Finite Element Solution of Poroelastoplasticity in Strain-softening Porous Media

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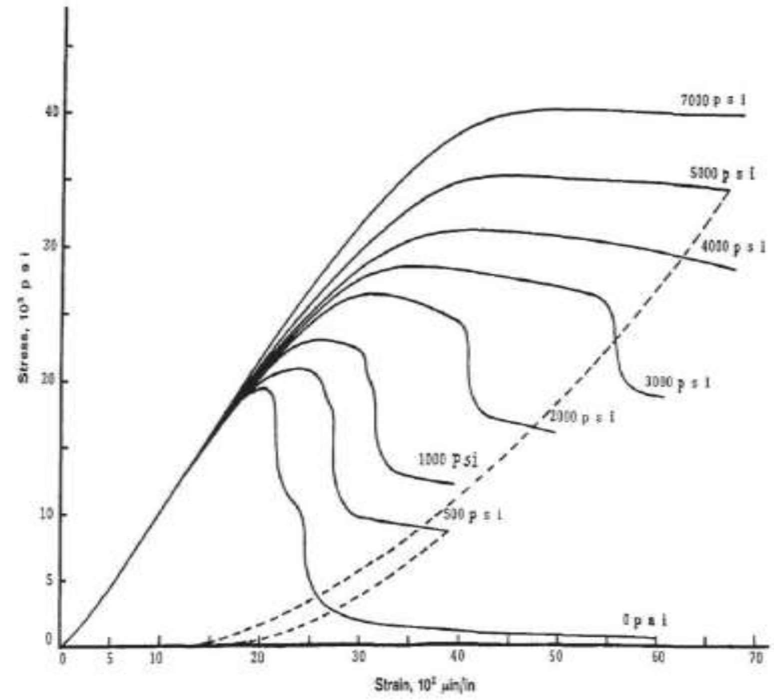
Outline

- Background
- FEM for elasto-plastic consolidation
- Strain-softening process
- Numerical results
- Conclusion

Background

- Laboratory triaxial compression experiment of geomaterials

- Strain-softening phenomenon is very common in Petroleum Geomechanics



FEM for elasto-plastic consolidation

- Field equations

$$G\nabla^2\boldsymbol{u} + (G+\lambda)\nabla div(\boldsymbol{u}) - \left(1-\frac{K}{K_m}\right)\nabla p = 0$$

$$\left(1-\frac{K}{K_m}\right) div(\boldsymbol{u}_t) + \left[\frac{1-\emptyset}{K_m} + \frac{\emptyset}{K_f} - \frac{1}{(3K_m)^2} i^T D i\right] p_t + \frac{k}{\mu}\nabla^2 p = 0$$

- Galerkin discretization

$$\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{C} \\ 0 & \boldsymbol{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ \boldsymbol{C}^T & \boldsymbol{S} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_t \\ \boldsymbol{p}_t \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}^u \\ \boldsymbol{f}^p \end{Bmatrix}$$

- Time discretization

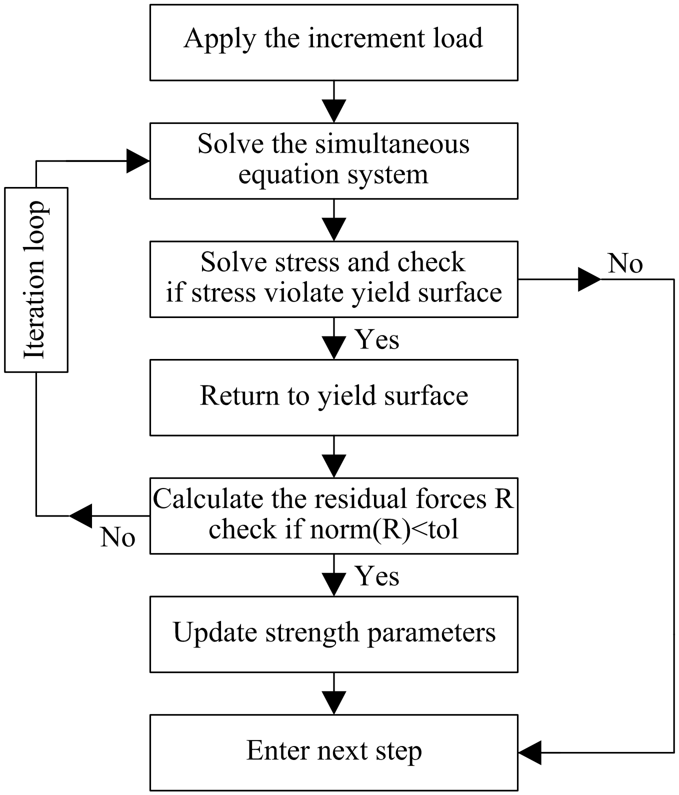
$$\begin{bmatrix} \theta\boldsymbol{M} & -\theta\boldsymbol{C} \\ \boldsymbol{C}^T & \boldsymbol{S} + \theta\Delta t\boldsymbol{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_{n+1} \\ \boldsymbol{p}_{n+1} \end{Bmatrix} = \begin{bmatrix} (\theta-1)\boldsymbol{M} & -(\theta-1)\boldsymbol{C} \\ \boldsymbol{C}^T & \boldsymbol{S} + (\theta-1)\Delta t\boldsymbol{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_n \\ \boldsymbol{p}_n \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{f}^u \\ \Delta t\boldsymbol{f}^p \end{Bmatrix}$$

FEM for elasto-plastic consolidation

- Initial stress method

$$\begin{bmatrix} \theta \boldsymbol{M} & -\theta \boldsymbol{C} \\ \boldsymbol{C}^T & \boldsymbol{S} + \theta \Delta t \boldsymbol{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_{n+1} \\ \boldsymbol{p}_{n+1} \end{Bmatrix} = \begin{bmatrix} (\theta - 1)\boldsymbol{M} & -(\theta - 1)\boldsymbol{C} \\ \boldsymbol{C}^T & \boldsymbol{S} + (\theta - 1)\Delta t \boldsymbol{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_n \\ \boldsymbol{p}_n \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{f}^u \\ \Delta t \boldsymbol{f}^p \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{R} \\ \boldsymbol{0} \end{Bmatrix}$$

- Numerical flow chart



Strain-softening process

- Stress changes in brittle-plastic plasticity

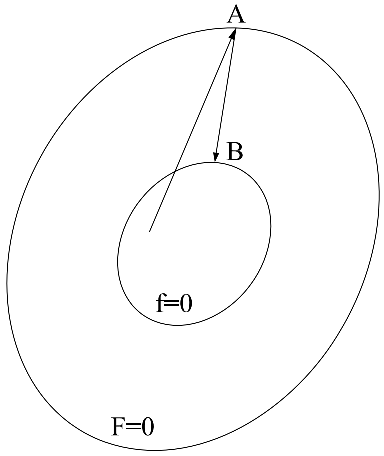
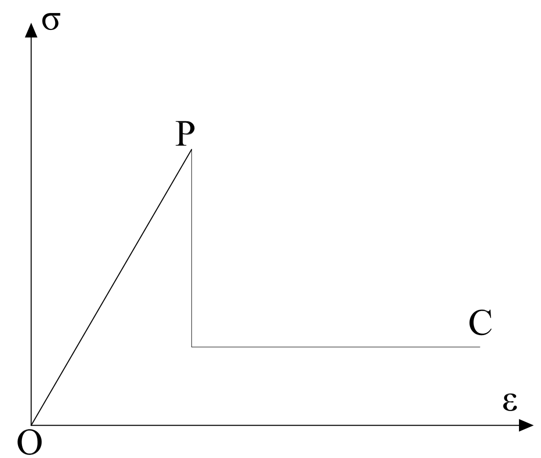
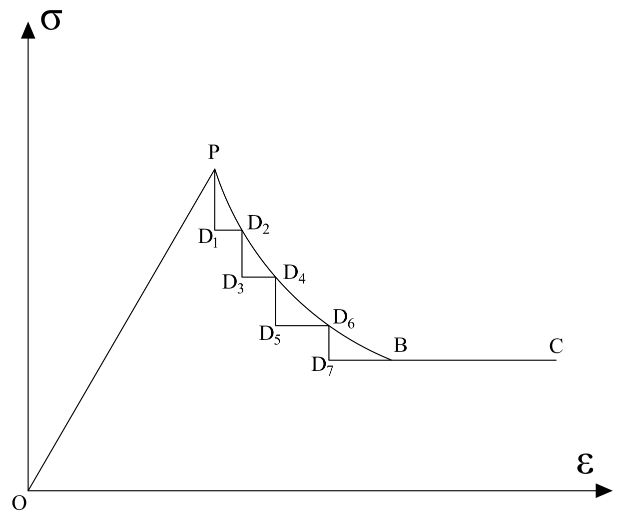


Diagram of stress-dropping



Brittle-plastic model

- Simplification of strain-softening process



Strain-softening process

- Stress changes in M-C

Assumption: constant minor principal stress

Peak stress surface: $F_p(\sigma_{1p}, \sigma_{3p}, \eta) = \sigma_{1p} - \alpha_p \sigma_{3p} - Y_p = 0$

Residual stress surface: $F_r(\sigma_{1r}, \sigma_{3r}, \eta) = \sigma_{1r} - \alpha_r \sigma_{3r} - Y_r = 0$

$$\Delta\sigma_3 = \sigma_{3r} - \sigma_{3p} = 0$$

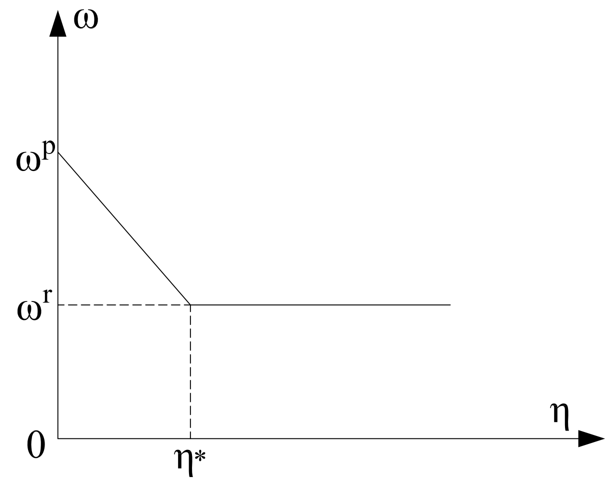
$$\Delta\sigma_1 = \sigma_{1r} - \sigma_{1p} = (\alpha_r - \alpha_p)\sigma_{3r} + Y_r - Y_p$$

Assume the ratio of difference of the principal stress is constant

$$\Delta\sigma_2 = \sigma_{2r} - \sigma_{2p} = \sigma_{3r} + \frac{\sigma_{2p} - \sigma_{3p}}{\sigma_{1p} - \sigma_{3p}}(\sigma_{1r} - \sigma_{3r}) - \sigma_{2p}$$

Strain-softening process

- Evolution of strength parameters



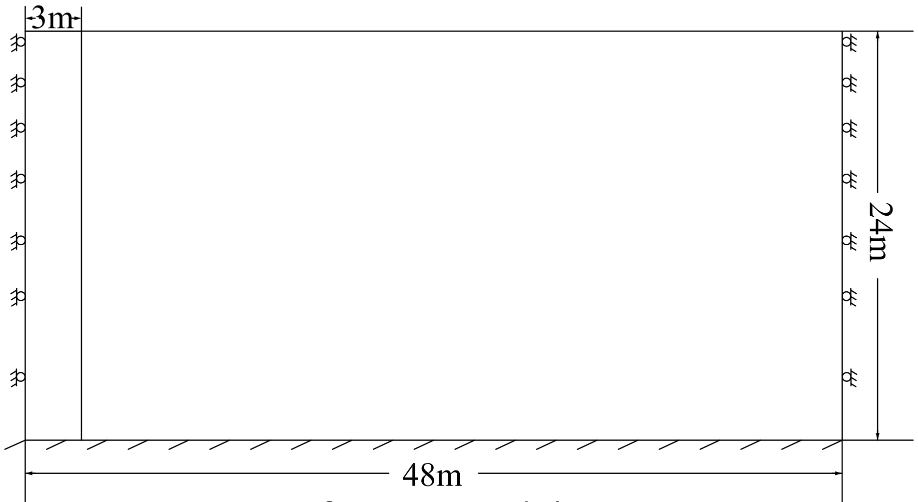
- Plastic shear strain ($\eta = \varepsilon_1^p - \varepsilon_3^p$)

is chosen to measure strength parameters

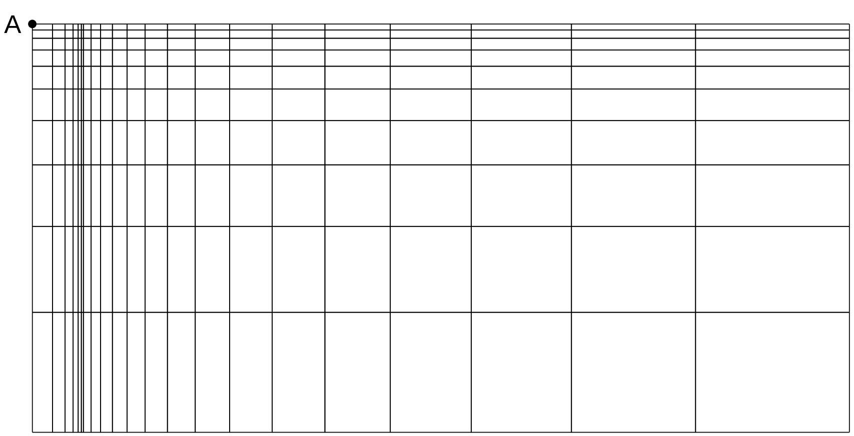
$$w(\eta) = \begin{cases} w_p - (w_p - w_r)(\eta/\eta^*) & 0 < \eta < \eta^* \\ w_r & \eta > \eta^* \end{cases}$$

Numerical results

Consolidation considering strain-softening



Strip footing model

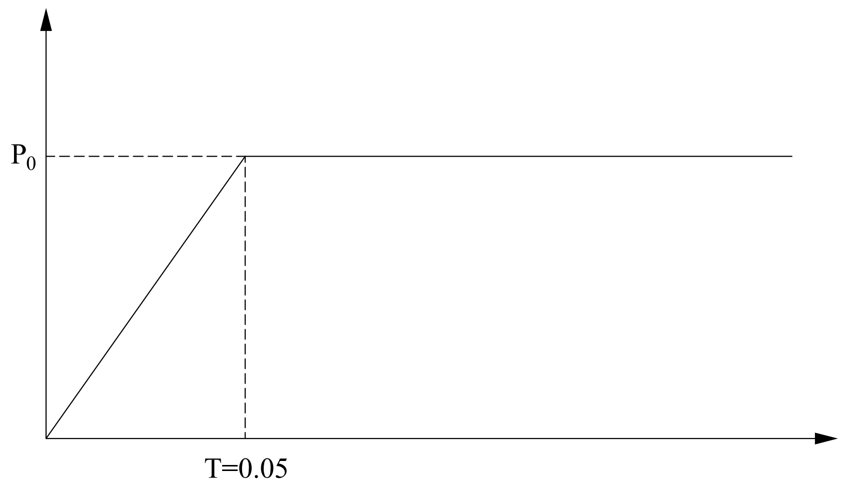


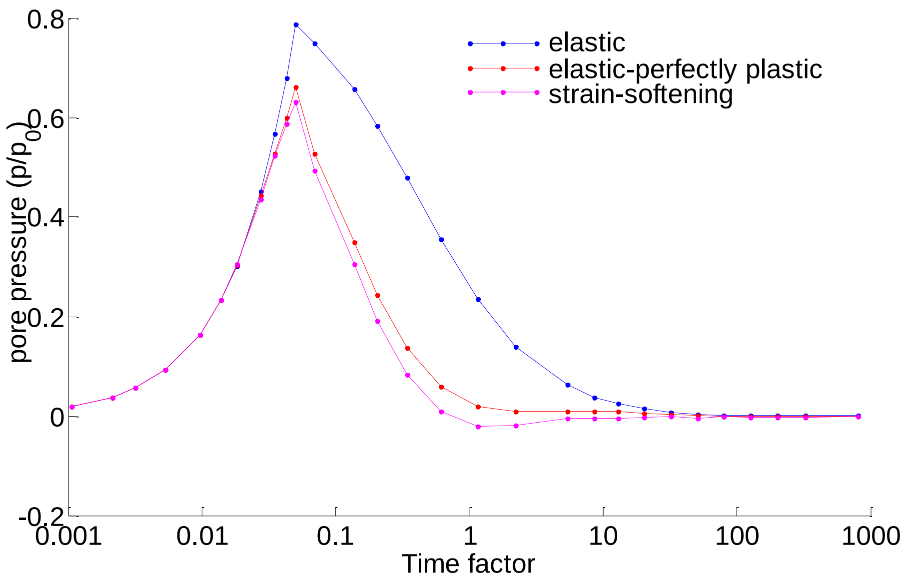
Finite element mesh

Half width of footing, a(m)	10
Young's modulus, E(Mpa)	2
Poisson's ratio, ν	0.3
η^*	0.06
c_p(Mpa)	0.01
c_r(Mpa)	0.006
φ_p(deg)	20
φ_r(deg)	14
ψ(deg)	20
Permeability $k(m/day)$	10^{-5}

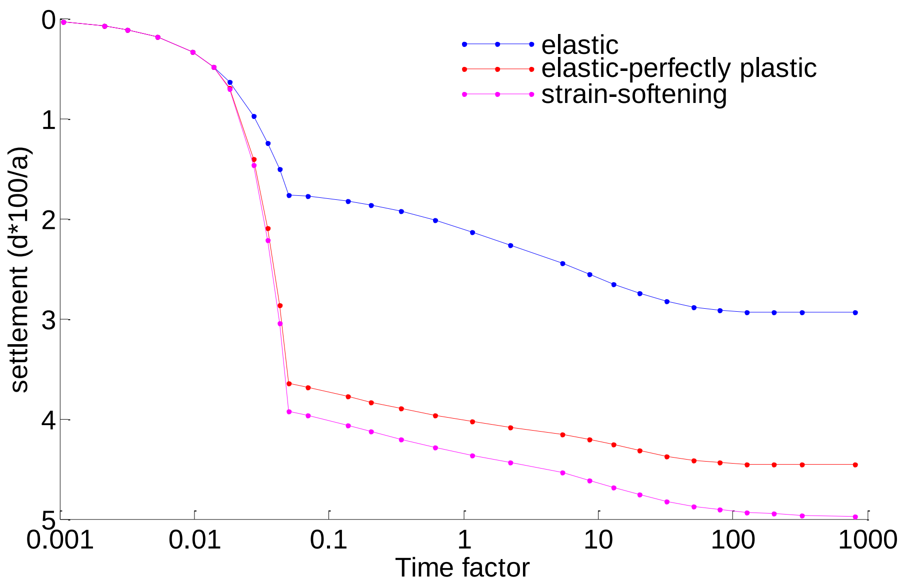
Numerical results

- Pressure





Consolidation settlement at centre of footing



Dissipation of pressure at point A

Conclusions

- In coupled hydro-mechanical (H-M) system, if material exhibits strain-softening behavior, the strain-softening model should be applied, otherwise, the results will deviate far from the real solution.

- Finite element method is capable of solving the coupled H-M problem with strain softening considered.