Mechanism and Theoretical Model of Intermediate Principal Stress Effect on Rock Strength

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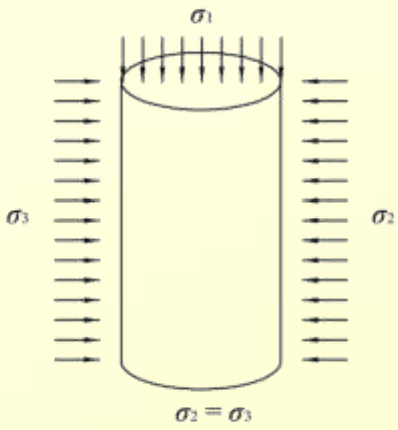

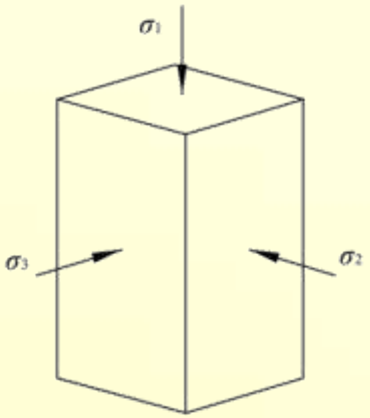

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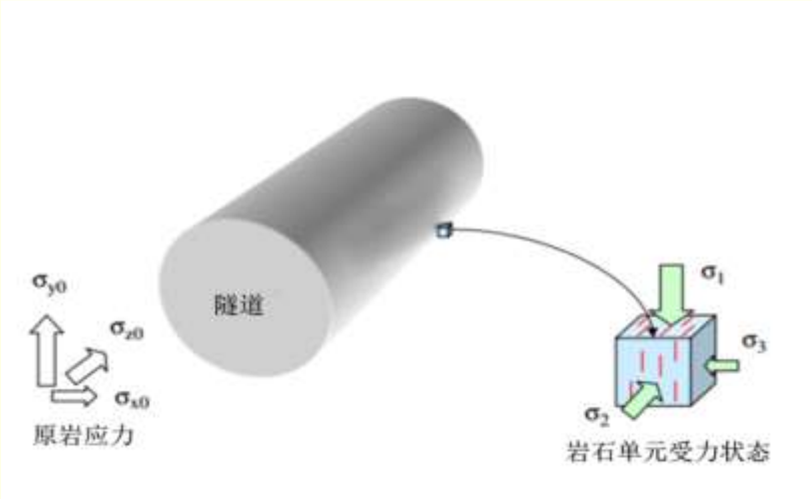
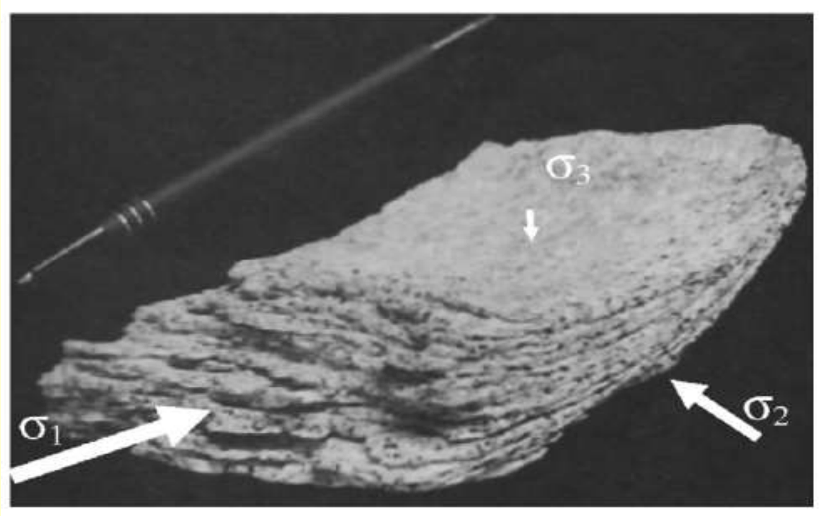
Contents

- 1. Engineering sig**nificance** of intermediate principal stress effect on rock strength
- 2. Experimental results and strength model of intermediate principal stress effect on rock strength
- 3. Mechanism discussion of intermediate principal stress effect on rock strength
- 4. Statistical model of intermediate principal stress effect on rock strength
- 5. Case study
- 6. Conclusions

➢1. Engineering significance of intermediate principal stress effect on rock strength

- three-dimensional unequal stress states are universal
- failure mechanism of rock material
- bring out more mechanical potentials of rock material

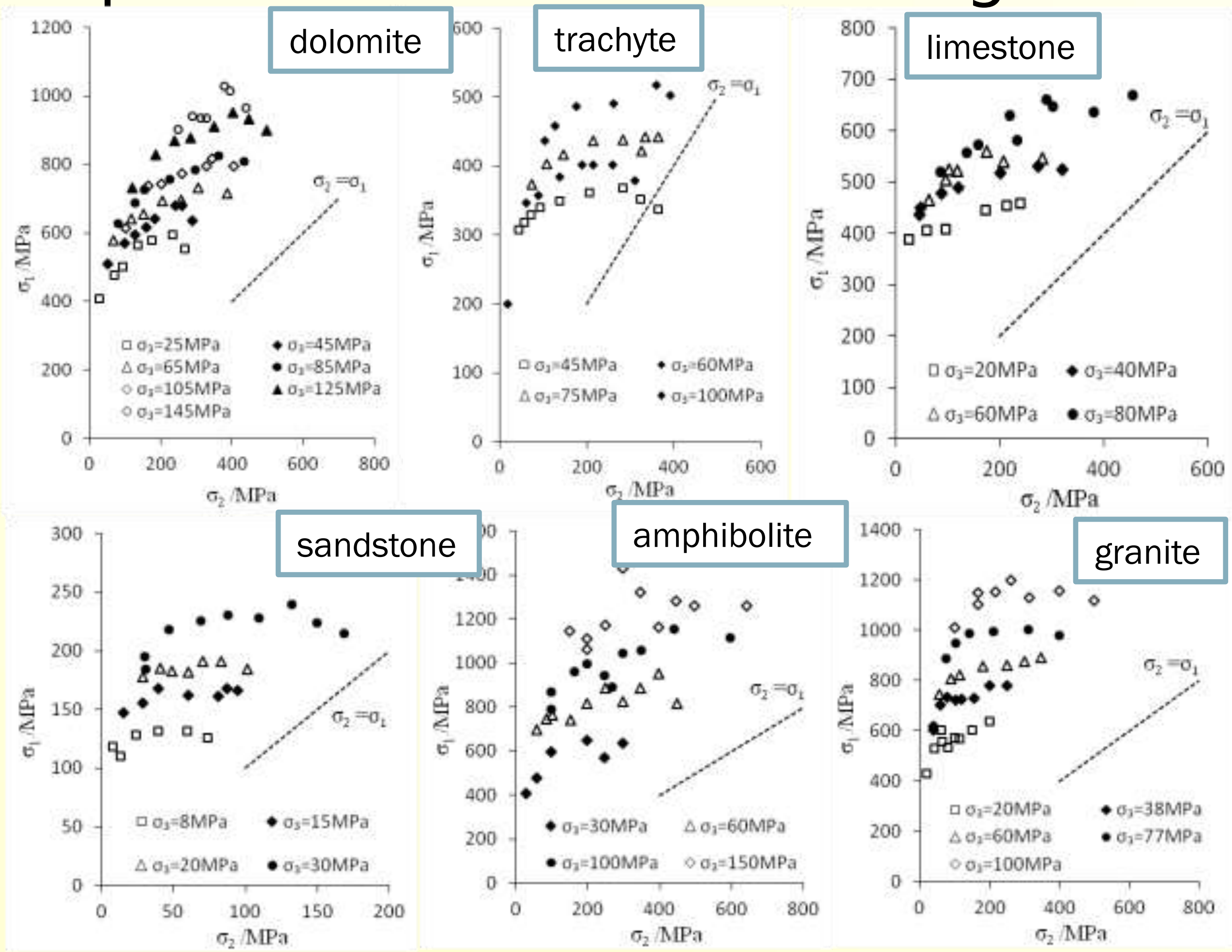




rock stress states and rock fracturing type

➢2. Experimental results and strength model of intermediate principal stress effect on rock strength

- Intermediate principal stress effect has been be observed in lots of experimental tests.
- Most tests agree that rock strength first increases and then reduces with the increase of σ_2.



Typical rock experimental tests results

2. Experimental results and strength model of intermediate principal stress effect on rock strength

Mohr-Coulomb Criterion

$$\frac{1}{2}(\sigma_1-\sigma_3)=\frac{1}{2}(\sigma_1+\sigma_3)\sin\varphi+c\cos\varphi$$

Drucker-Prager Criterion

$$\alpha I_1+\sqrt{J_2}+k=0$$

3D Griffith Criterion

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2=24T_0(\sigma_1+\sigma_2+\sigma_3)$$

Twin Shear Strength Criterion

$$\left.\begin{array}{l} F=\sigma_1-\frac{\alpha}{2}(\sigma_2+\sigma_3)=\sigma_t,\sigma_2\le\frac{\sigma_1+\alpha\sigma_3}{1+\alpha} \\ F'=\frac{1}{2}(\sigma_1+\sigma_2)-\alpha\sigma_3=\sigma_t,\sigma_2\ge\frac{\sigma_1+\alpha\sigma_3}{1+\alpha} \end{array}\right\}$$

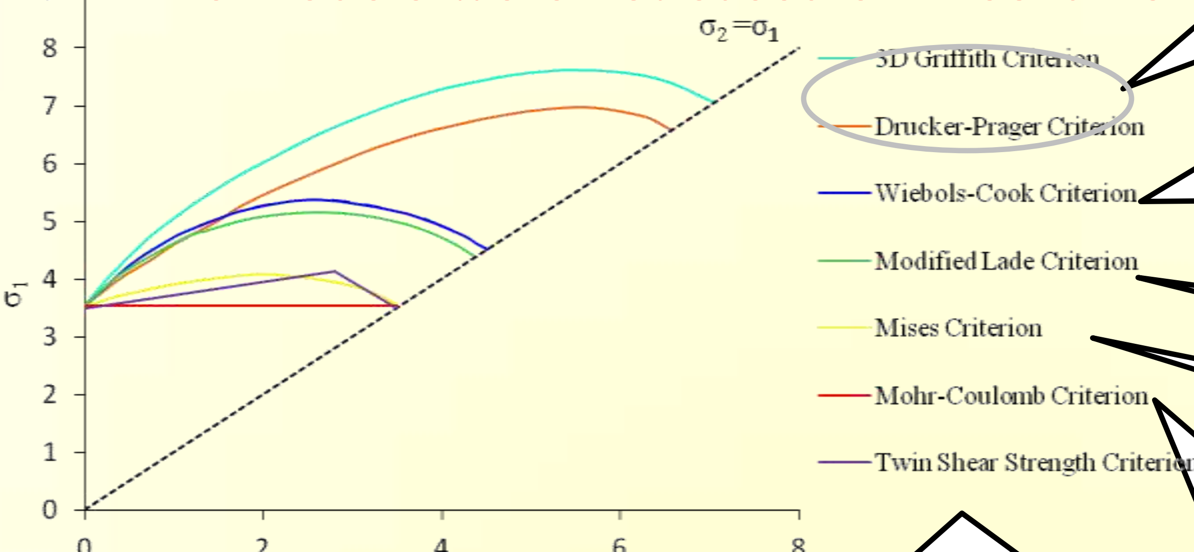
Modified Lade Criterion

$$I_1^3/I_3=27-\eta$$

There are **quali**tative discussion but not **quanti**tative analysis

➢2. Experimental results and strength model of intermediate principal stress effect on rock strength

- The influence of intermediate principal stress on rock strength can not be fully reflected.
- Mechanism of intermediate principal stress effect on rock strength is not clear.
- The models were not based on mechanism



not base on a physical argument except for symmetry in principal stress space

For general values of the principal stresses the criterion is difficult to evaluate and has a complicated form

no explicit physical explanation

it does not depend on the mean stress, contrary to the behavior observed for most rocks

does not depend on the intermediate principal stress, contrary to the experimental observations

sharp corners ; surface is not differentiable at the corners; it sometimes causes problems in numerical calculations involving the criterion

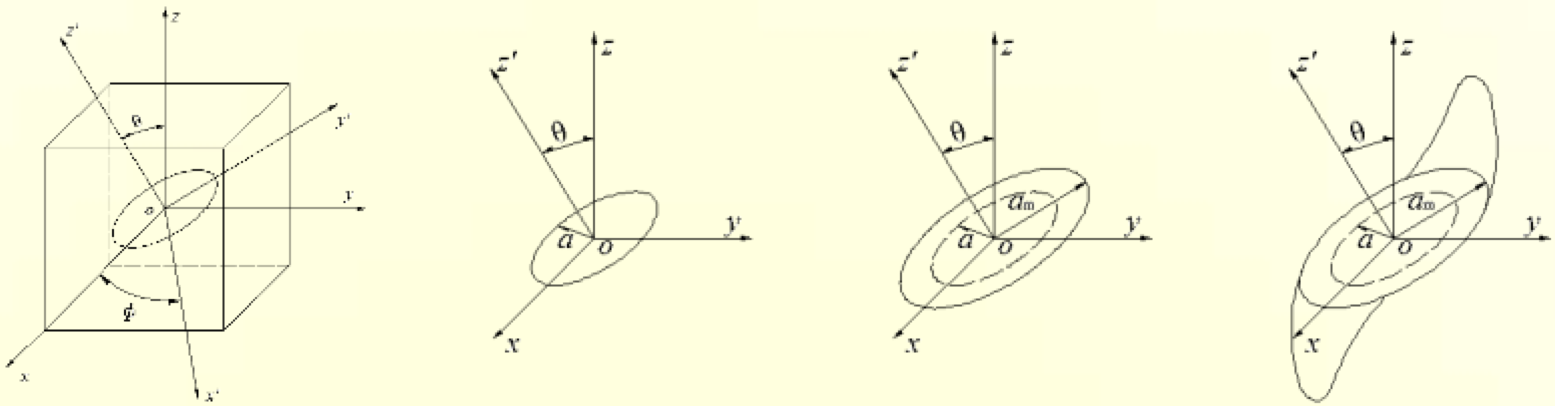
Common strength models

➢3. Mechanism discussion

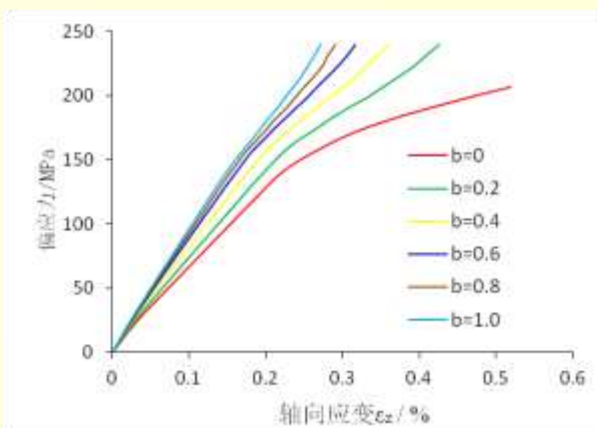
of intermediate principal stress effect on rock strength

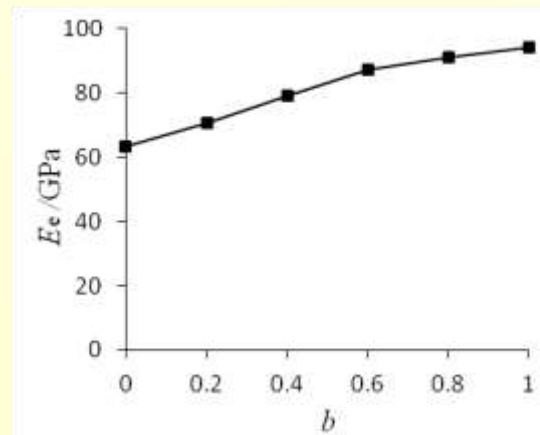
Micro cracks system

Single crack deformation



Calculating results of 800 micro cracks





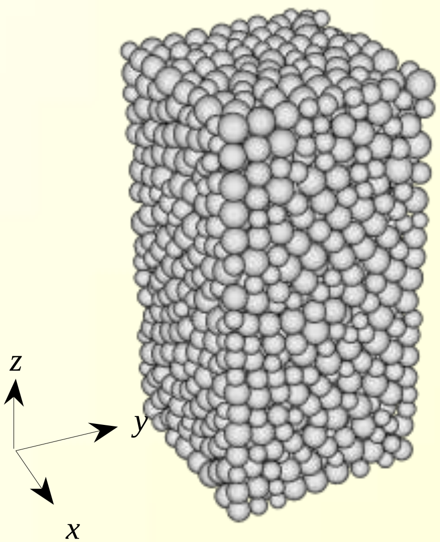
When b is near 0,increase of effective elasticity modulus is **apparent**;

When b is near 1,increase of effective elasticity modulus is **inapparent**.

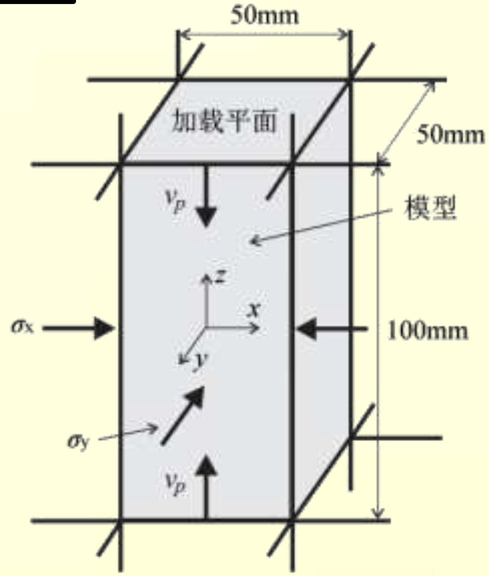
➢3. Mechanism discussion（2）

of intermediate principal stress effect on rock strength

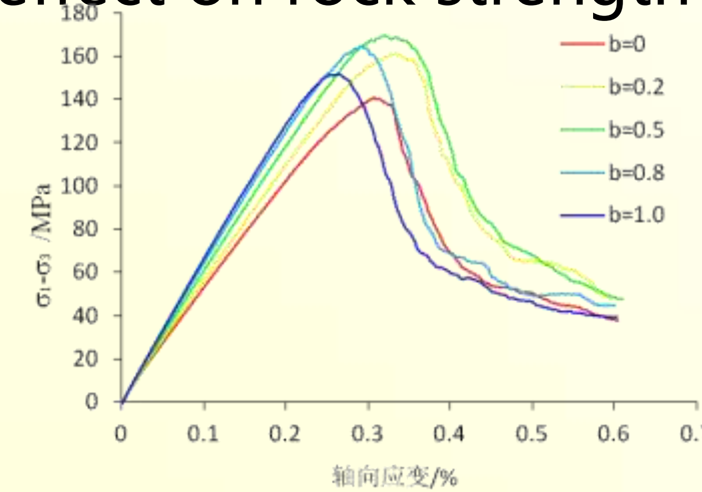
Particle system



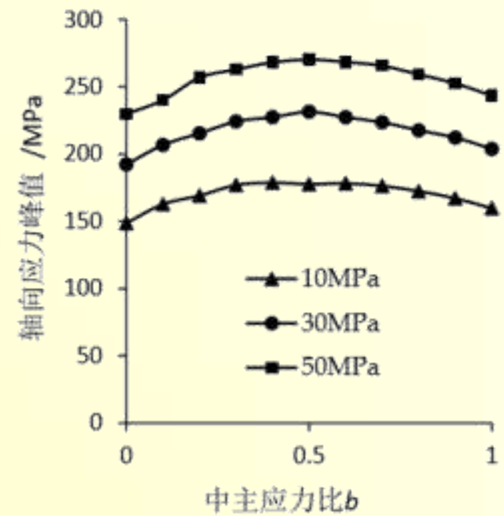
PFC model



loading method



stress-strain relationship



Peak stress vs *b*

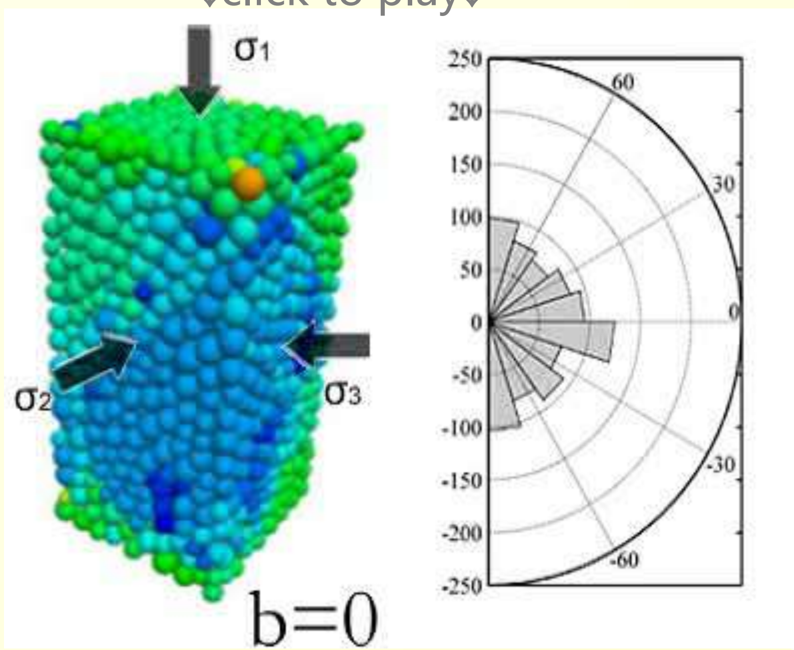
- When b<0.5, 0,increase of effective elasticity modulus is apparent;
- Peak strength: first increase and then decrease; increase rate 20%

➢3. Mechanism discussion (3)

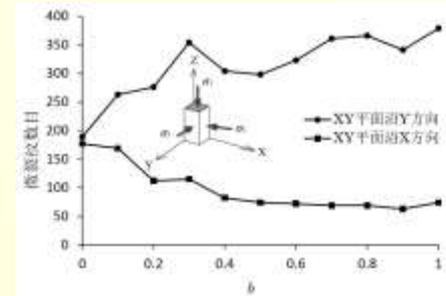
of intermediate principal stress effect on rock strength

Particle system

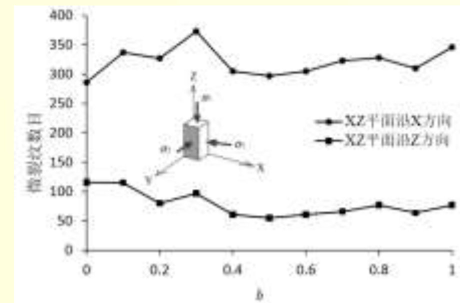
↓click to play↓



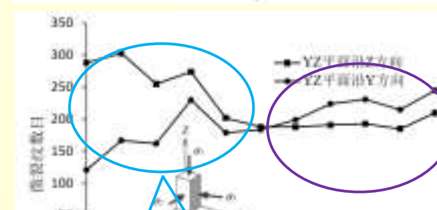
$\sigma_2 - \sigma_3$



$\sigma_1 - \sigma_3$



$\sigma_1 - \sigma_2$



σ2 **restrains** the expansion of micro cracks normal to σ2 direction(or with component in this direction), which makes rock strength **increases**. On the other side, σ2 **promotes** the expansion of micro cracks normal to σ3 direction(or with component in this direction), which makes rock strength **decreases**.

➢3. Mechanism discussion（4）

of intermediate principal stress effect on rock strength

Micro cracks system	Effective elasticity modulus varies in a different way as b in different interval, due to **different orientations** of micro cracks.
Particle system	The restraint and promotion of micro cracks in different directions lead to the intermediate principal stress effect.

The shear planes in rock samples are considered as potential failure planes. In order to calculate the probability of each direction, each potential shear failure plane is regarded as a micro-unit. The effect of the intermediate principal stress can quantitatively be estimated by calculating the failure probabilities for all the shear planes and combining these into the total probability for failure.

Weibull distribution is used to describe the heterogeneities of micro units strength.

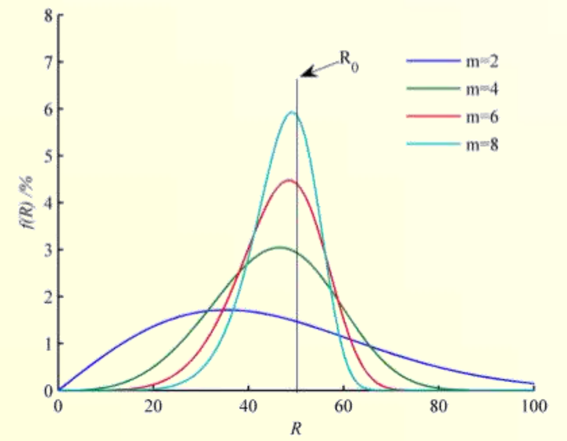
➢4. Statistical model of intermediate principal stress effect on rock strength

Weibull distribution

Stress threshold

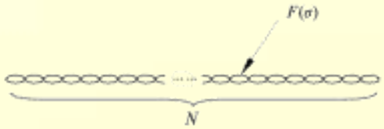
$$f(R) = \frac{m}{R_0}\left\langle \frac{R-R_u}{R_0} \right\rangle^{m-1} \exp\left(-\left\langle \frac{R-R_u}{R_0} \right\rangle^{m}\right)$$

scale parameter



m is the shape parameter and it can be considered as the uniformity coefficient

weakest link theory



$$P(\sigma) = 1 - [1 - F(\sigma)]^N$$

failure probability

$$F(R) = \sum_k^\infty F(k|V) = 1 - F(0|V) = 1 - \exp\left(-V \int_0^R n(s)\, ds\right)$$

Nonuniform force field

Weibull distribution

➢4. Statistical model （2）

of intermediate principal stress effect on rock strength

Considering the nonuniform force field and using Weibull distribution

Integral failure probability $$F(R) = 1 - \exp\left(-\frac{1}{V_0}\int_V \left\langle\frac{Rf(x,y,z)-R_u}{R_0}\right\rangle^m \mathrm{d}V\right)$$

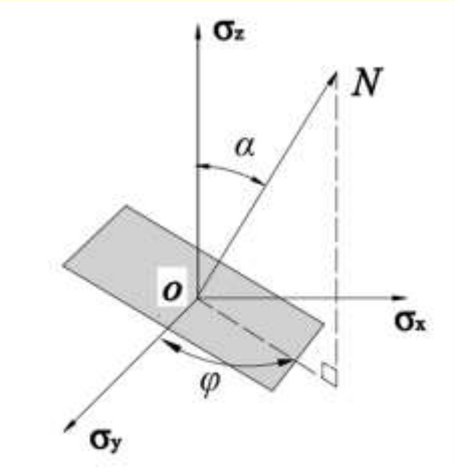
Mean strength $$\bar{R}_* = R_0\left(\frac{V_0}{V_*}\right)^{\frac{1}{m}}\Gamma\left(1+\frac{1}{m}\right)$$ ➡ $$\frac{\bar{R}_2}{\bar{R}_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{m}}$$

Size effect

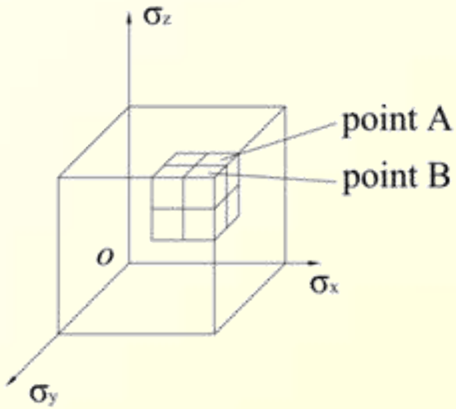
Variation coefficient $$\omega_* = \frac{s_*}{\bar{R}_*} = \sqrt{\frac{\Gamma(1+\frac{2}{m})}{\Gamma^2(1+\frac{1}{m})} - 1}$$

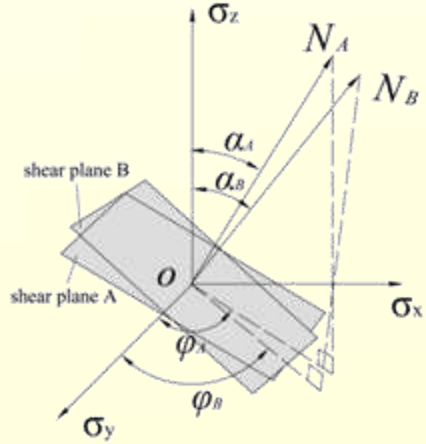
Equivalent volume $$V_* = \int_V f^m(x,y,z)\mathrm{d}V$$

Each potential shear failure plane is regarded as a micro-unit



4. Statistical model (3) of intermediate principal stress effect on rock strength





$V = \iiint [f(x, y, z)]^m \mathrm{d}x\mathrm{d}y\mathrm{d}z$

⇩

size effect

$V = \int_0^{\pi/2} \int_0^{\pi/2} f^m(\alpha, \varphi) \mathrm{d}\alpha \mathrm{d}\varphi$

⇩

intermediate principal stress effect

In **volume considerations**, materials heterogeneity means the different properties between **different points**, such as point A (x_A, y_A, z_A) and point B (x_B, y_B, z_B).

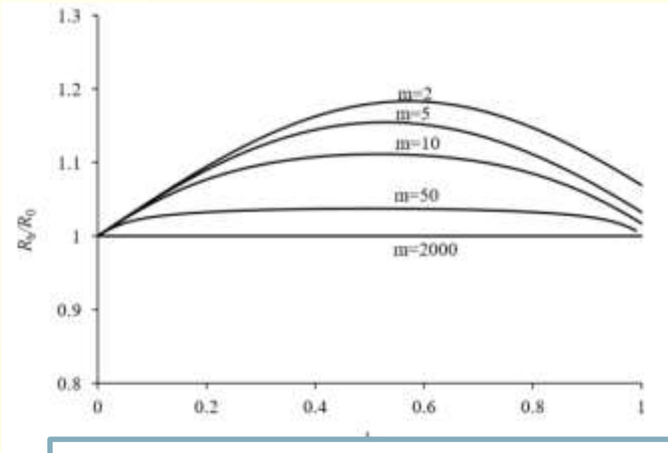
In **direction considerations**, materials heterogeneity means the different properties between **different shear planes**, such as plane A (α_A, φ_A) and plane B (α_B, φ_B).

$$\bar{R}_b = \bar{R}_0 \left(\frac{V_0}{V_b}\right)^{\frac{1}{m}}$$

the generalized volume $V_b = \int_0^{2\pi} \int_0^{\pi/2} f^m(\varphi, \alpha)\, d\alpha\, d\varphi$

➢4. Statistical model （4）

of intermediate principal stress effect on rock strength

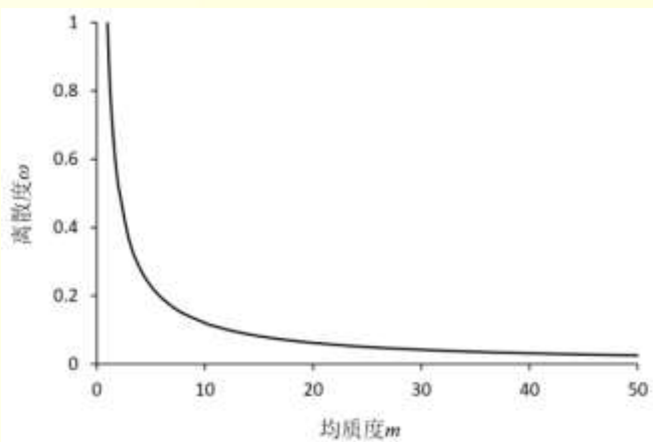


enhancement coefficient of strength

表 3.2 从强度的尺寸效应而求得的均匀性系数*m*值的例

岩 石 名	*m* 值	试验种类	测 定 者
花岗岩	12	单轴压缩	Lundborg[63]
秋吉大理石	33	单轴压缩	茂 木[64]
三城目安山岩	18.3	单轴压缩	西松他[61]
三城目安山岩	16.6	单轴拉伸	西松他[61]
荻野凝灰岩	20.2	单轴压缩	西松他[61]
英国煤炭	9.4～18	单轴压缩	Evans& Pomeroy[65]
日本煤炭	5.5～12	单轴压缩	会田，冈本[66]
水泥砂浆	11.7～20.7	单轴压缩	会田，冈本[66]
钢铁	23.3	单轴拉伸	Davidenkov[67]
(低温脆性断裂)	25.4	弯曲试验	Davidenkov[67]

(Yamaguchi Metaro，1982)



Variation coefficient

•As m increases, the materials become more homogeneous and the effects of intermediate principal stress become less prominent and the variation coefficients of the results are lower

•When m → ∞， the materials are absolutely homogeneous, there is no intermediate principal stress effect.

➢4. Statistical model （5）

of intermediate principal stress effect on rock strength

Mohr-Coulomb Criterion

$$R_0 = \frac{\sigma_{10} - \sigma_3}{2} = \frac{\sigma_3 \sin\varphi + c\cos\varphi}{1 - \sin\varphi} \Rightarrow \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_3 \sin\varphi + c\cos\varphi}{1 - \sin\varphi}\left(\frac{V_0}{V_b}\right)^{\frac{1}{m}}$$

New strength Criterion

$$\tau_{13} = f_1(\sigma_3) \cdot f_2(\mu_\sigma)$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$f_1(\sigma_3) = \frac{\sigma_3 \sin\varphi + c\cos\varphi}{1 - \sin\varphi}$$

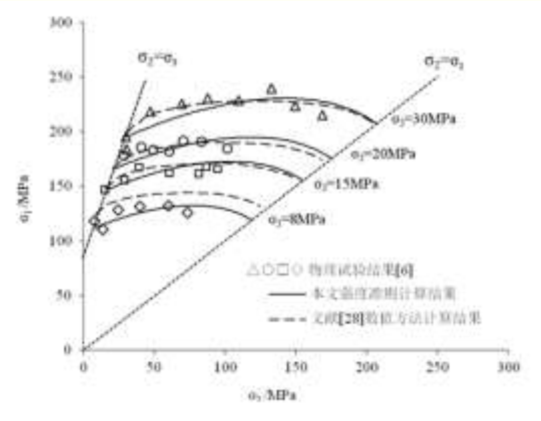
$$f_2(\mu_\sigma) = \left[\frac{2\pi\int_0^{\pi/2} \sin^m(2\alpha)\,\mathrm{d}\alpha}{\int_0^{2\pi}\int_0^{\pi/2} f_3^{\,m}(\varphi,\alpha,\mu_\sigma)\,\mathrm{d}\alpha\,\mathrm{d}\varphi}\right]^{1/m}$$

$$f_3(\varphi,\alpha,\mu_\sigma) = 2\sqrt{\sin^2\alpha\left[\frac{(1-\mu_\sigma)^2}{4}\cos^2\varphi + \sin^2\varphi\right] - \sin^4\alpha\left[\frac{1-\mu_\sigma}{2}\cos^2\varphi + \sin^2\varphi\right]^2}$$

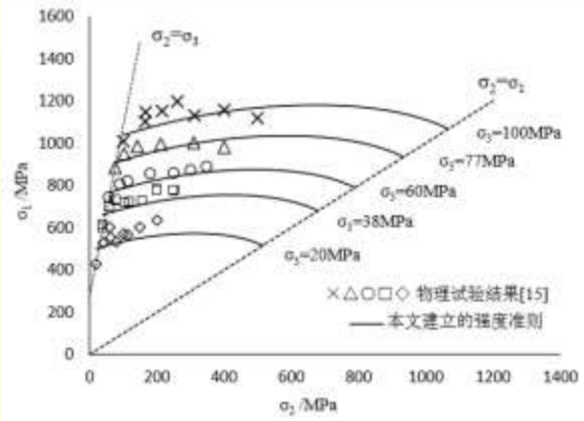
$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$ Lode参数

$$m \to \infty \Rightarrow f_2(\mu_\sigma) = 1 \Rightarrow \tau_{13} = f_1(\sigma_3)$$ Mohr-Coulomb Criterion

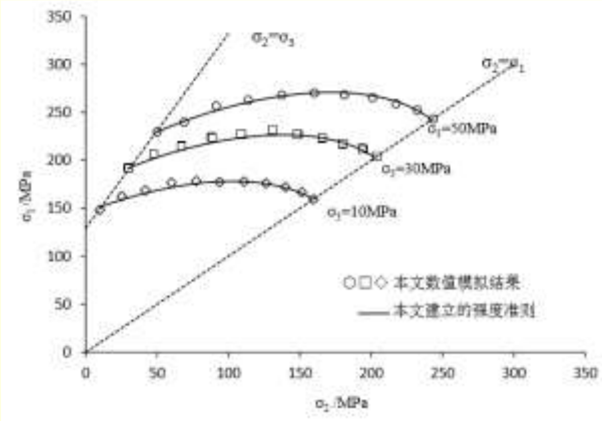
➢4. Statistical model （6）
of intermediate principal stress effect on rock strength



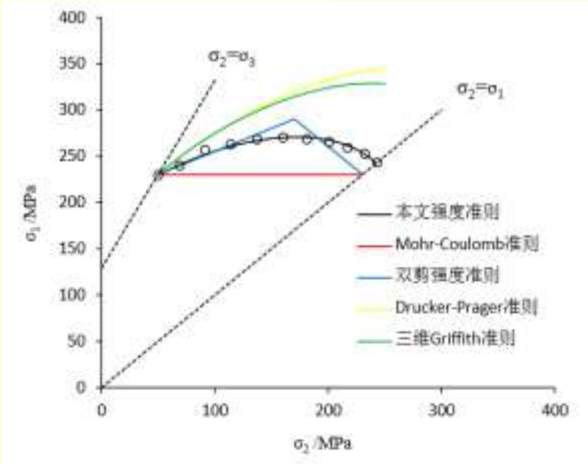
Shirahama sandstone
(Takahashi，1989)
m=2.5



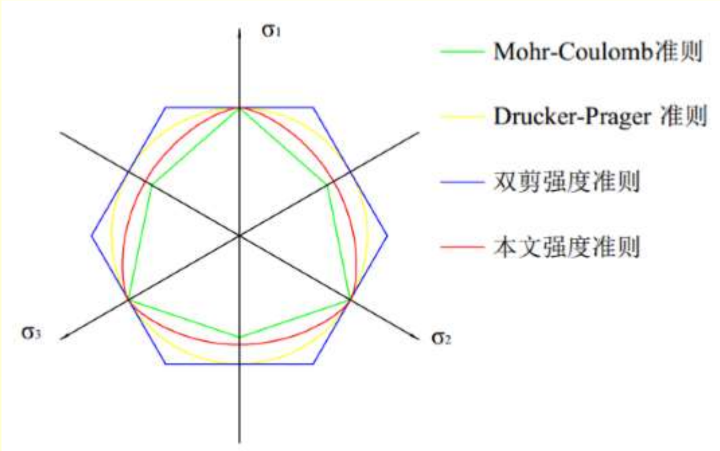
Westley granite
（Chang & Haimson,2000）
m=5.2



Numerical simulation
m=2.7



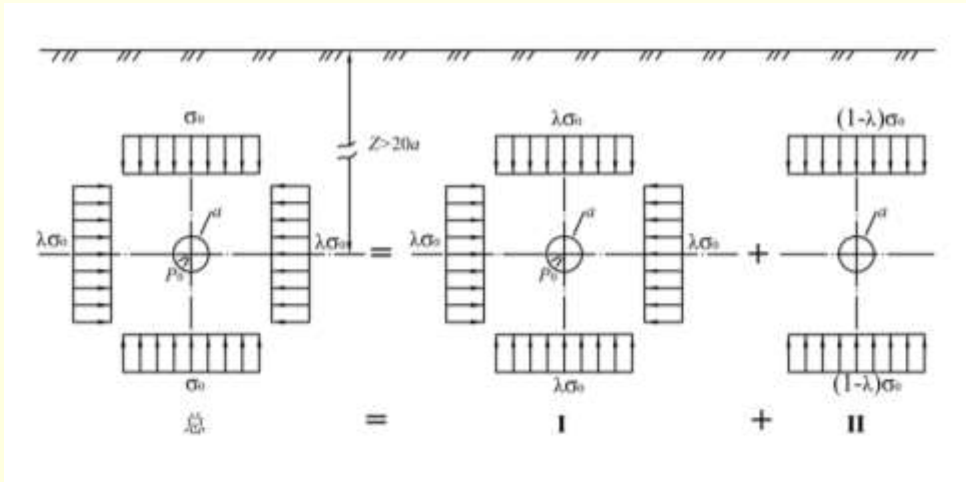
Comparison with Common Strength Criteria



failure curves on deviatoric stress plane

➢5. Case study

——plastic zone of round tunnel under non-uniform stress filed



Stress state

Yield criterion

$$\left.\begin{aligned}\sigma_r &= \frac{1}{2}(1+\lambda)\sigma_0(1-\frac{a^2}{r^2})-\frac{1}{2}(1-\lambda)\sigma_0(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4})\cos 2\theta+p_0\frac{a^2}{r^2}\\ \sigma_\theta &= \frac{1}{2}(1+\lambda)\sigma_0(1+\frac{a^2}{r^2})+\frac{1}{2}(1-\lambda)\sigma_0(1+3\frac{a^4}{r^4})\cos 2\theta-p_0\frac{a^2}{r^2}\\ \tau_{r\theta} &= \frac{1}{2}(1-\lambda)\sigma_0(1+2\frac{a^2}{r^2}-3\frac{a^4}{r^4})\sin 2\theta\end{aligned}\right\}$$

insert

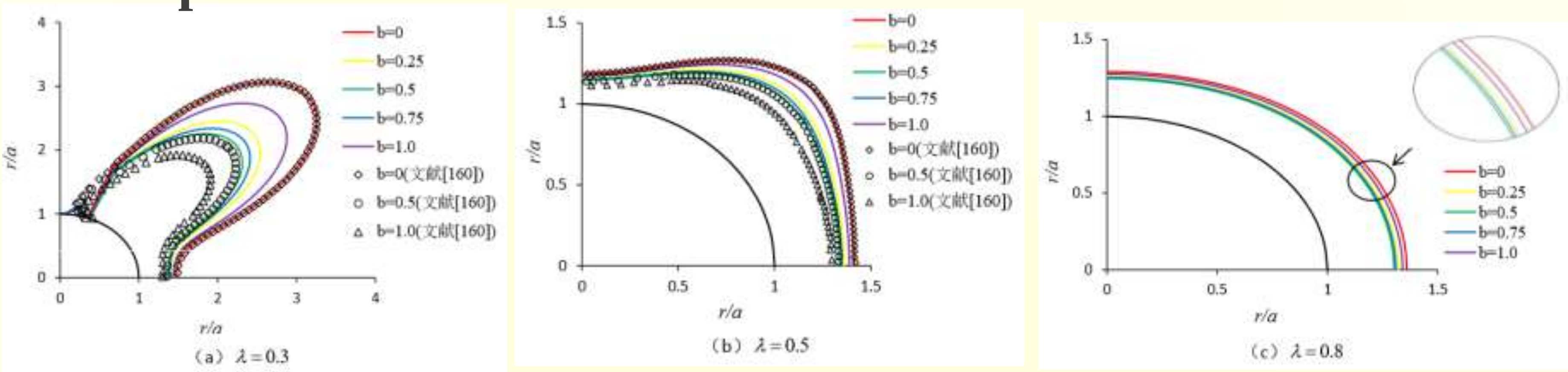
$$\sqrt{(\frac{\sigma_r-\sigma_\theta}{2})^2+\tau_{r\theta}{}^2}=\frac{(\sigma_r+\sigma_\theta)\sin\varphi+2c\cos\varphi}{2(\frac{1-\sin\varphi}{f_2(\mu_\sigma)}+\sin\varphi)}$$

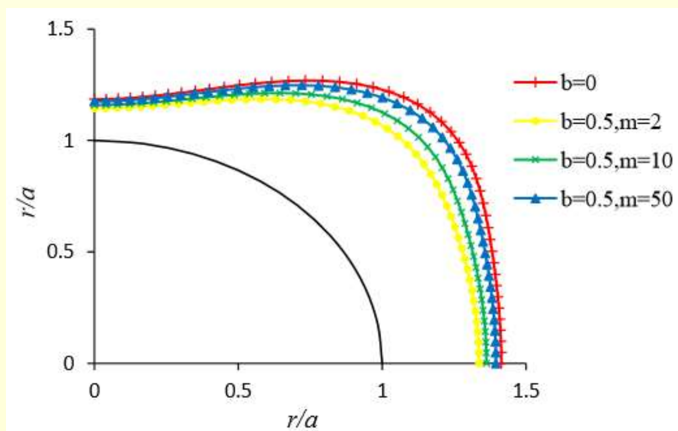
$$f(k)=a_0+a_1k+a_2k^2+a_3k^3+a_4k^4=0 \qquad k=a^2/r^2$$

Using Newton iteration method to solve

➢5. Case study

——plastic zone of round tunnel under non-uniform stress filed





Expression of boundary line of surrounding rock mass plastic zone

6. Conclusions

1

The restraint and promotion of micro cracks induced by σ_2 make the difference of failure probabilities in different directions, which leads to the intermediate principal stress effect.

2

Each potential shear failure plane is regarded as a micro-unit. Weibull distribution is used to describe the heterogeneities of micro units strength.

The effect of σ_2 can quantitatively be estimated by calculating the failure probabilities for all the shear planes and combining these into the total probability for failure.

3

New strength criterion is developed to quantitatively describe the effect of σ_2 on rock strength. When uniformity coefficient *m* is infinitely large, the new criterion is equivalent to Mohr-Coulomb criterion. Therefore, the proposed strength criterion can be regarded as a modified Mohr-Coulomb criterion that can reflect the effect of intermediate principal stress.

Thank you！