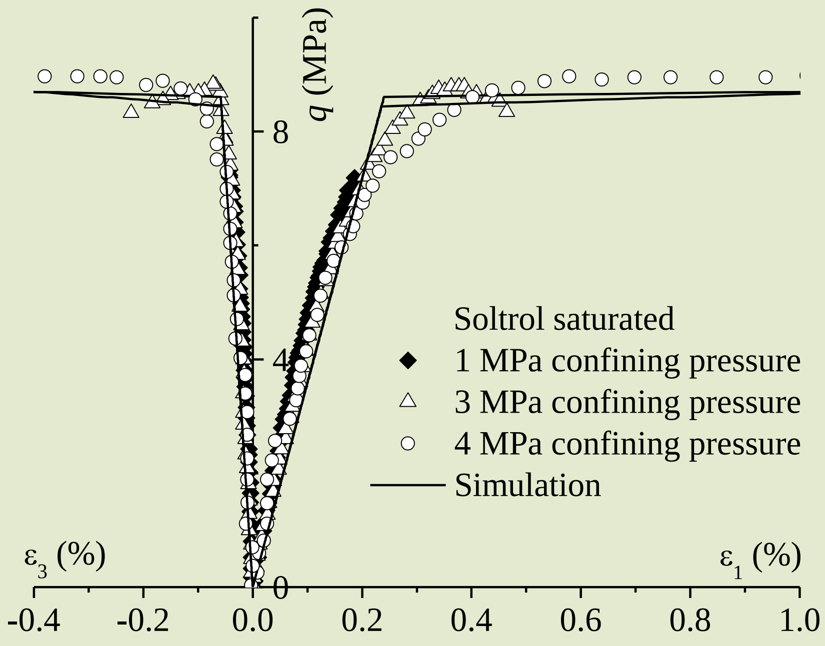

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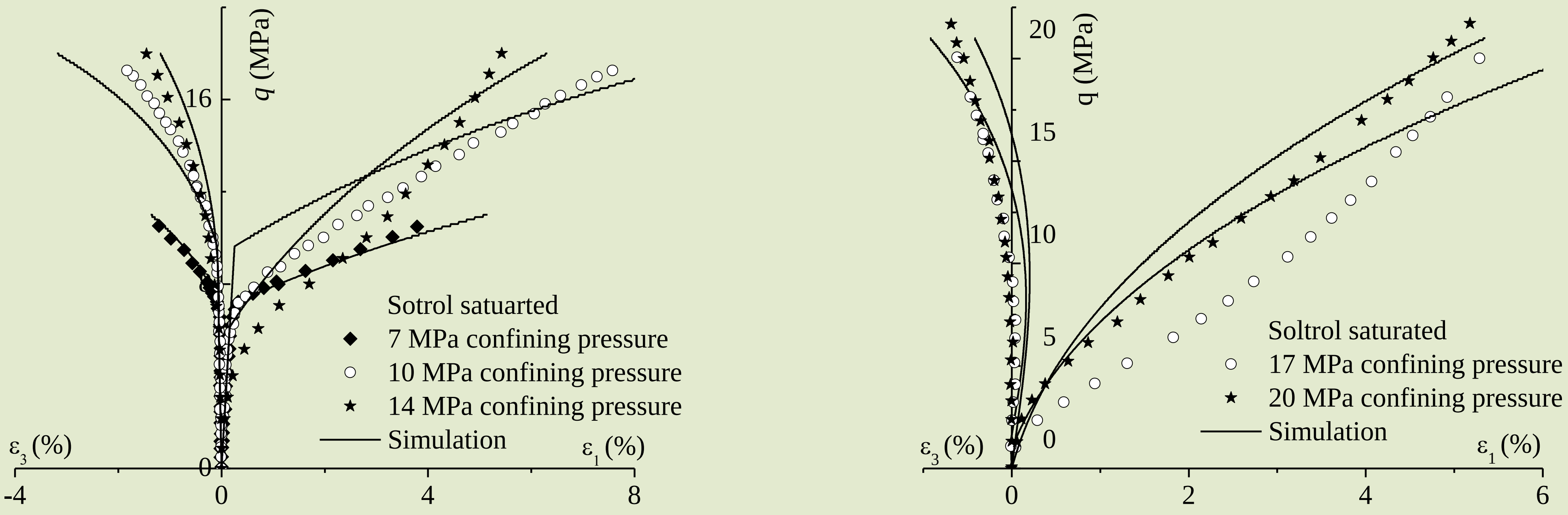

5. Concluding remarks

- **Physicochemical effect** remains elusive in the classical theory of geomechanics, due to its microscopic nature, which is closely related to many unresolved geomechanical problems.
- A **theoretical continuum framework** has been recently developed for describing coupled physical and chemical processes (C. Wei, 2014, Vadose Zone Journal).
- The proposed theory addresses well the **constitutive behavior** of chalk materials.

Thank you for your attention!