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Shakedown analysis of lined rock cavern for compressed air energy storage

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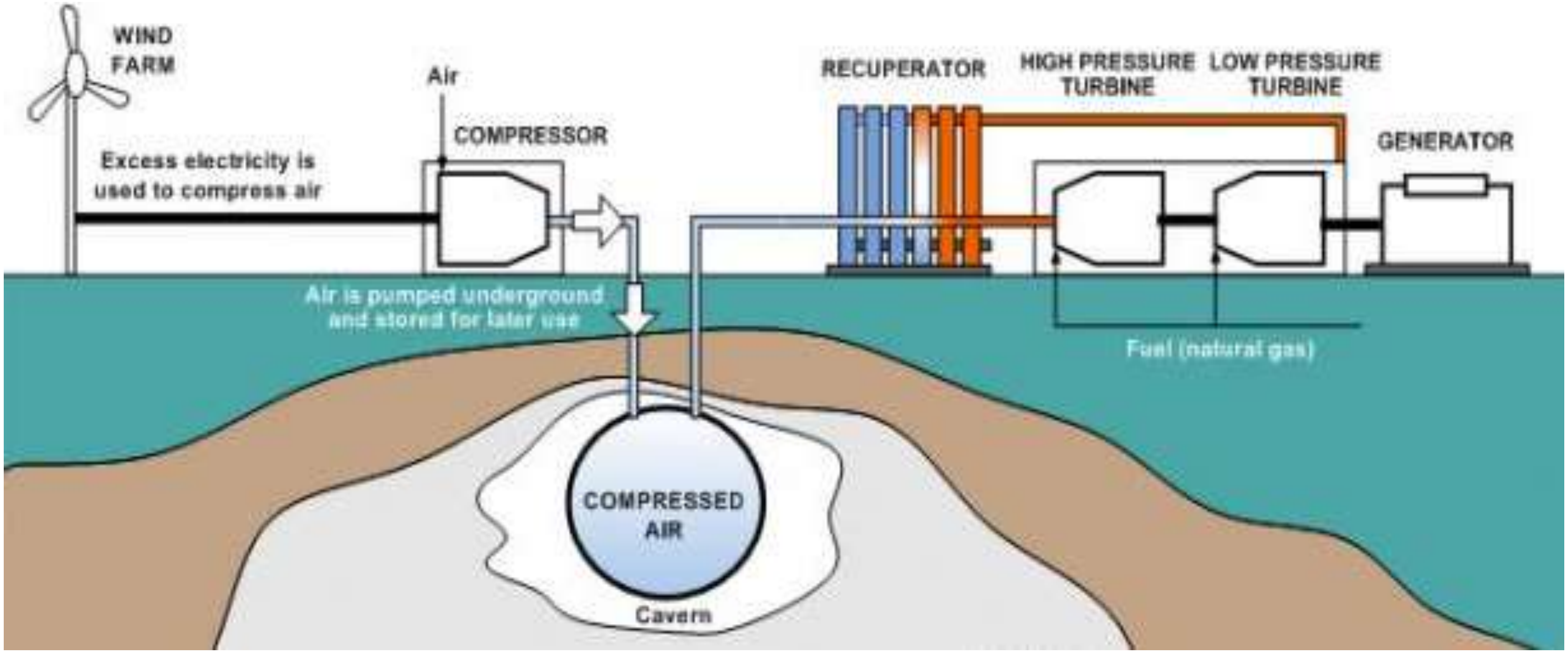
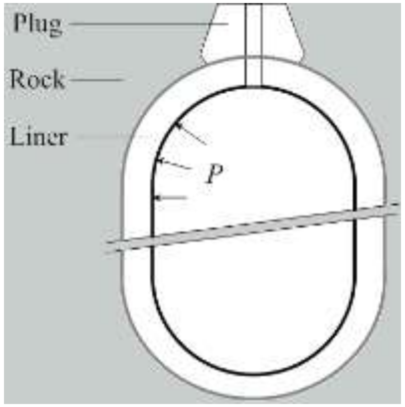
Contents

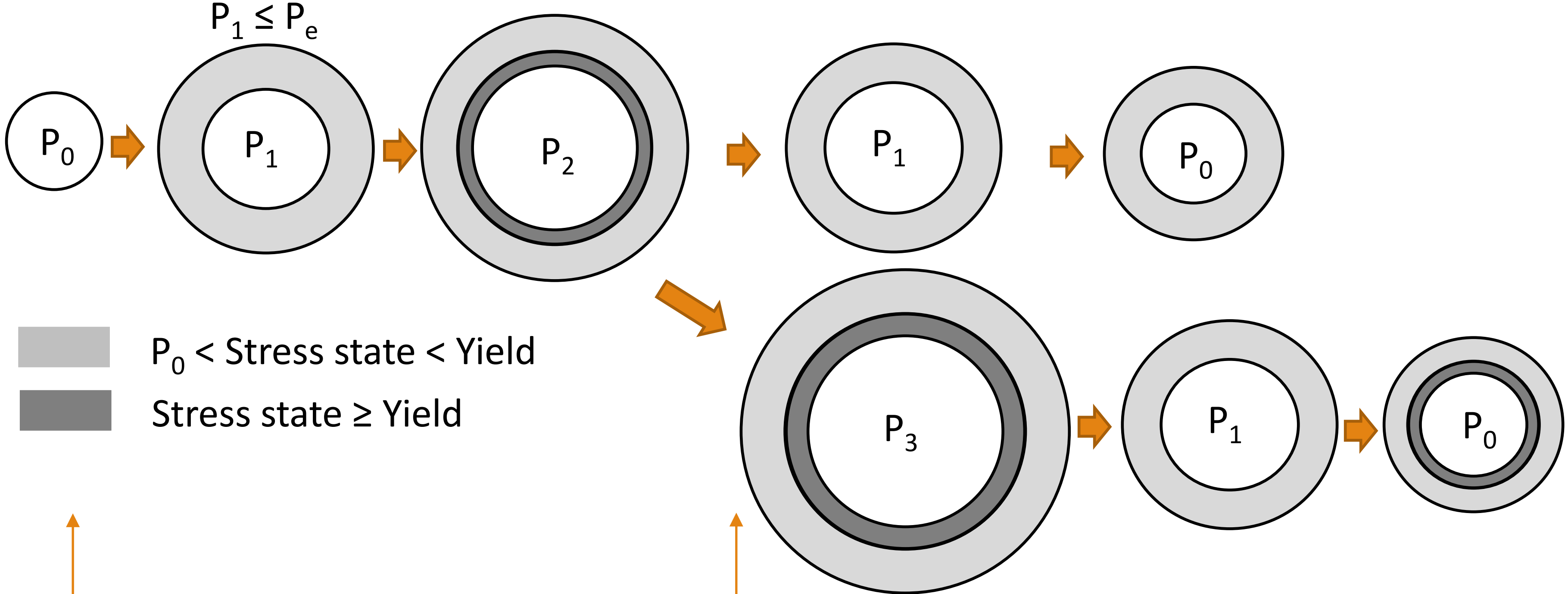
Background - CAES

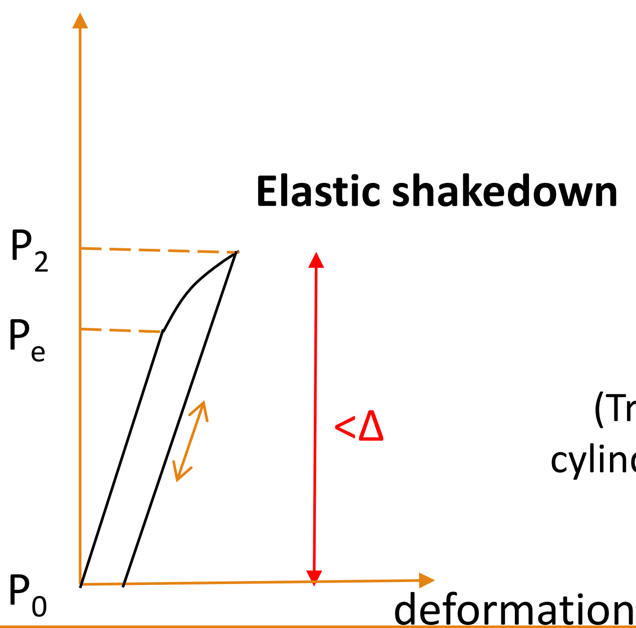


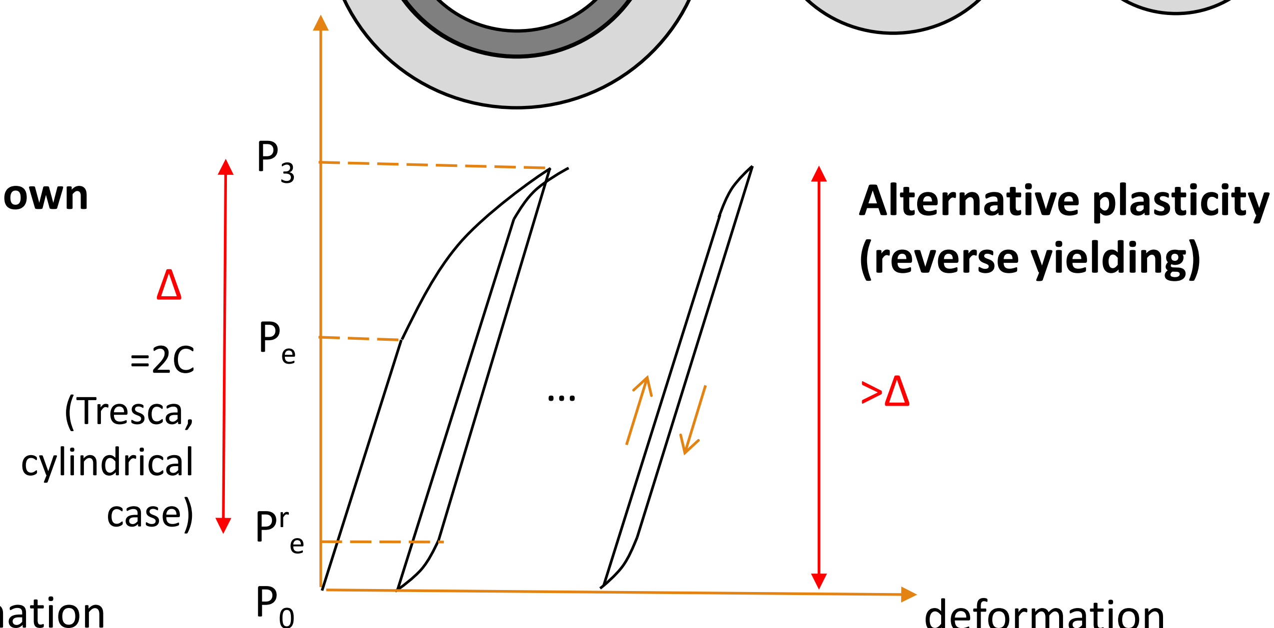
- Compressed air energy storage (CAES) systems
- Lined or unlined rock caverns
- Gas pressure: 10 to 30 MPa
- Repeated compression-decompression cycles



Background – carven behavior







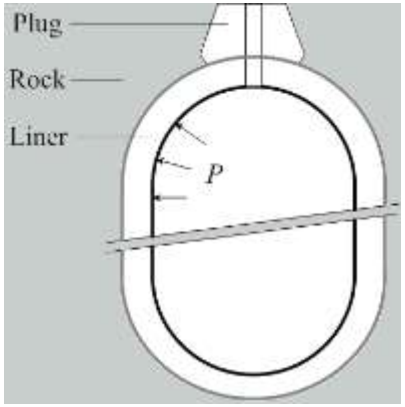
Creeping effect is not considered

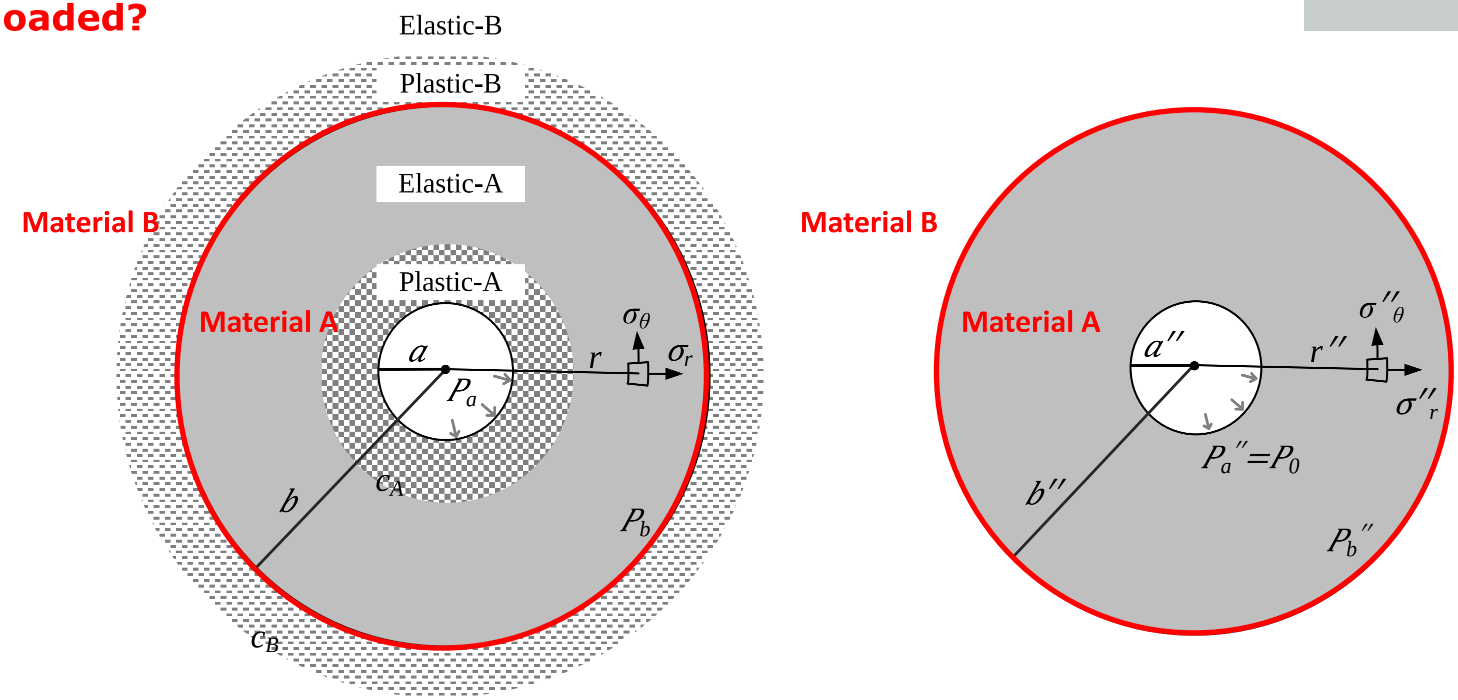
Background

- The cavern may exhibit plastic strain during each loading cycle as yielding occurs in loading and unloading processes.
- **Reverse yielding** can lead to **low-cycle fatigue** of elastic-plastic materials. E.g. failure will occur in steel specimen after 1000 cycles for a strain range of around 0.02 (Stephens, 1980).
- Johansson (2003) suggested to check low-cycle fatigue of steel lining of lined rock cavern.
- Low-cycle fatigue needs to be considered in the survivability analysis of rock openings subjected to multiple attack (Balachandra, 1978).
- **Shakedown analysis** can provide the maximum capacity of a structure against reverse yielding under cyclic loads based on elastic-plastic theory.
- Melan's shakedown theorem: $f(\sigma_e + \sigma_r) \leq Y$
- For the CAES carven problem, if the stress state after unloading is smaller than the yield criterion, shakedown will occur; otherwise, reverse yielding will happen.

An analytical approach – problem definition

- Two layers of isotropic homogenous linear elastic cohesive-frictional materials (Mohr-Coulomb criterion)
- Combine both cylindrical scenario (k=1) and spherical scenario (k=2)
- **Reverse yielding occurs or not in both materials when fully unloaded?**





(a) Loading

(b) Unloading

An analytical approach

- **Yield or not?**

$$\alpha\sigma_r - \sigma_\theta = Y$$

$$\alpha = \frac{1 + sin\phi}{1 - sin\phi}$$

$$Y = \frac{2C\ cos\phi}{1 - sin\phi}$$

- **When loaded: (Mo et al., 2014)**

$$\alpha_A\sigma_{r,A} - \sigma_{\theta,A} = Y_A \quad \sim f(P_a, r)$$

$$\alpha_B\sigma_{r,B} - \sigma_{\theta,B} = Y_B \quad \sim f(P_b, r)$$

$$\alpha_n = \frac{1 + sin\phi_n}{1 - sin\phi_n}$$

$$Y_n = \frac{2C_n\ cos\phi_n}{1 - sin\phi_n}$$

where n = A or B

- **When unloaded:**

$$\alpha_A\sigma''_{r,A} - \sigma''_{\theta,A} = Y_A$$

$$\alpha_B\sigma''_{r,B} - \sigma''_{\theta,B} = Y_B$$

in which:

$$\sigma''_{r,A} = \sigma_{r,A} + \Delta\sigma_{r,A}$$

$$\sigma''_{\theta,A} = \sigma_{\theta,A} + \Delta\sigma_{\theta,A}$$

$f(P_a, r)$?

$$\sigma''_{r,B} = \sigma_{r,B} + \Delta\sigma_{r,B}$$

$$\sigma''_{\theta,B} = \sigma_{\theta,B} + \Delta\sigma_{\theta,B}$$

$f(P_b, r)$?

An analytical approach

- **For material A:**

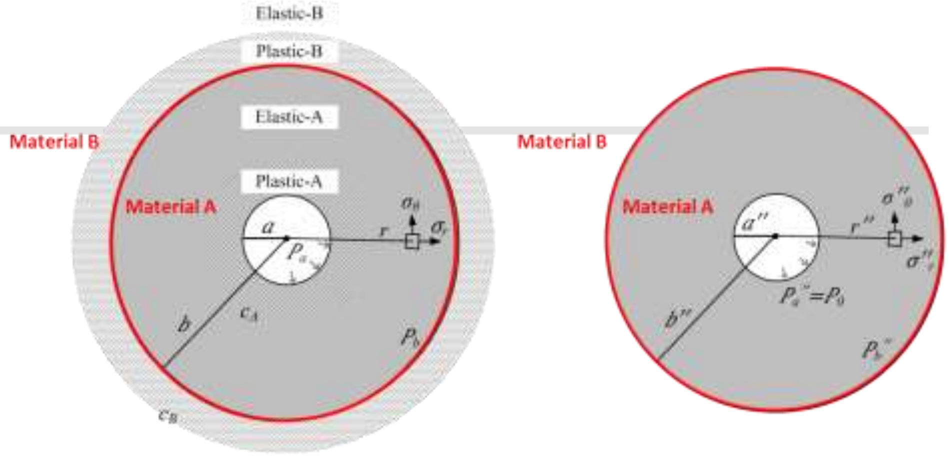
For cavity **elastic** expansion/contraction from p_0:

$$\Delta\sigma_{r,A} = A + \frac{B}{r^{k+1}} - p_0$$

$$\Delta\sigma_{\theta,A} = A - \frac{B}{kr^{k+1}} - p_0$$

$$\Delta\sigma_{r,A} = f(P_a, r)$$

$$\Delta\sigma_{\theta,A} = f(P_a, r)$$



(a) Loading

(b) Unloading

$$\Delta\sigma_{r,A}\Big|_{r=a} = (-P_a'') - (-P_a) = P_a - P_0$$

$$\Delta\sigma_{r,A}\Big|_{r=b} = (-P_b'') - (-P_b) = P_b - P_b''$$

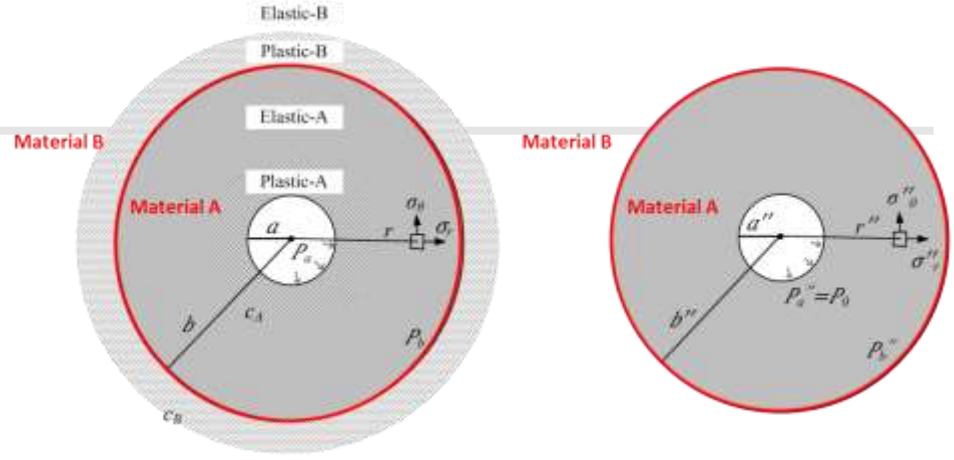
$$\Delta\varepsilon_{\theta,A}\Big|_{r=b} = \Delta\varepsilon_{\theta,B}\Big|_{r=b}$$

$$M_n = \frac{E_n}{1 - {\nu_n}^2(2-k)}$$

$$\Delta\varepsilon_{\theta,A}\Big|_{r=b} = \frac{1}{M_A}\left\{-\frac{\nu_A}{1-\nu_A(2-k)}\Delta\sigma_{r,A}\Big|_{r=b} + [1-\nu_A(k-1)]\,\Delta\sigma_{\theta,A}\Big|_{r=b}\right\}$$

$$\Delta\varepsilon_{\theta,B}\Big|_{r=b} = \frac{1}{M_B}\left\{-\frac{\nu_B}{1-\nu_B(2-k)}\Delta\sigma_{r,B}\Big|_{r=b} + [1-\nu_B(k-1)]\,\Delta\sigma_{\theta,B}\Big|_{r=b}\right\}$$

An analytical approach



(a) Loading (b) Unloading

- **Yield or not?**

$$\alpha\sigma_r - \sigma_\theta = Y$$

- **When loaded: (Mo et al., 2014)**

$$\alpha_A\sigma_{r,A} - \sigma_{\theta,A} = Y_A \qquad \sim f(P_a, r)$$

$$\alpha_B\sigma_{r,B} - \sigma_{\theta,B} = Y_B \qquad \sim f(P_b, r)$$

- **When unloaded:**

$$\alpha_A\sigma''_{r,A} - \sigma''_{\theta,A} = Y_A$$

$$\alpha_B\sigma''_{r,B} - \sigma''_{\theta,B} = Y_B$$

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$f(P_a, r)$?

$$\sigma''_{r,B} = \sigma_{r,B} + \Delta\sigma_{r,B}$$

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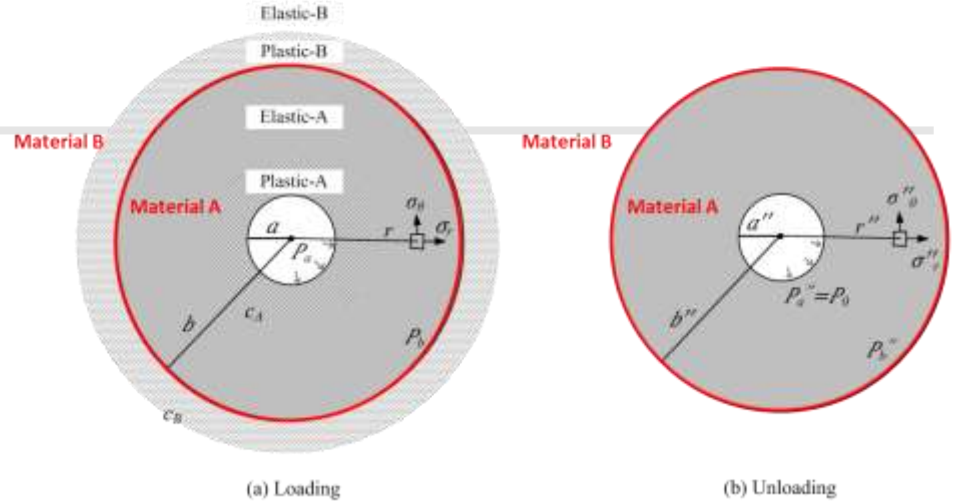
$f(P_b, r)$?

An analytical approach



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$$\alpha_B \sigma_{r,B} - \sigma_{\theta,B} = Y_B \qquad \sim f(P_b, r)$$

- **When unloaded:**

$$\alpha_A \sigma''_{r,A} - \sigma''_{\theta,A} = Y_A \xrightarrow{r=a} P_{a,sA}$$

$$\alpha_B \sigma''_{r,B} - \sigma''_{\theta,B} = Y_B$$

in which:

$$\sigma''_{r,A} = \sigma_{r,A} + \Delta\sigma_{r,A}$$

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$$f(P_a, r) \quad f(P_a, r) \quad f(P_a, r)$$

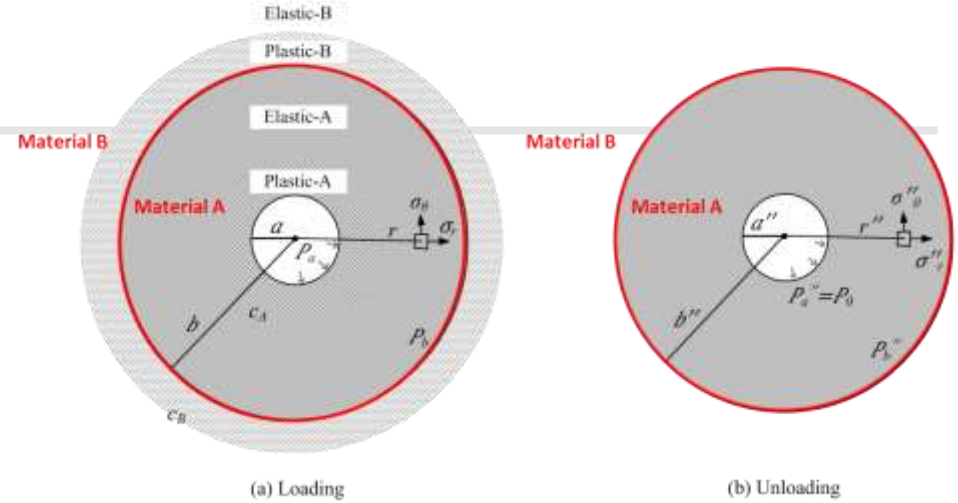
$$\sigma''_{r,B} = \sigma_{r,B} + \Delta\sigma_{r,B}$$

$$\sigma''_{\theta,B} = \sigma_{\theta,B} + \Delta\sigma_{\theta,B}$$

$$f(P_b, r) \quad ?$$

An analytical approach

- **For material B:**



For cavity **elastic** expansion/contraction from p_0:

$$\Delta\sigma_{r,B} = A + \frac{B}{r^{k+1}} - p_0$$

$$\Delta\sigma_{\theta,B} = A - \frac{B}{kr^{k+1}} - p_0$$

$$\Delta\sigma_{r,B}\Big|_{r=b} = (-P_b'') - (-P_b) = P_b - P_b''$$

$$\Delta\sigma_{r,B}\Big|_{r=\infty} = (-P_0) - (-P_0) = 0$$

$$\Delta\varepsilon_{\theta,A}\Big|_{r=b} = \Delta\varepsilon_{\theta,B}\Big|_{r=b}$$

$$M_n = \frac{E_n}{1 - {\nu_n}^2(2-k)}$$

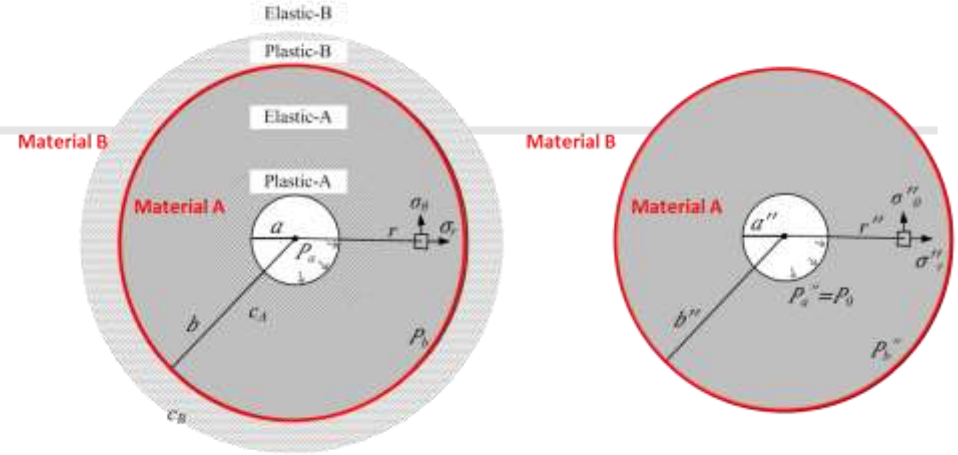
$$\Delta\varepsilon_{\theta,A}\Big|_{r=b} = \frac{1}{M_A}\left\{-\frac{\nu_A}{1-\nu_A(2-k)}\Delta\sigma_{r,A}\Big|_{r=b} + [1-\nu_A(k-1)]\,\Delta\sigma_{\theta,A}\Big|_{r=b}\right\}$$

$$\Delta\varepsilon_{\theta,B}\Big|_{r=b} = \frac{1}{M_B}\left\{-\frac{\nu_B}{1-\nu_B(2-k)}\Delta\sigma_{r,B}\Big|_{r=b} + [1-\nu_B(k-1)]\,\Delta\sigma_{\theta,B}\Big|_{r=b}\right\}$$

$$\Delta\sigma_{r,B} = f(P_a, r)$$

$$\Delta\sigma_{\theta,B} = f(P_a, r)$$

An analytical approach



(a) Loading (b) Unloading

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$$\alpha_B \sigma_{r,B} - \sigma_{\theta,B} = Y_B \qquad \sim f(P_b, r)$$

- **When unloaded:**

$$\alpha_A \sigma''_{r,A} - \sigma''_{\theta,A} = Y_A \xrightarrow{r = a} P_{a,sA}$$

$$\alpha_B \sigma''_{r,B} - \sigma''_{\theta,B} = Y_B$$

in which:

$$\sigma''_{r,A} = \sigma_{r,A} + \Delta\sigma_{r,A}$$

$$\sigma''_{\theta,A} = \sigma_{\theta,A} + \Delta\sigma_{\theta,A}$$

$f(P_a, r)$ $f(P_a, r)$ $f(P_a)$

$$\sigma''_{r,B} = \sigma_{r,B} + \Delta\sigma_{r,B}$$

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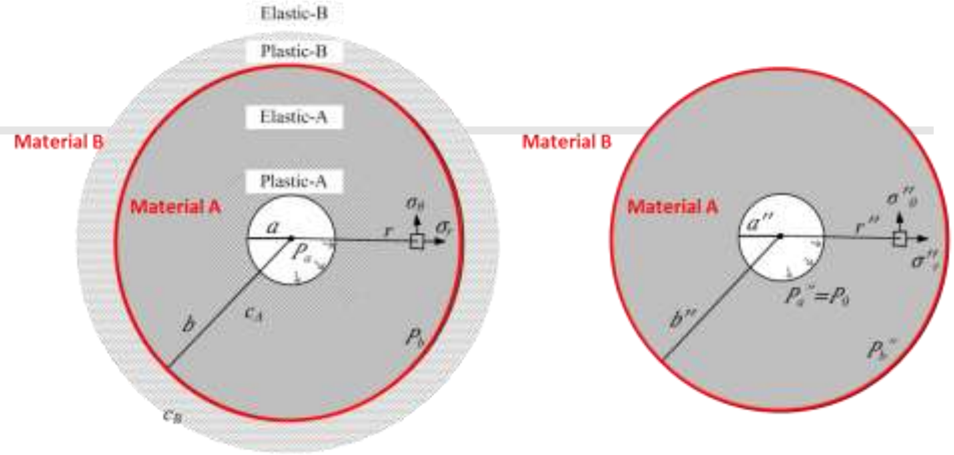
$f(P_b, r)$?

An analytical approach



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$$\alpha\sigma_r - \sigma_\theta = Y$$

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$$\alpha_B \sigma_{r,B} - \sigma_{\theta,B} = Y_B \qquad \sim f(P_b, r)$$

- **When unloaded:**

$$\alpha_A \sigma''_{r,A} - \sigma''_{\theta,A} = Y_A \xrightarrow{r=a} P_{a,sA}$$

$$\alpha_B \sigma''_{r,B} - \sigma''_{\theta,B} = Y_B \xrightarrow{r=b} P_{a,sB}$$

$$P_{a,s} = min\{P_{a,sA}, P_{a,sB}\}$$

in which:

$$\sigma''_{r,A} = \sigma_{r,A} + \Delta\sigma_{r,A}$$

$$\sigma''_{\theta,A} = \sigma_{\theta,A} + \Delta\sigma_{\theta,A}$$

$$f(P_a, r) \quad f(P_a, r) \quad f(P_a)$$

Assume $P''_b = P_0$

$$\sigma''_{r,B} = \sigma_{r,B} + \Delta\sigma_{r,B}$$

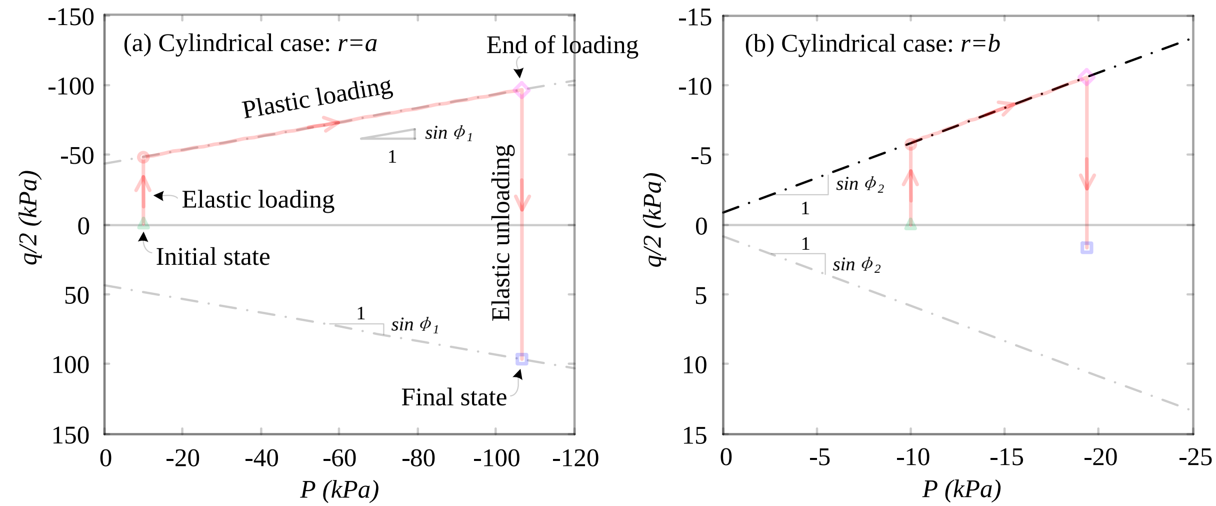
$$\sigma''_{\theta,B} = \sigma_{\theta,B} + \Delta\sigma_{\theta,B}$$

$$f(P_a, P_b, r) \quad f(P_b, r) \quad f(P_a, r)$$

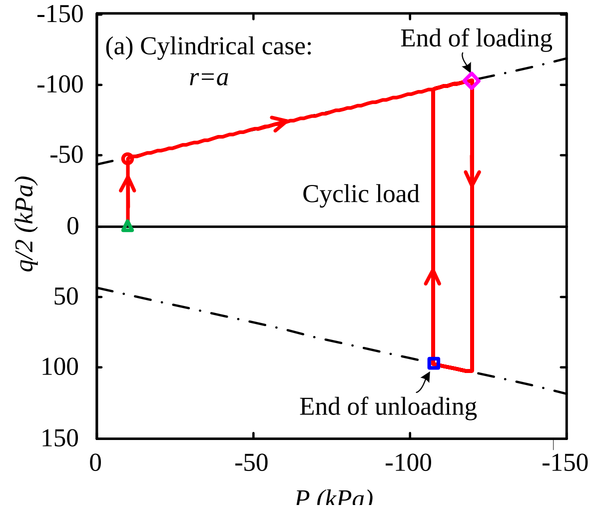
Results – numerical validation (cylindrical case)

$$p = \frac{\sigma_r + \sigma_\theta}{2};\ q = (\sigma_r - \sigma_\theta)$$

$P_a = 20.32 P_0$



$P_a = 20.32 P_0 \times 1.1$

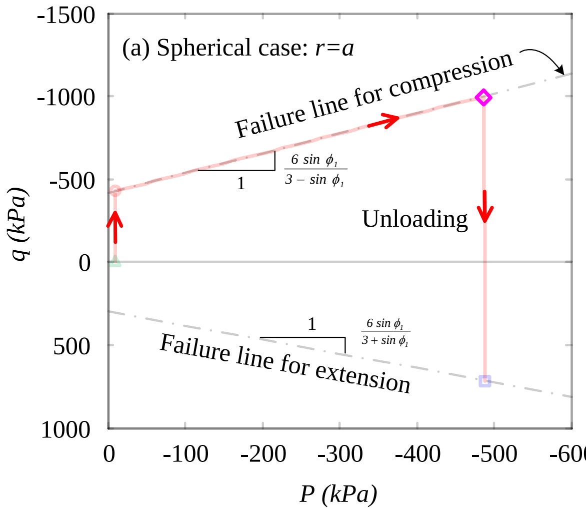


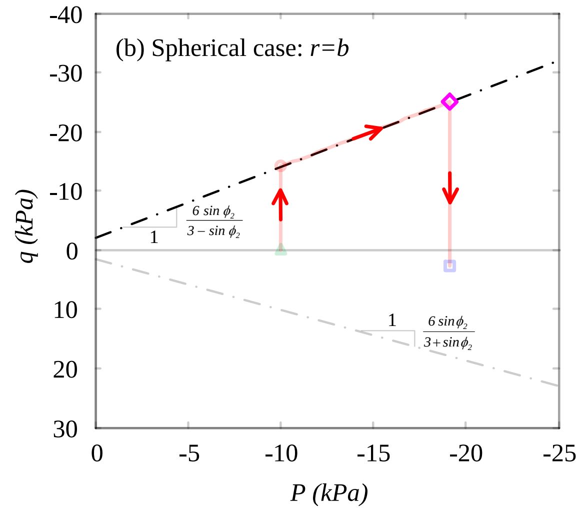
$b/a = 4$
$C_A/C_B = 50$
$\phi_A = \phi_B = 30°$
$E_A = E_B = 100\ MPa$
$\nu_A = \nu_B = 0.2$
$P_0 = 10 kPa$

$\boldsymbol{P_{a,s}/P_0 = 20.32}$

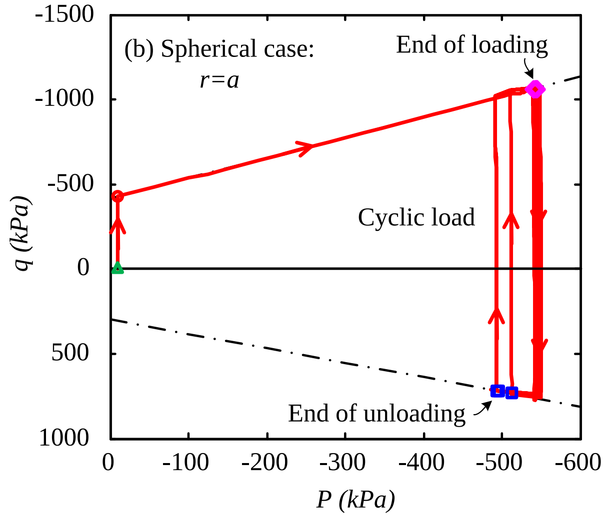
Results – numerical validation (spherical case)

$$p = \frac{\sigma_r + 2\sigma_\theta}{3};\ q = (\sigma_r - \sigma_\theta)$$





$P_a = 115.05 P_0$



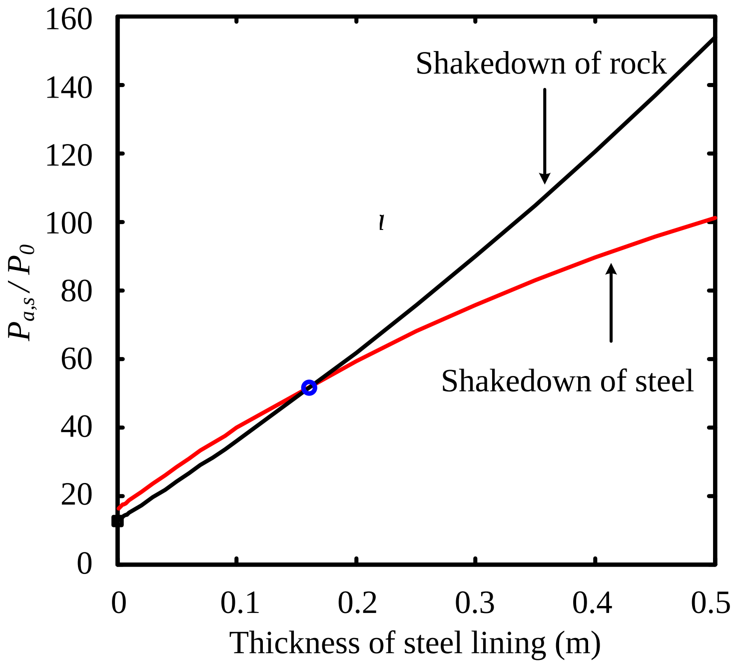
$P_a = 115.05 P_0 \times 1.1$

$b/a = 4$
$C_A/C_B = 200$
$\phi_A = \phi_B = 30°$
$E_A = E_B = 100\ MPa$
$\nu_A = \nu_B = 0.2$
$P_0 = 10kPa$

$\boldsymbol{P_{a,s}/P_0 = 115.05}$

Results – Lined rock carven

- P_0 = 2.5MPa (around 100m depth)
- Rock friction angle ϕ = 37°
- Rock unconfined compression strength σ_c = 10MPa
- Rock cohesion = $\sigma_c(1 - sin\phi)/(2cos\phi)$ = $\sigma_c/4$
- Rock tensile strength = $\sigma_c/3$
- Rock Young's Modulus = 10GPa
- Rock Poisson's ratio = 0.3
- Steel yield strength = 355MPa
- Steel Young's Modulus = 198GPA
- Steel Poisson's ratio = 0.32
- Cavern radius = 2m



Shakedown limit for a spherical rock cavern with steel lining

Concluding remarks

- Analytical solutions for shakedown conditions of a cavity in two layers of cohesive-frictional materials are developed.
- It has been proved by numerical simulations that the proposed analytical solutions provide admissible cyclic loading conditions for the cavity to avoid reverse yielding.
- The shakedown limit pressure of underground lined rock caverns for the CAES systems, could then be determined by using the developed solutions, which can avoid low-cycle fatigue.
- Further work will be conducted considering concrete lining and better estimation of P_b''.

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Thank You