Contact Theory

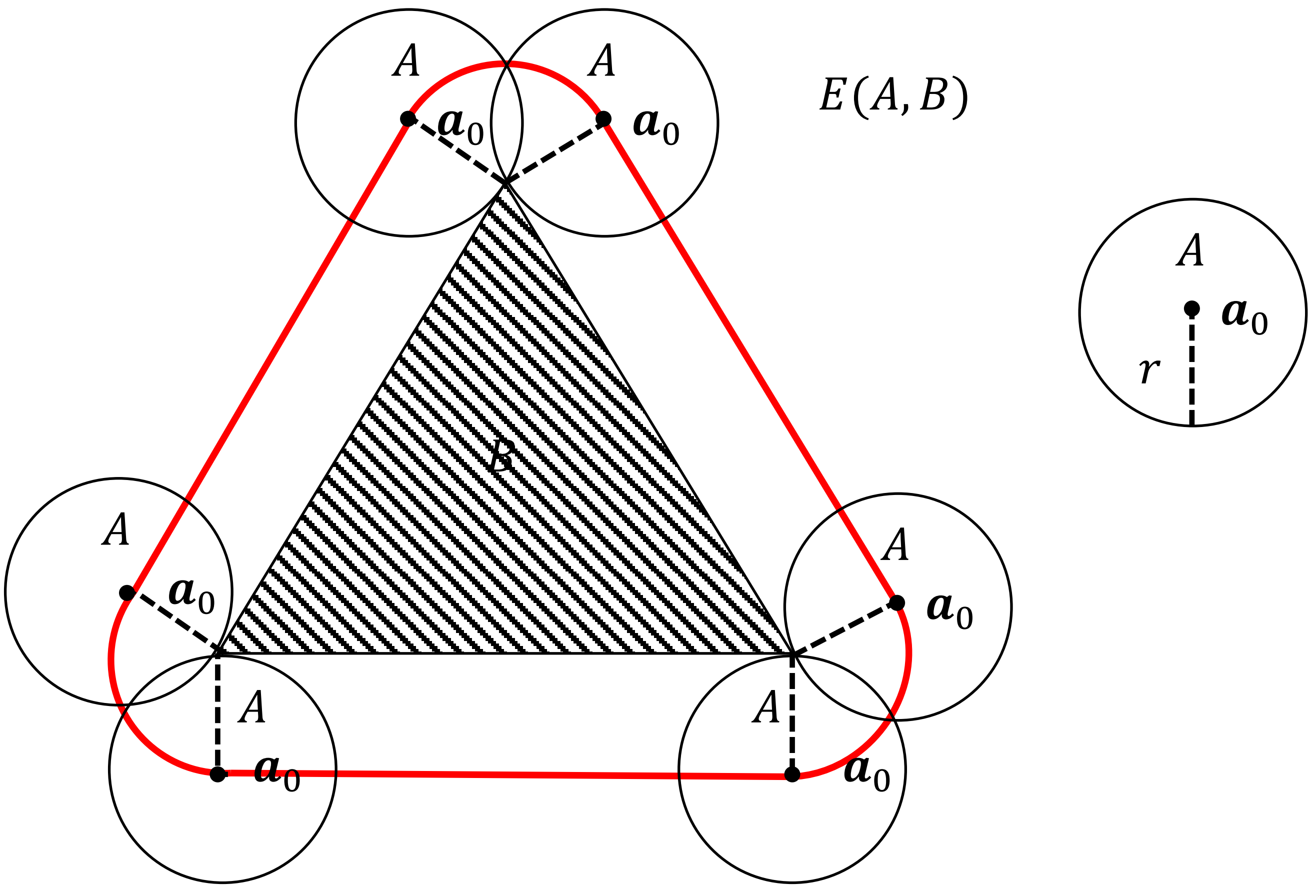
Two Blocks

A and B

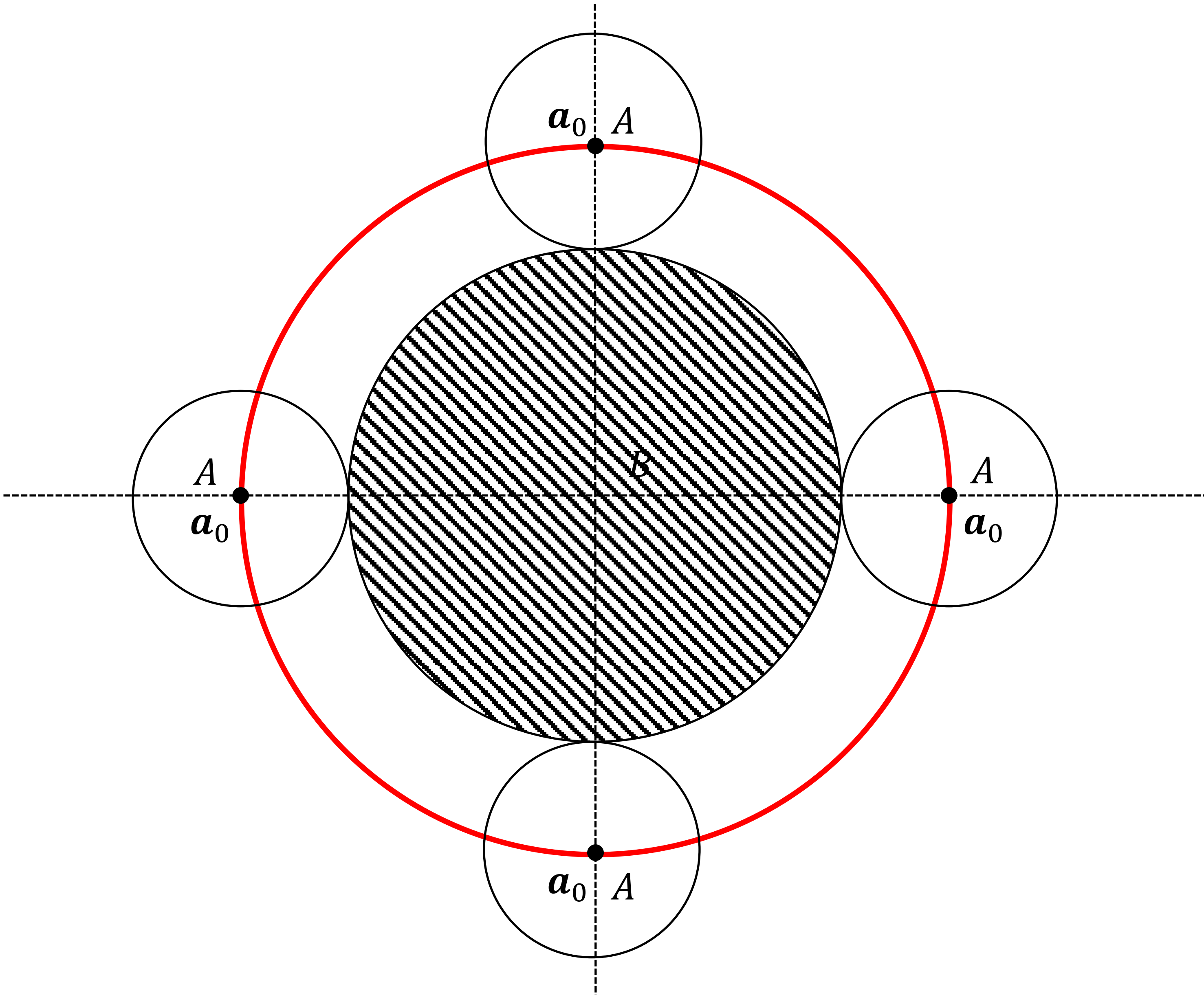
$\Rightarrow$

One point $\boldsymbol{a_0}$

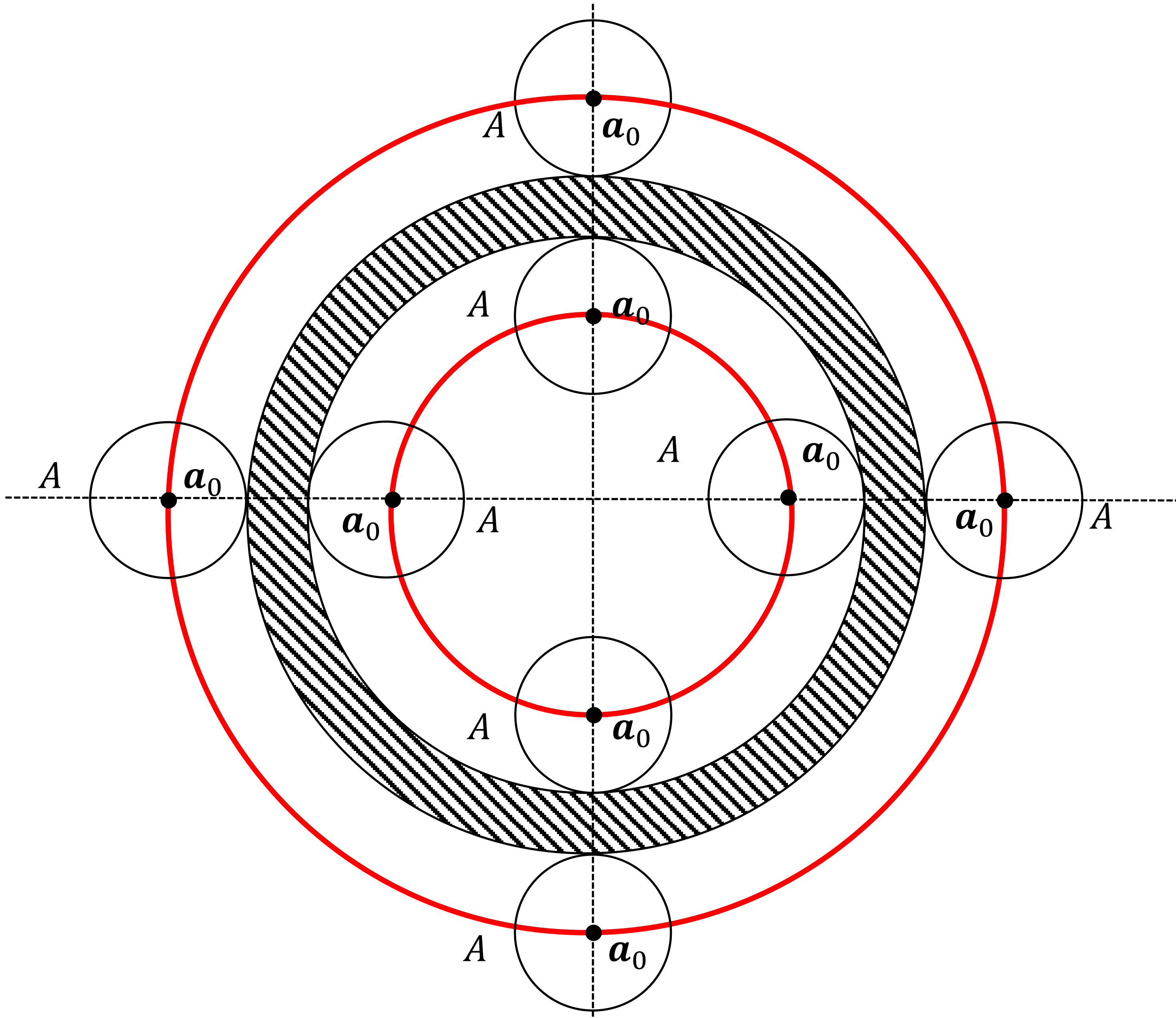
one block $E(A,B)$



$E(A, B)$
A
$\boldsymbol{a}_0$
B
r

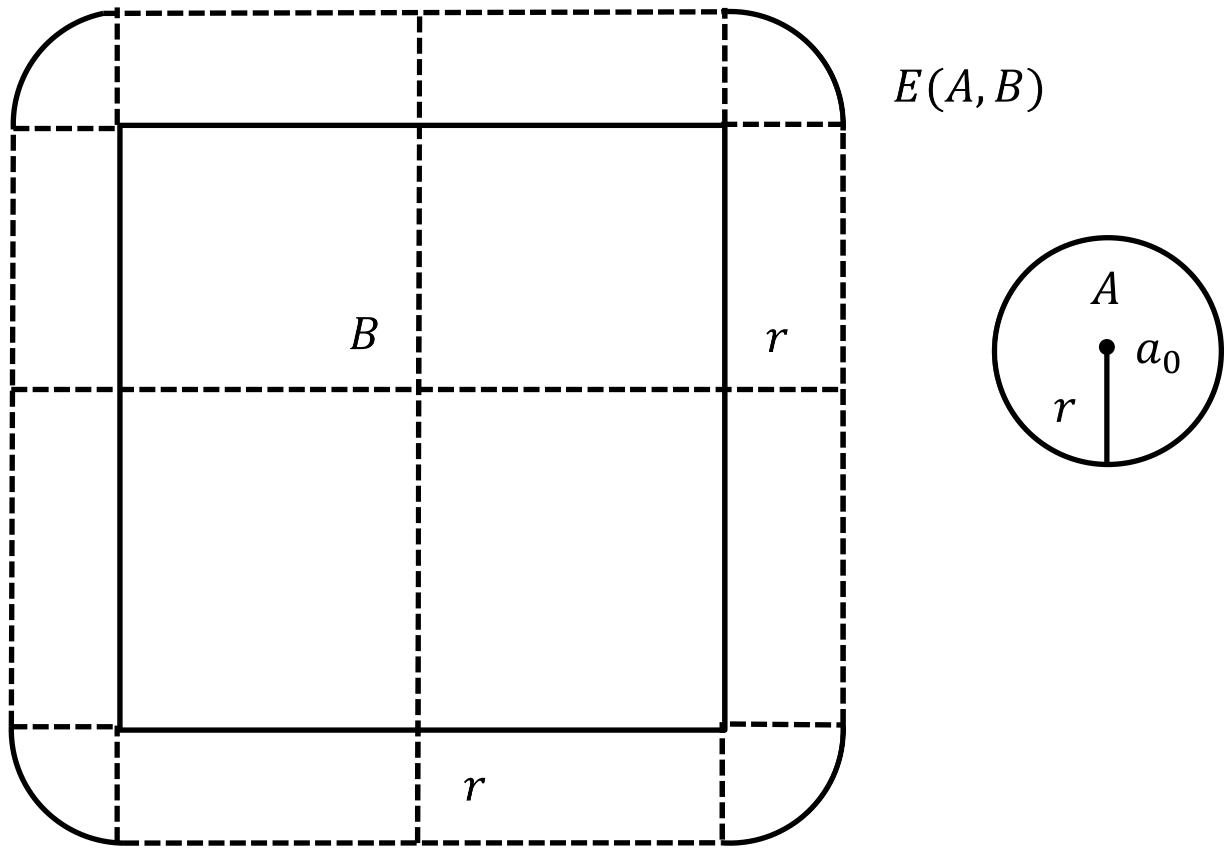


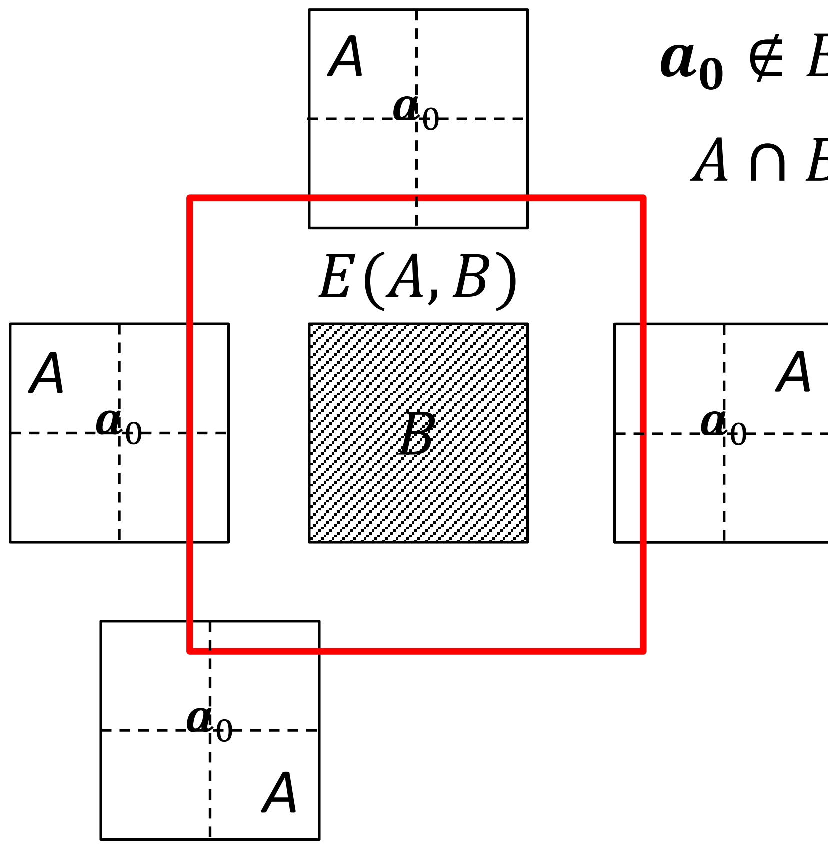
a_0
A
A
a_0
B
A
a_0
a_0
A



A
a_0
A
a_0
A
a_0
A
a_0
A
a_0
A
a_0
A
a_0
A
a_0
A

Entrance Block and Distance

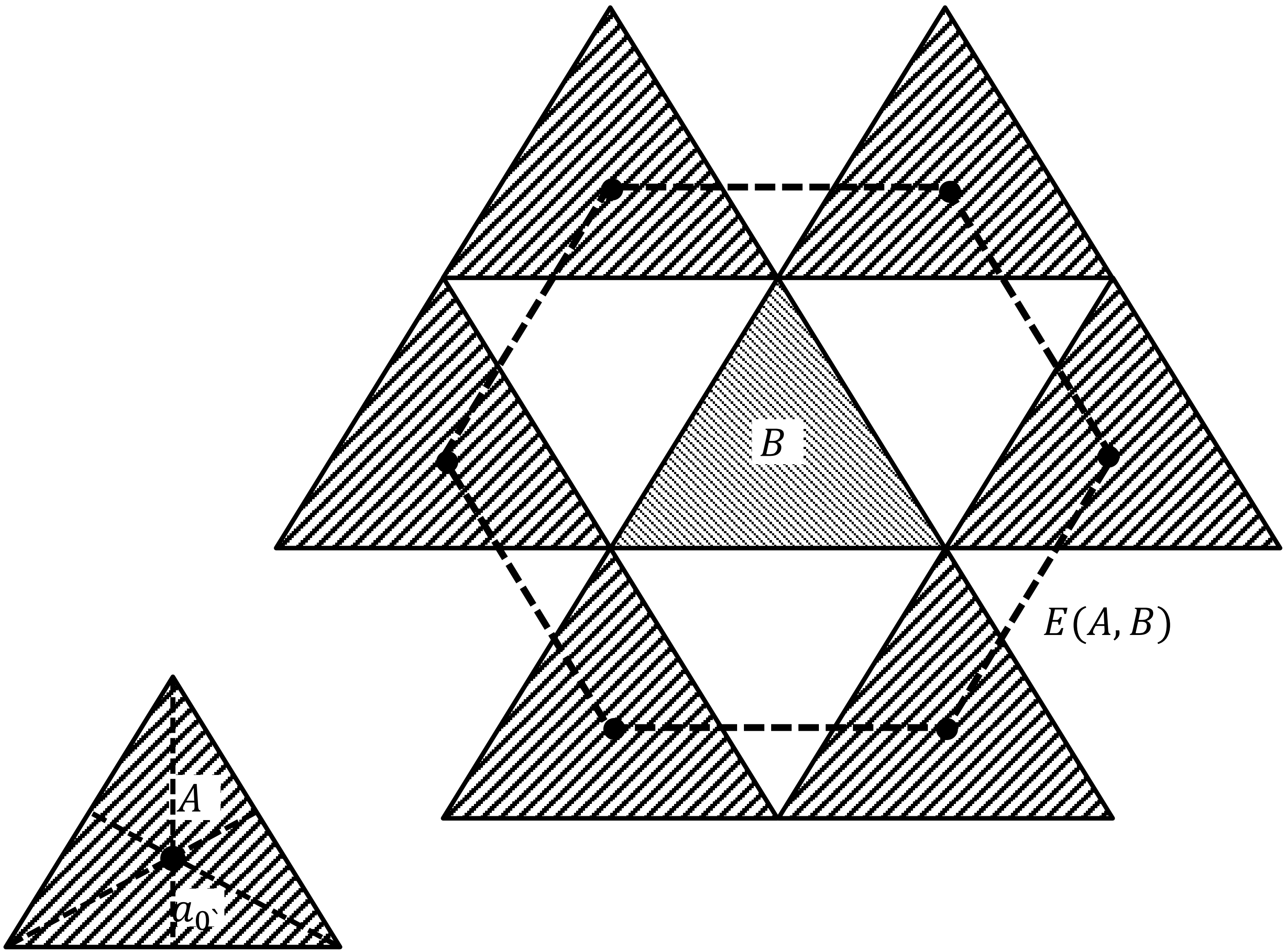




$$\boldsymbol{a_0} \notin E(A,B)$$

$$A \cap B = \phi$$

2D Convex Entrance Block

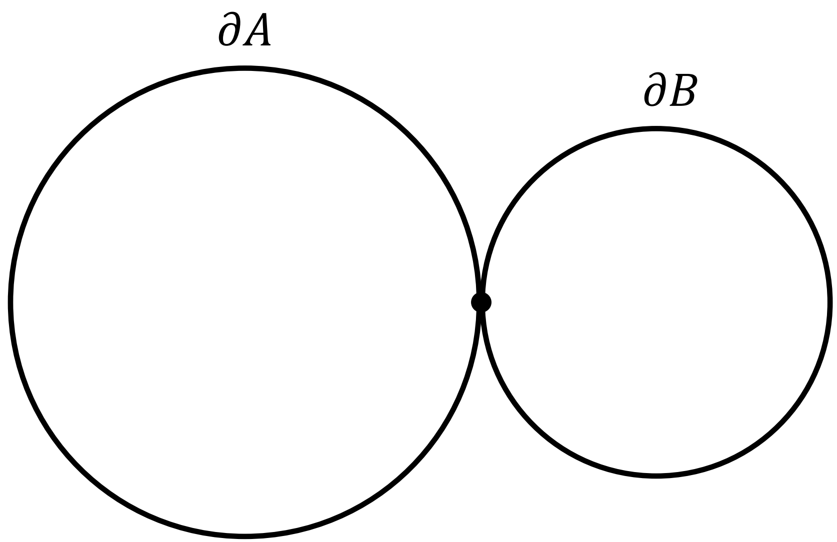


Definition
of
entrance block

Definition of contact

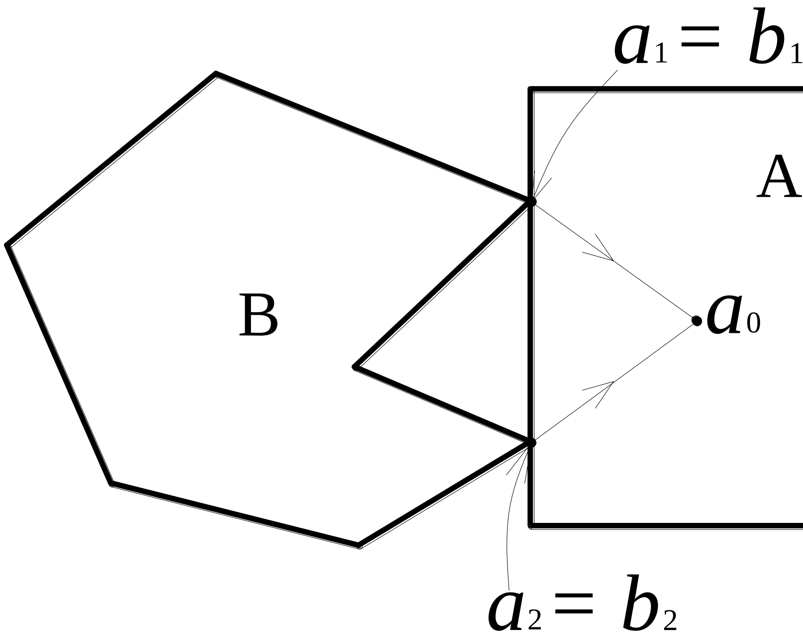
$$(A \cap B) \subset (\partial A \cap \partial B) \neq \emptyset$$

Contact condition



$A \cap B \subset \partial A$

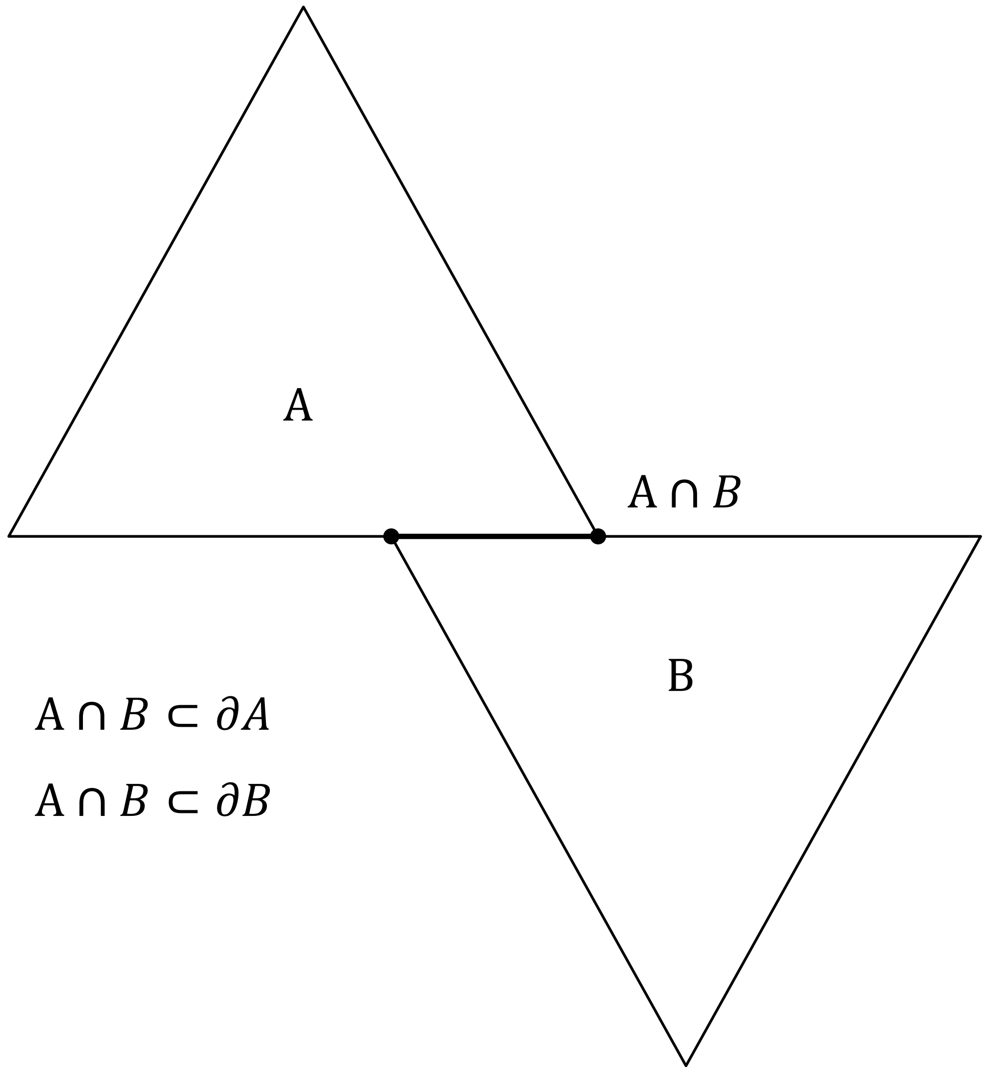
$A \cap B \subset \partial B$



$$E(a_1, b_1) = b_1 + (a_0 - a_1) = a_0$$

$$E(a_2, b_2) = b_2 + (a_0 - a_2) = a_0$$

Contact Condition

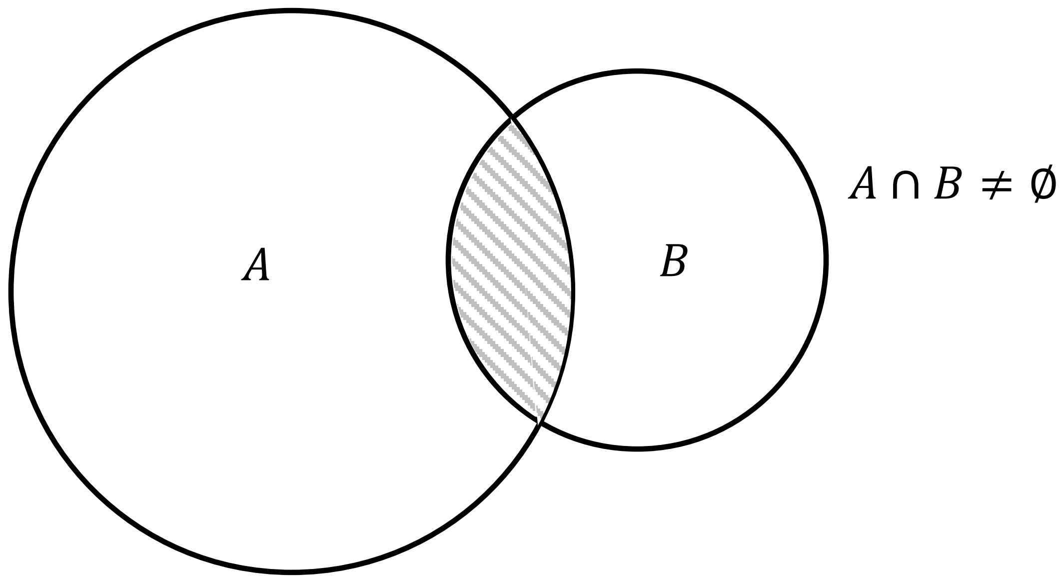


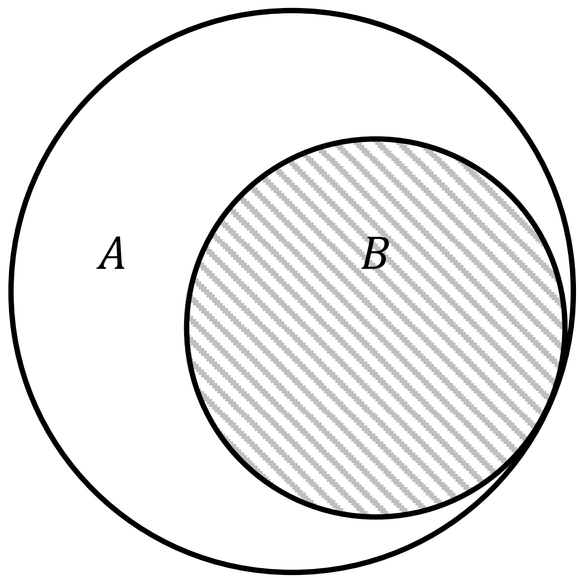
$$\mathrm{A} \cap B \subset \partial A$$

$$\mathrm{A} \cap B \subset \partial B$$

Definition of penetration

$$int(A \cap B) \neq \emptyset$$


A
B
A ∩ B ≠ ∅



$A \cap B \neq \emptyset$

Definition of entrance

$$A \cap B \neq \emptyset$$

$$\Leftrightarrow$$

$$int(A \cap B) \neq \emptyset$$

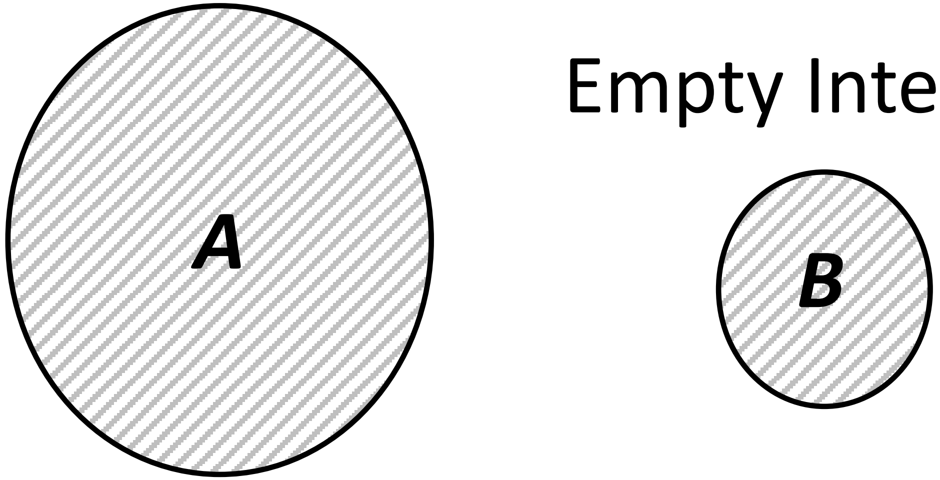
or

$$(A \cap B) \subset (\partial A \cap \partial B) \neq \emptyset$$

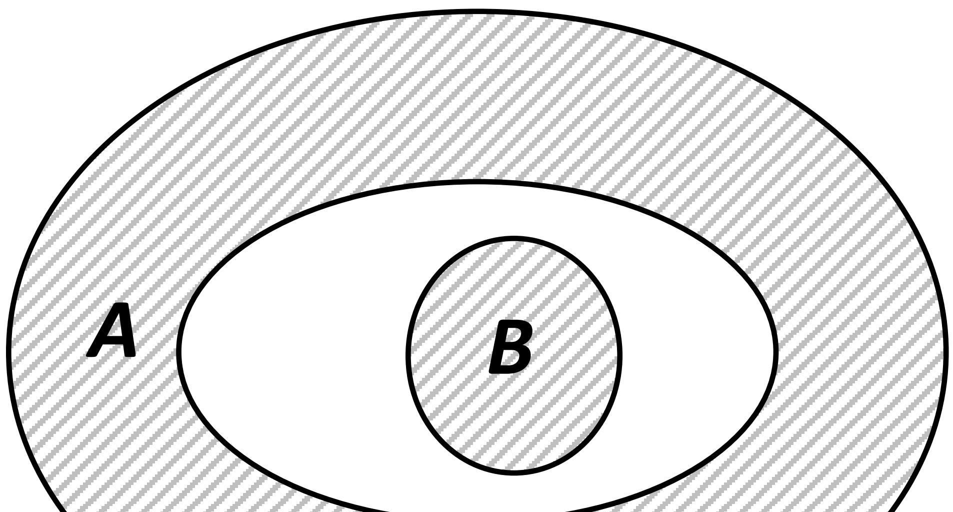
Definition of separation

$$A \cap B = \emptyset$$

Empty Intersection



$$A \cap B = \emptyset$$



Geometric definition Of Entrance block

$$E(A,B) = \{\boldsymbol{a}_0 + \boldsymbol{x} \mid (A + \boldsymbol{x}) \cap B \neq \emptyset\}$$

$$E(A,B) = \bigcup_{(A+x)\cap B \neq \emptyset} (\boldsymbol{x} + \boldsymbol{a}_0)$$

Algebraic Definition of Entrance block

$$E(A, B) = B - A + \boldsymbol{a_0}$$

$$E(A,B) = \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} (\boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a_0})$$

$$E(A,B) = \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} E(\boldsymbol{a}, \boldsymbol{b})$$

$$(A + \boldsymbol{x}) \cap B \neq \emptyset \iff$$

$$\exists\ \boldsymbol{a} \in A, \qquad \exists \boldsymbol{b} \in B, \qquad \boldsymbol{b} = \boldsymbol{a} + \boldsymbol{x} \iff$$

$$\boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a}_0 = \boldsymbol{x} + \boldsymbol{a}_0 \iff$$

$$\bigcup_{(A + x) \cap B \neq \emptyset} (\boldsymbol{x} + \boldsymbol{a}_0)$$

$$= \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} (\boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a}_0)$$

$$E(A, B) = B - A + \boldsymbol{a_0}$$

$$\forall\, \boldsymbol{a} = \boldsymbol{b}$$

$$\boldsymbol{a} \in A$$

$$\boldsymbol{b} \in B$$

$$\Longrightarrow$$

$$E(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a_0} = \boldsymbol{a_0}.$$

Minkowski sum (1910)

$$A + B = \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} (\boldsymbol{a} + \boldsymbol{b})$$

Define

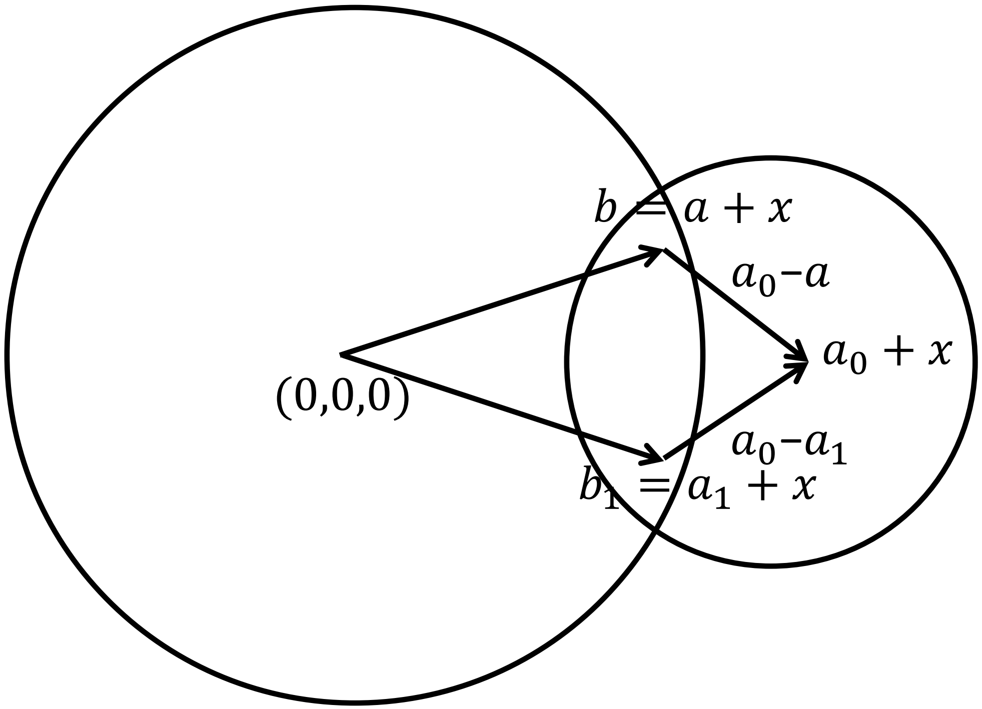
$$-B = \bigcup_{\boldsymbol{b} \in B} (-\boldsymbol{b})$$

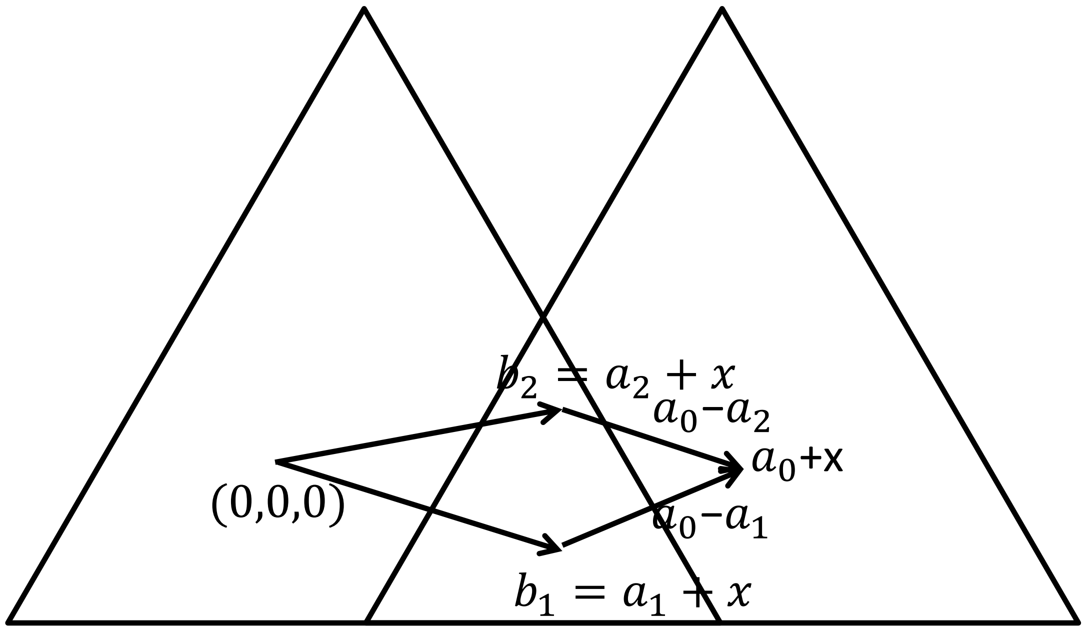
$$A - B = A + (-B)$$

$$A + B = \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} (\boldsymbol{a} + \boldsymbol{b})$$

$$A - B = \bigcup_{\boldsymbol{a} \in A, \boldsymbol{b} \in B} (\boldsymbol{a} - \boldsymbol{b})$$

Entrance Point




$b_2 = a_2 + x$
$a_0 - a_2$
a_0+x
(0,0,0)
$a_0 - a_1$
$b_1 = a_1 + x$

Examples
of
Entrance Block

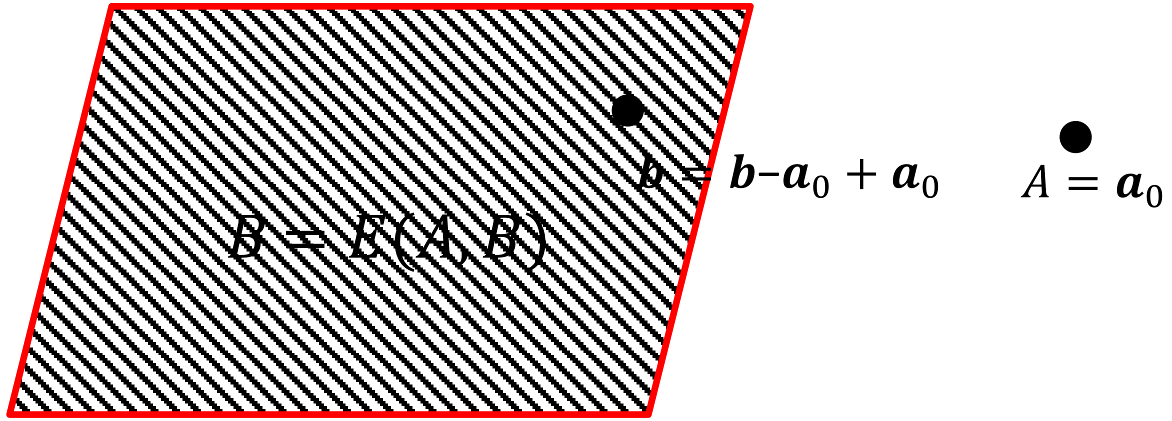
$$A = \boldsymbol{a_0}$$

$$E(A, B)$$

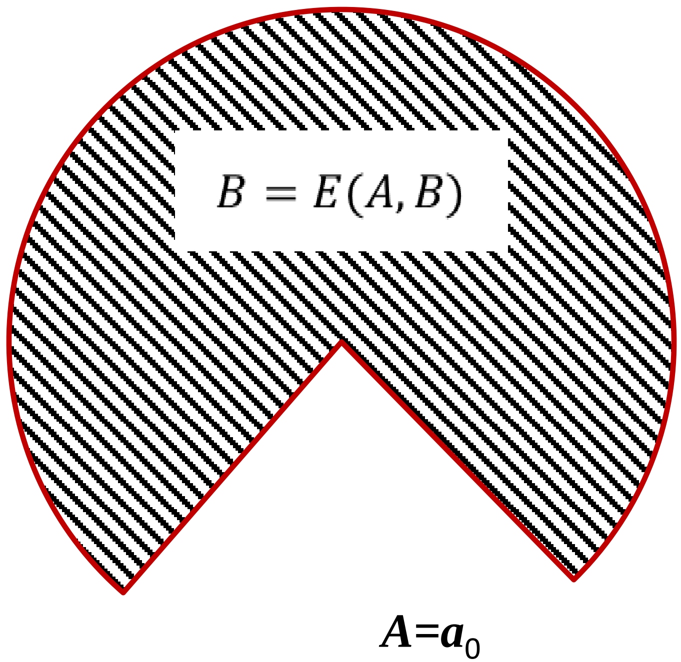
$$= B - A + \boldsymbol{a_0}$$

$$= B - \boldsymbol{a_0} + \boldsymbol{a_0}$$

$$= B$$

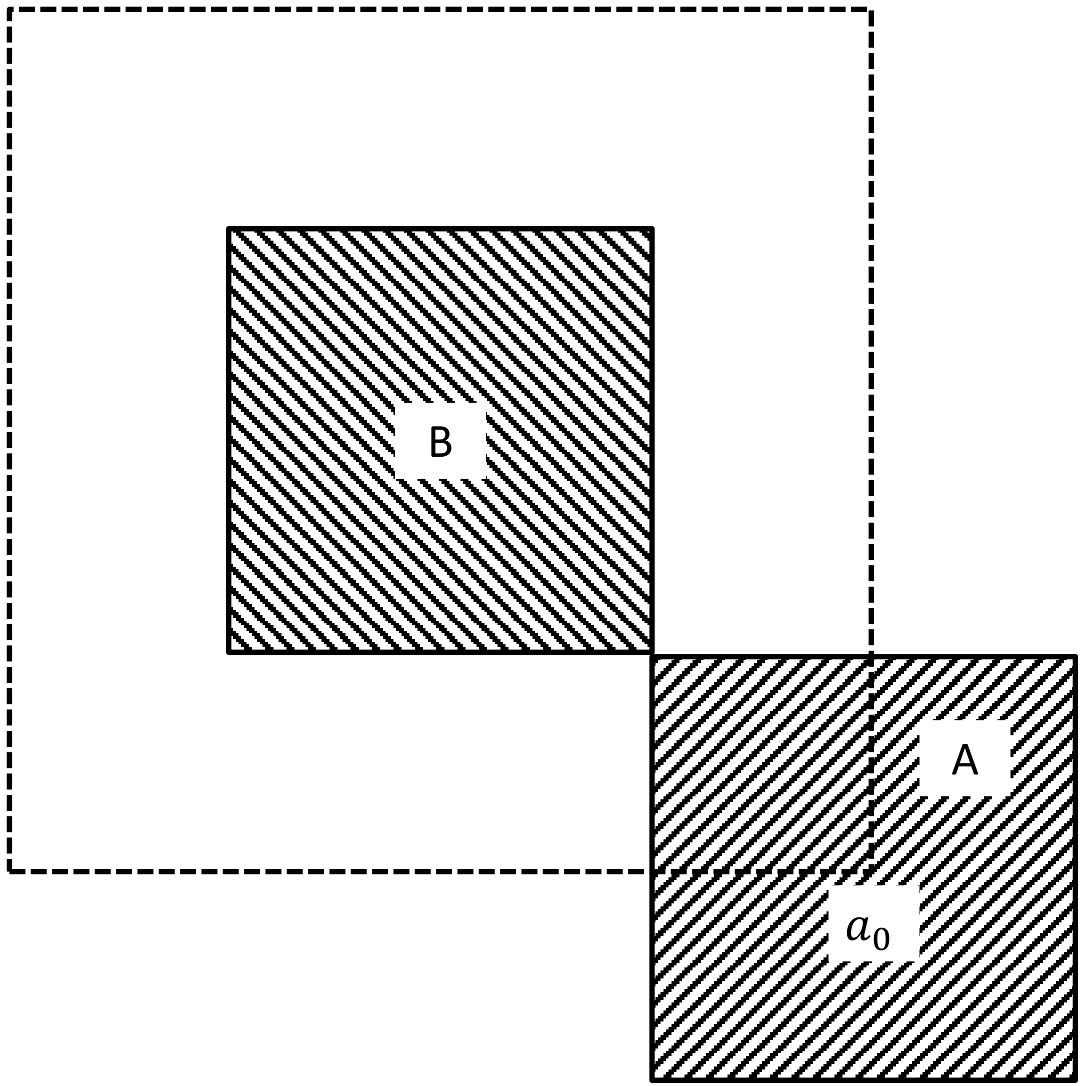


$B = E(A, B)$
$\boldsymbol{b} = \boldsymbol{b} - \boldsymbol{a}_0 + \boldsymbol{a}_0$
$A = \boldsymbol{a}_0$

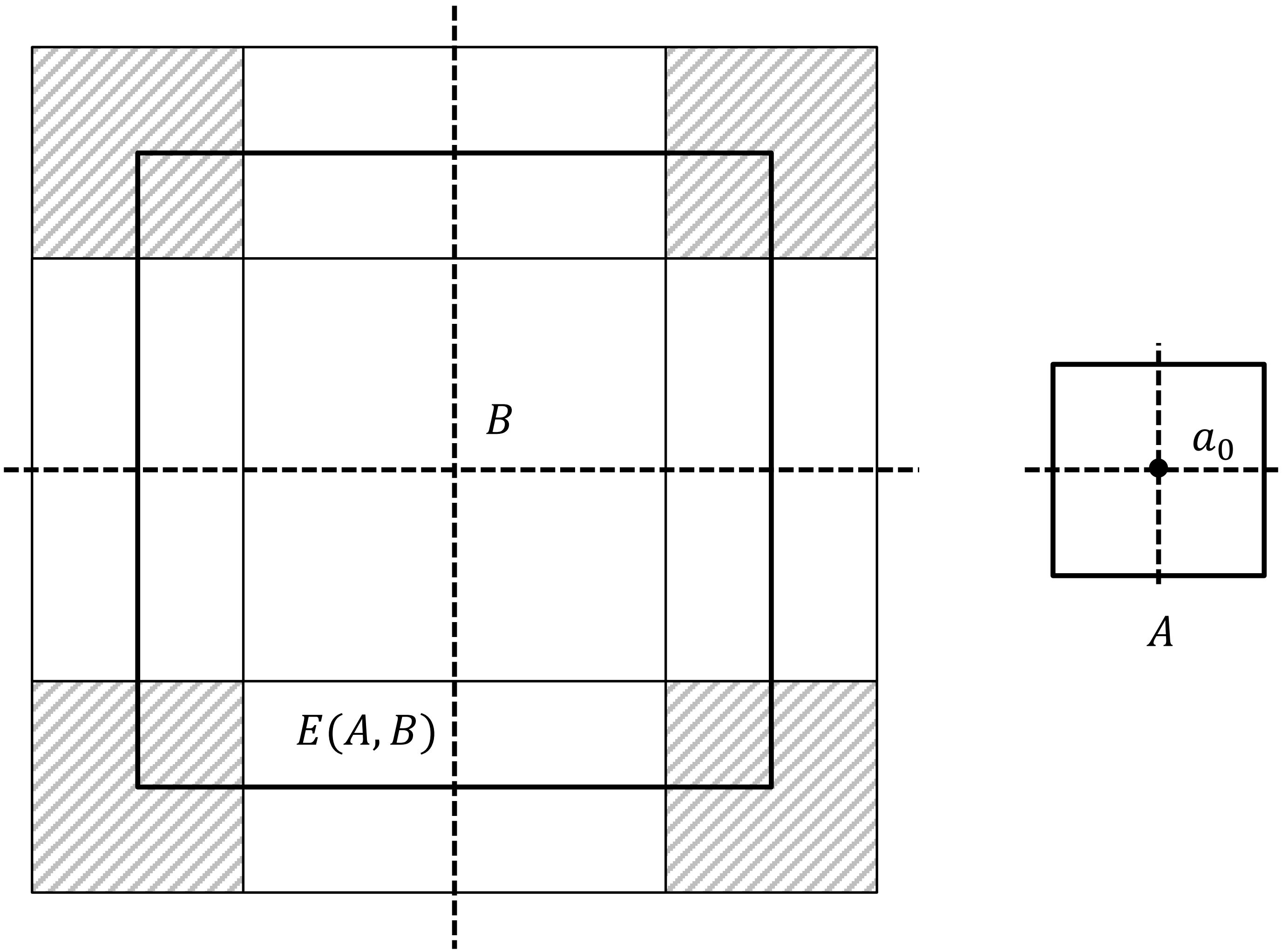


B = E(A, B)
A=a0

Entrance Block



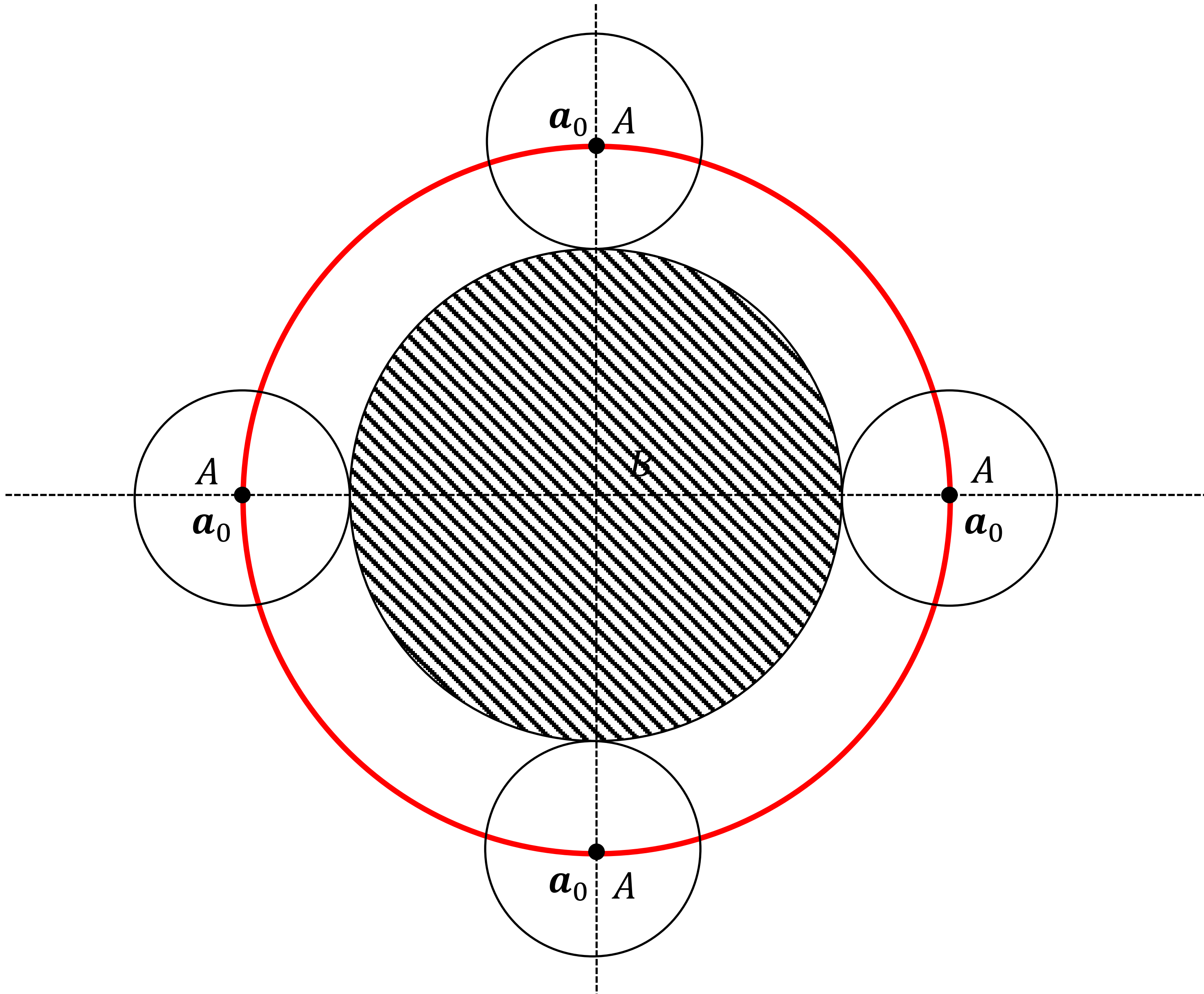
2D Convex Entrance Block



$$A = \{|\boldsymbol{x} - \boldsymbol{a}_0| \leq r_1\},$$

$$B = \{|\boldsymbol{x} - \boldsymbol{b}_0| \leq r_2\},$$

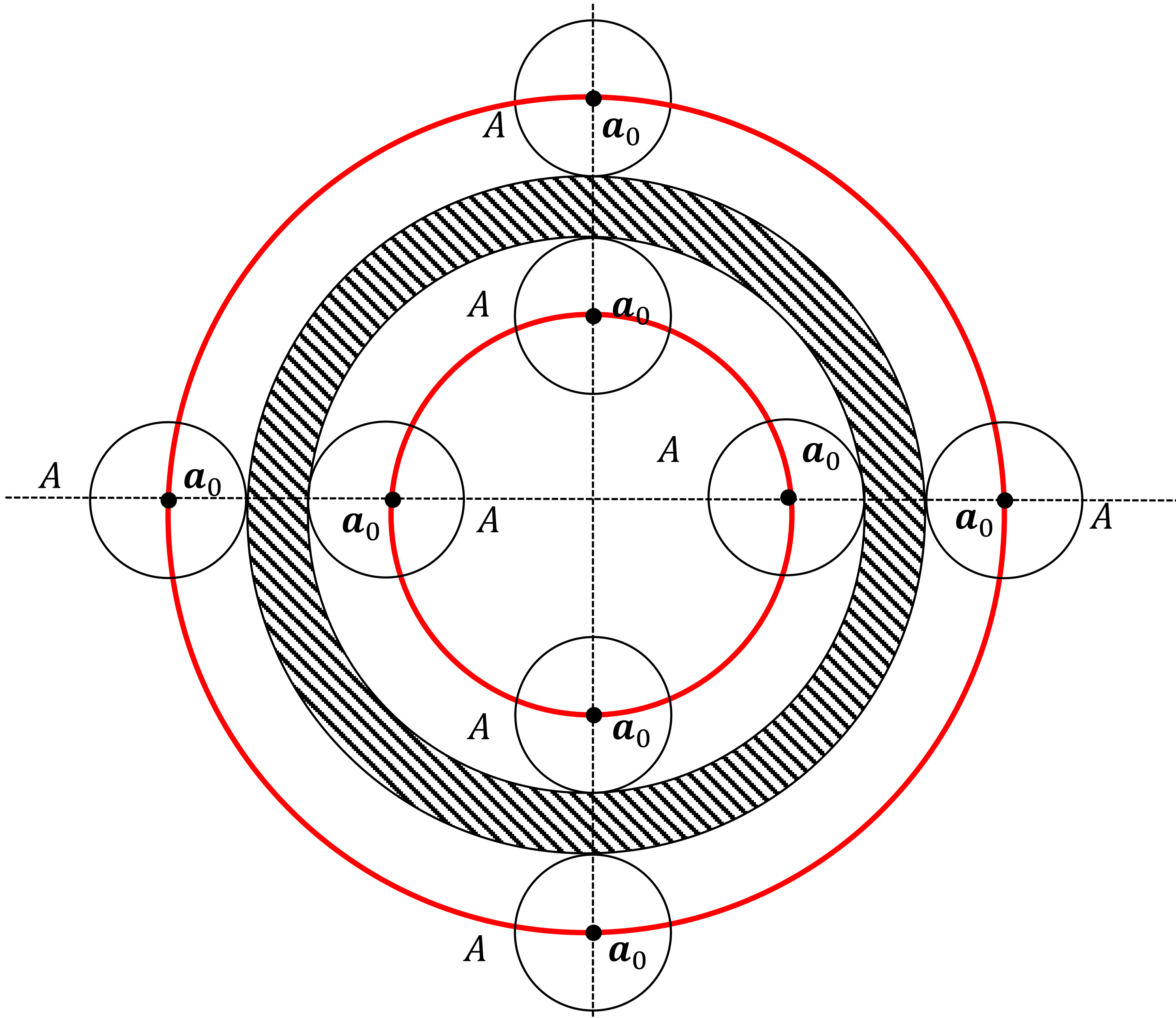
$$C = \{|\boldsymbol{x} - \boldsymbol{b}_0| \leq r_1 + r_2\}.$$



$\boldsymbol{a}_0$
A
A
$\boldsymbol{a}_0$
B
A
$\boldsymbol{a}_0$
$\boldsymbol{a}_0$
A

$$\forall\, \boldsymbol{a} \in A, \boldsymbol{b} \in B, E(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a}_0,$$
$$|E(\boldsymbol{a}, \boldsymbol{b}) - \boldsymbol{b}_0| = |(\boldsymbol{b} - \boldsymbol{b}_0) - (\boldsymbol{a} - \boldsymbol{a}_0)|$$
$$\leq |\boldsymbol{b} - \boldsymbol{b}_0| + |\boldsymbol{a} - \boldsymbol{a}_0| \leq r_2 + r_1$$
$$\Rightarrow E(A, B) \subset C.$$

$$|\boldsymbol{c} - \boldsymbol{b}_0| \leq (r_1 + r_2),$$
$$\exists\, \boldsymbol{a} \in A, \boldsymbol{b} \in B,$$
$$\boldsymbol{c} - \boldsymbol{b}_0 = \boldsymbol{b} - \boldsymbol{b}_0 - (\boldsymbol{a} - \boldsymbol{a}_0) \Rightarrow$$
$$E(A, B) \supset E(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{b} - \boldsymbol{a} + \boldsymbol{a}_0 = \boldsymbol{c}$$
$$\Rightarrow E(A, B) \supset C.$$



A
a_0
A
a_0
A
a_0
A
a_0
a_0
A
A
a_0
a_0
A
A
a_0
A
a_0

Study
of
Entrance Block

Entrance block and parallel movement

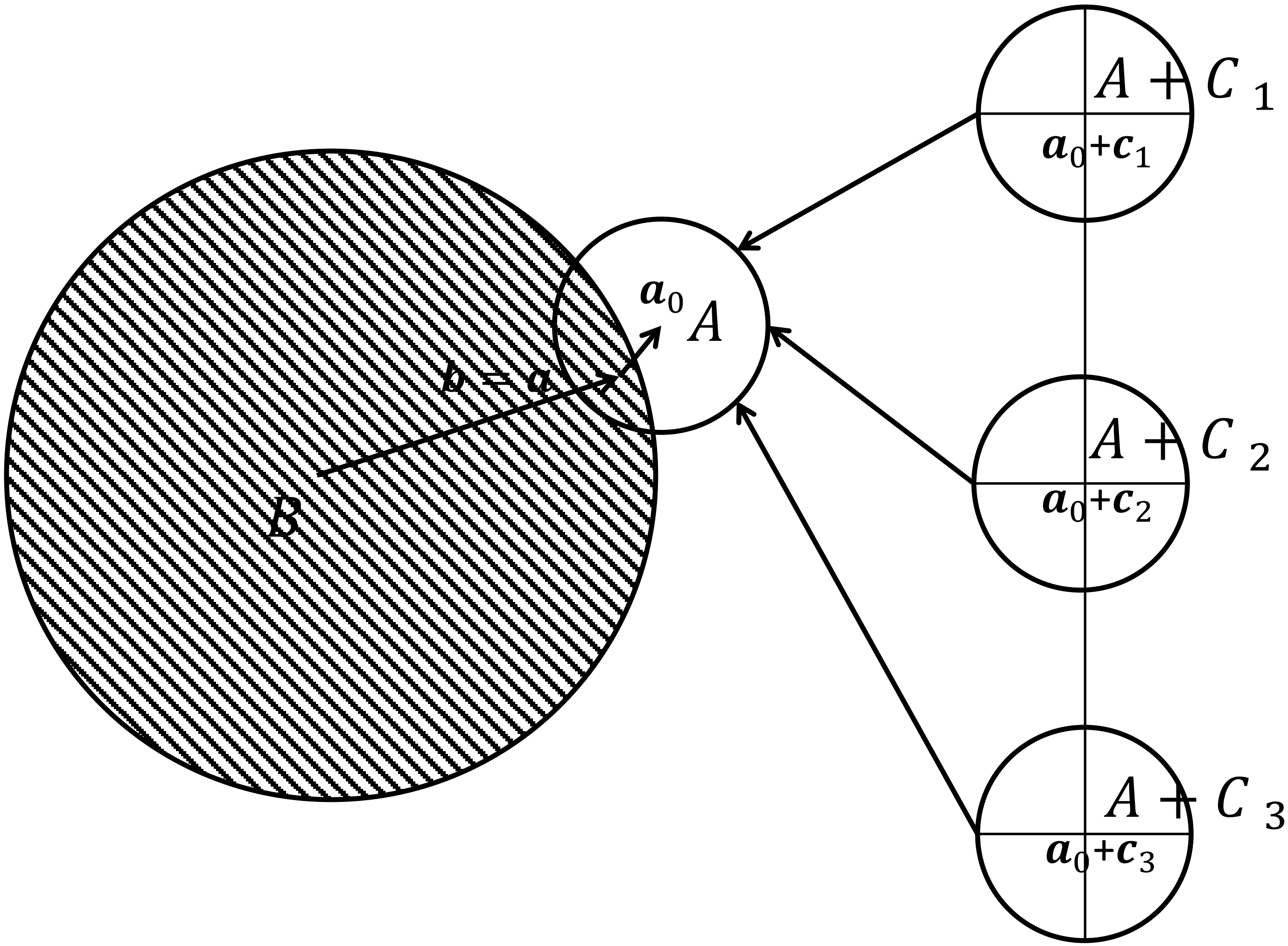
$$E(A + \boldsymbol{x}, B)$$

$$= B - (A + \boldsymbol{x}) + (\boldsymbol{a}_0 + \boldsymbol{x})$$

$$= B - A + \boldsymbol{a}_0$$

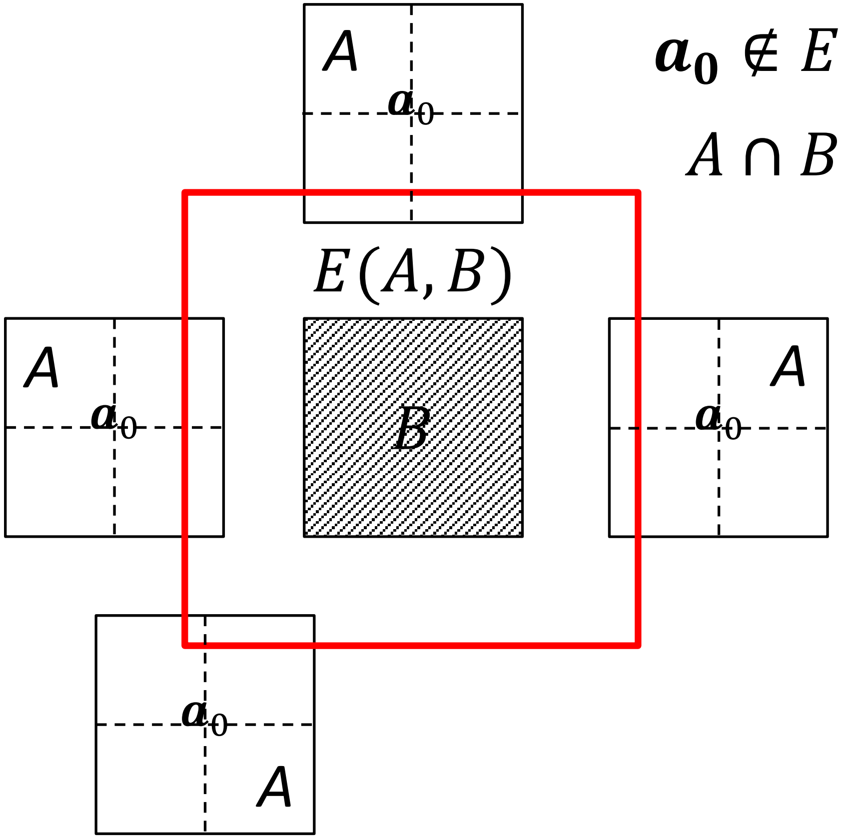
$$= E(A, B),$$

$$E(A + \boldsymbol{x}, B) = E(A, B)$$


A + C 1
a0+c1
a0
A
b − a
B
A + C 2
a0+c2
A + C 3
a0+c3

Entrance block and separation

$$A \cap B = \emptyset \iff$$
$$\boldsymbol{a}_0 \notin E(A, B)$$



$$\boldsymbol{a_0} \notin E(A,B)$$

$$A \cap B = \phi$$

Entrance block and entrance

$$A \cap B \neq \emptyset \iff$$
$$\boldsymbol{a}_0 \in E(A, B)$$

Entrance block and union

$$A = A_1 \cup A_2 \Rightarrow$$
$$E(A_1, B) \cup E(A_2, B) = E(A, B)$$
$$B = B_1 \cup B_2 \Rightarrow$$
$$E(A, B_1) \cup E(A, B_2) = E(A, B)$$

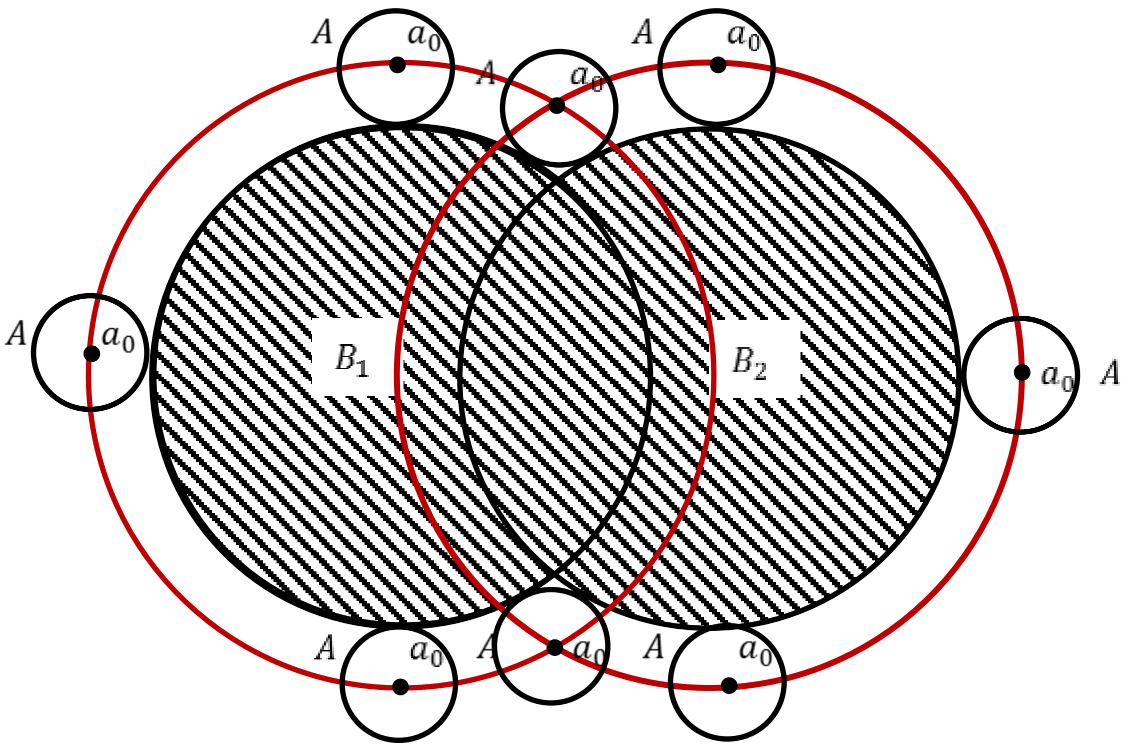
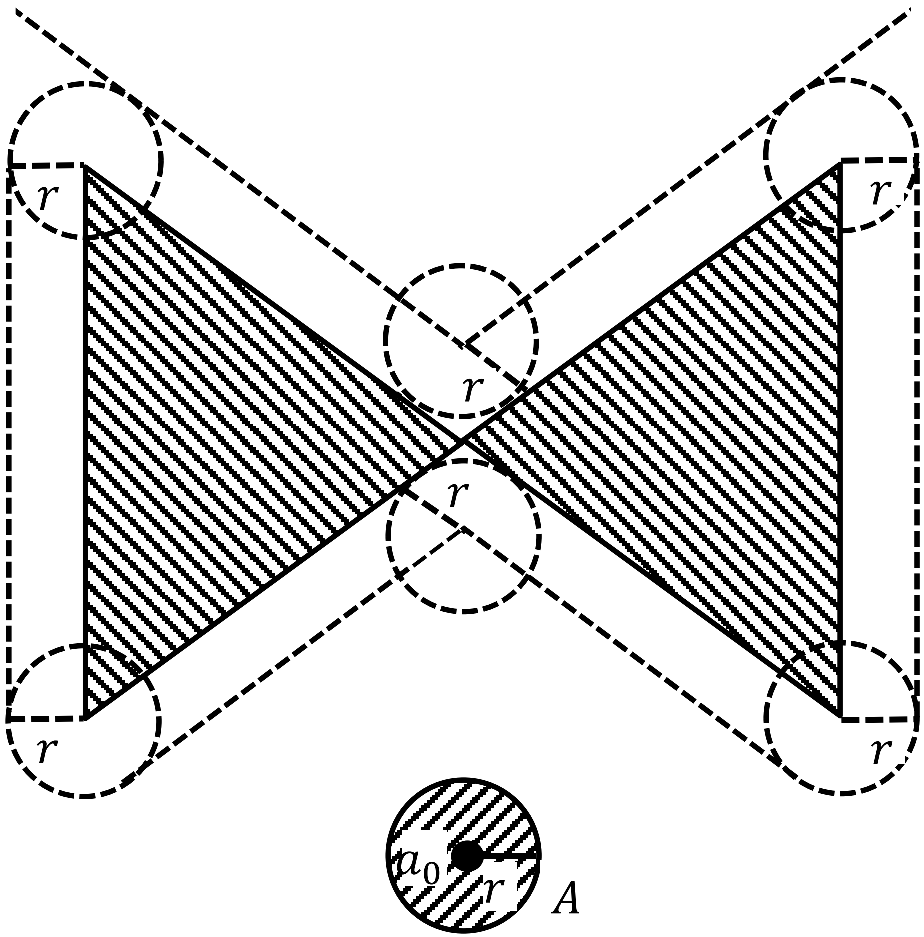
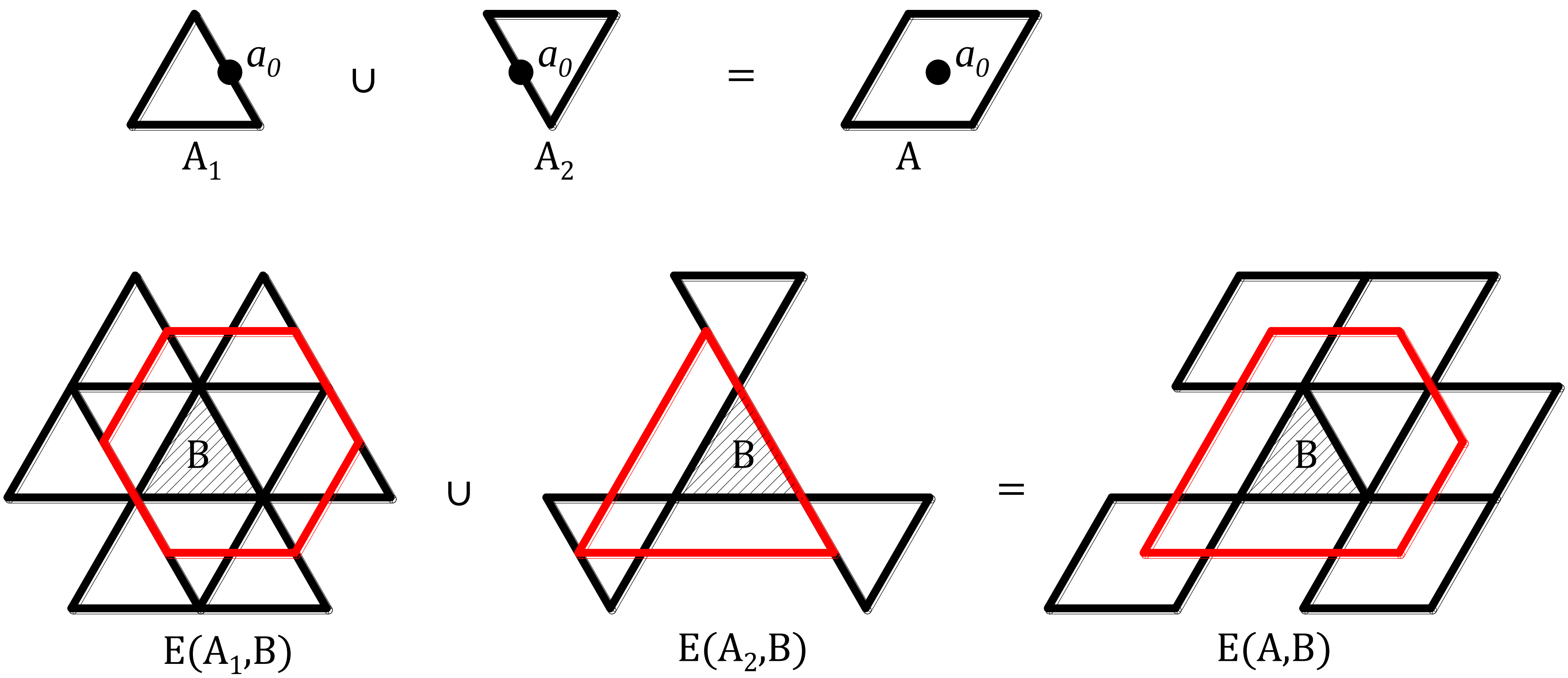


Figure 24a

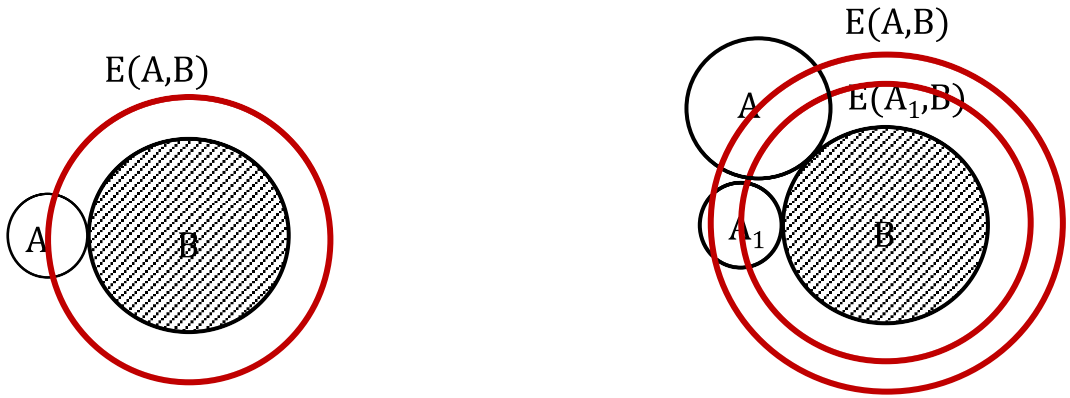
Entrance block of concave block

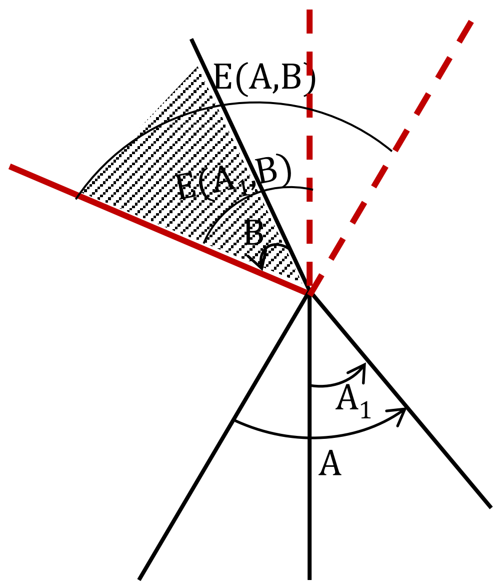



a_0
a_0
a_0
A_1
A_2
A
B
B
B
$E(A_1,B)$
$E(A_2,B)$
$E(A,B)$

Entrance block and including

$$A_1 \subset A \Rightarrow$$
$$E(A_1, B) \subset E(A, B)$$
$$B_1 \subset B \Rightarrow$$
$$E(A, B_1) \subset E(A, B).$$

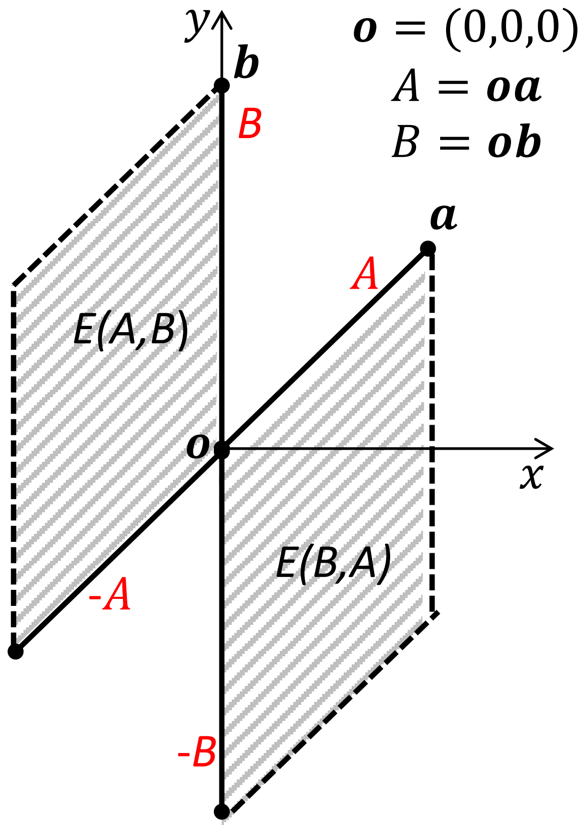

E(A,B)
A
B
E(A,B)
A
E(A1,B)
A1
B

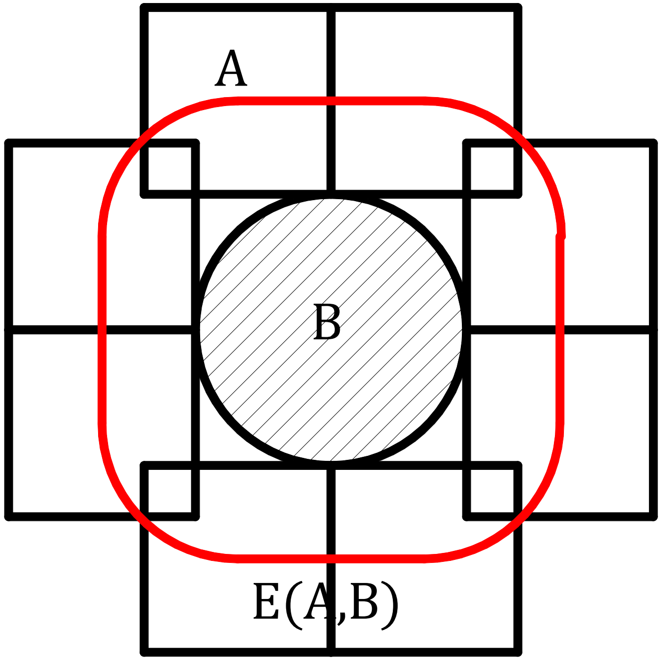

E(A,B)
E(A1,B)
B
A1
A

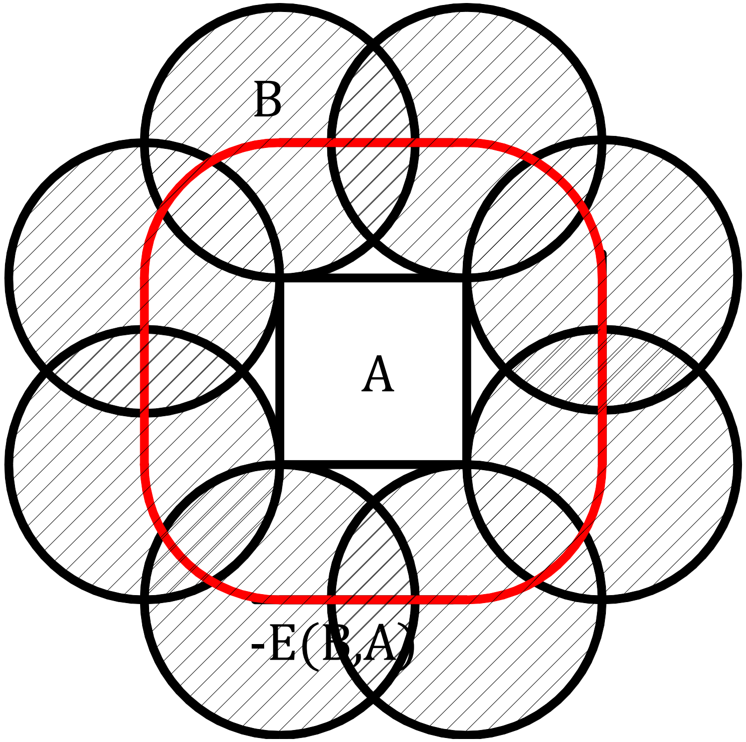
Entrance block and symmetry

$$\boldsymbol{a}_0 = (0, 0, 0)$$
$$\boldsymbol{b}_0 = (0, 0, 0)$$

$$E(A, B) = -E(B, A)$$


y
b
o = (0,0,0)
A = oa
B = ob
B
a
A
E(A,B)
o
x
E(B,A)
-A
-B


A
B
E(A,B)


B
A
-E(B,A)

Entrance block and removability

$$\boldsymbol{a_0} \in \boldsymbol{\partial E(A, B)} \Leftrightarrow$$

$$(A \cap B) \subset (\partial A \cap \partial B) \neq \emptyset$$

$$\forall\, \boldsymbol{\varepsilon} > \boldsymbol{0}, \exists|\boldsymbol{\delta}| < \boldsymbol{\varepsilon}$$

$$(\boldsymbol{A} + \boldsymbol{\delta}) \cap \boldsymbol{B} = \emptyset$$

Entrance block and contact

$$\boldsymbol{a_0} \in \partial E(A, B)$$

$$\Rightarrow$$

$$A \cap B \subset (\partial A \cap \partial B) \neq \emptyset$$

Entrance block and lock

$$(A \cap B) \subset (\partial A \cap \partial B) \neq \emptyset$$

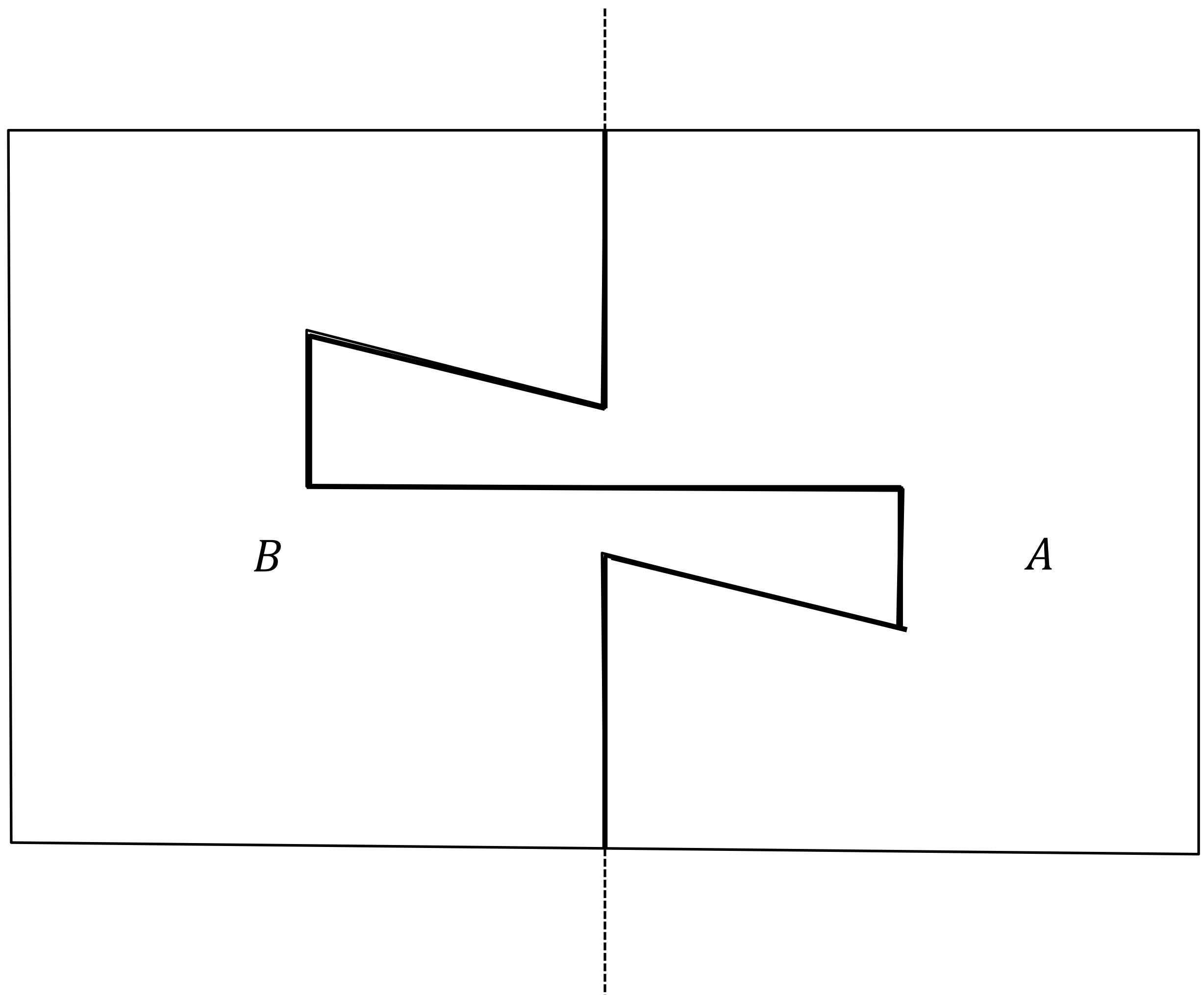
$$\boldsymbol{a}_0 \in int\big(E(A, B)\big) \Leftrightarrow$$

$$(A \cap B) \subset (\partial A \cap \partial B) \neq \emptyset$$

$$\exists\ \varepsilon > 0, \forall |\boldsymbol{\delta}| < \varepsilon,$$

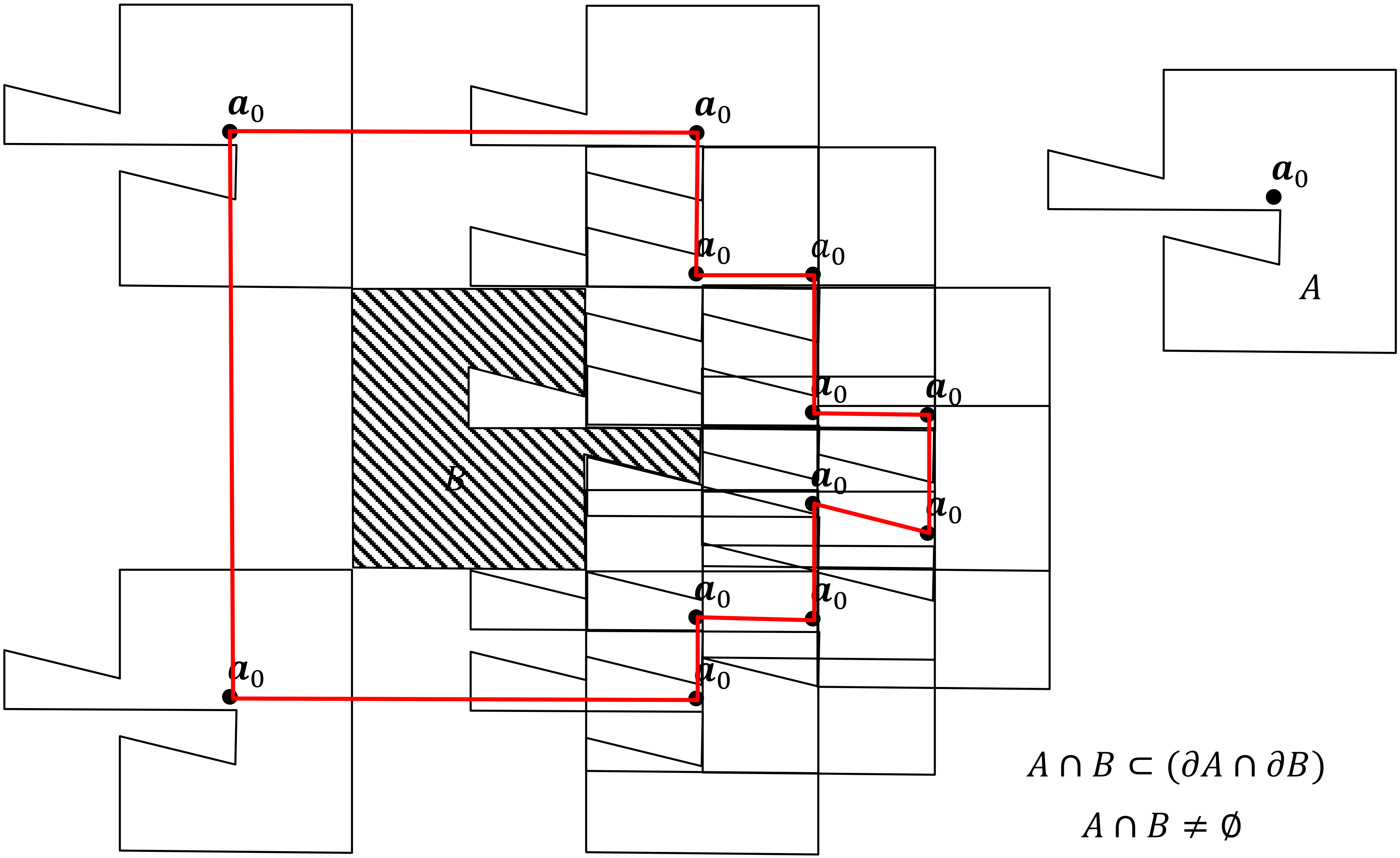
$$(A + \boldsymbol{\delta}) \cap B \neq \emptyset$$

Contact and Locked

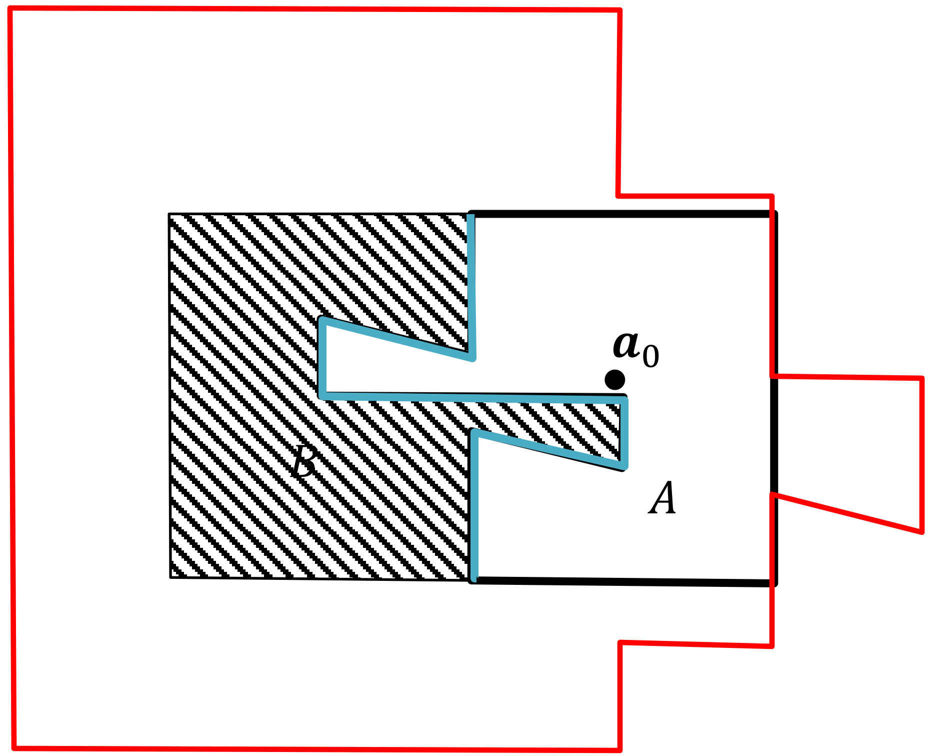


$$A \cap B \subset (\partial A \cap \partial B)$$

$$A \cap B \neq \emptyset$$



a_0
A
B
$A \cap B \subset (\partial A \cap \partial B)$
$A \cap B \neq \emptyset$



$$A \cap B \subset (\partial A \cap \partial B)$$

$$A \cap B \neq \emptyset$$

$$\boldsymbol{a_0} \in int E(A, B)$$

Entrance block and distance

$$A \cap B = \emptyset$$

or

$$(A \cap B) \subset (\partial A \cap \partial B).$$

$$|A, B| =$$
$$min\{|\boldsymbol{b} - \boldsymbol{a}| \setminus \forall\, \boldsymbol{a} \in A,\ \forall\, \boldsymbol{b} \in B\}.$$

$$|A, B| = min\{|\boldsymbol{x}| \setminus (A + \boldsymbol{x}) \cap B \neq\ \emptyset\}.$$

Basic Theorem
of
Entrance Block

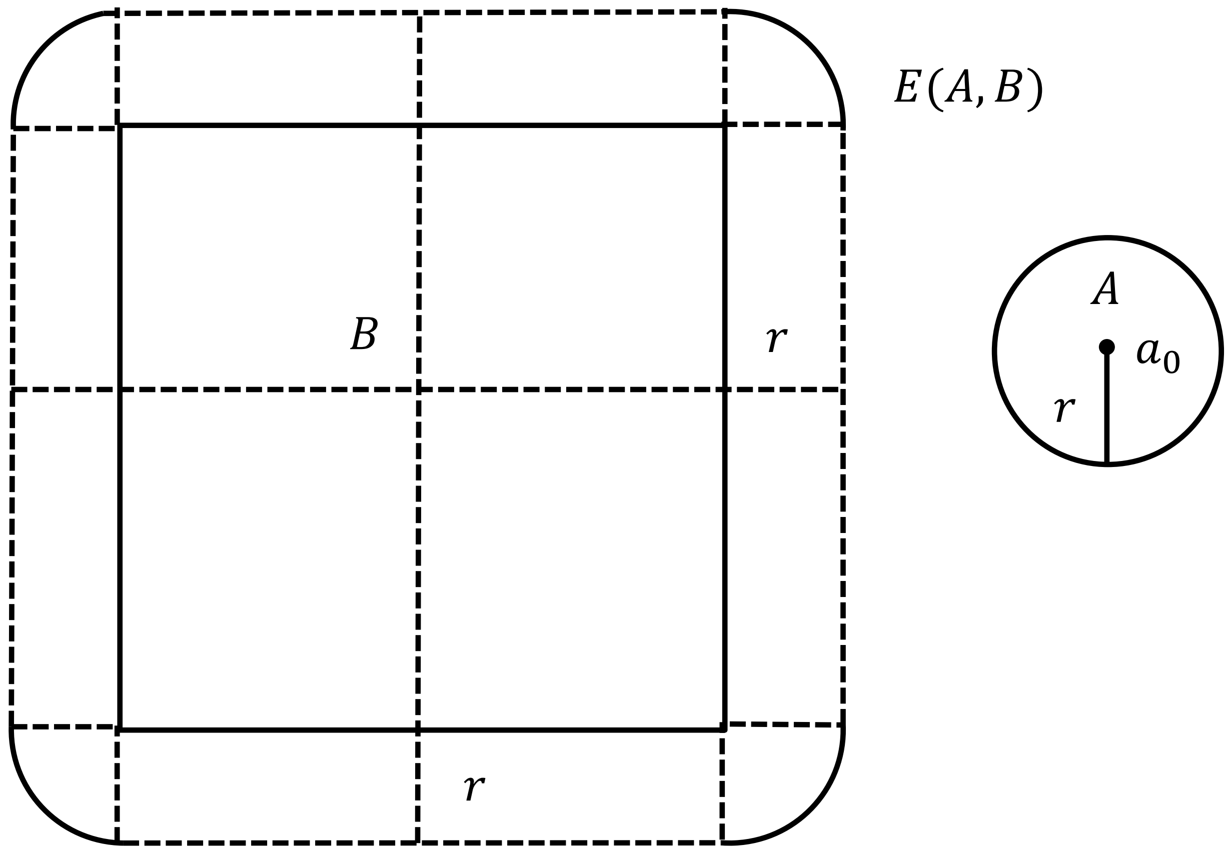
1. Theorem of entrance

$$A \cap B = \emptyset,$$

$$|A, B| = \varepsilon \Leftrightarrow |\boldsymbol{a}_0, E(A, B)| = \varepsilon \Leftrightarrow$$

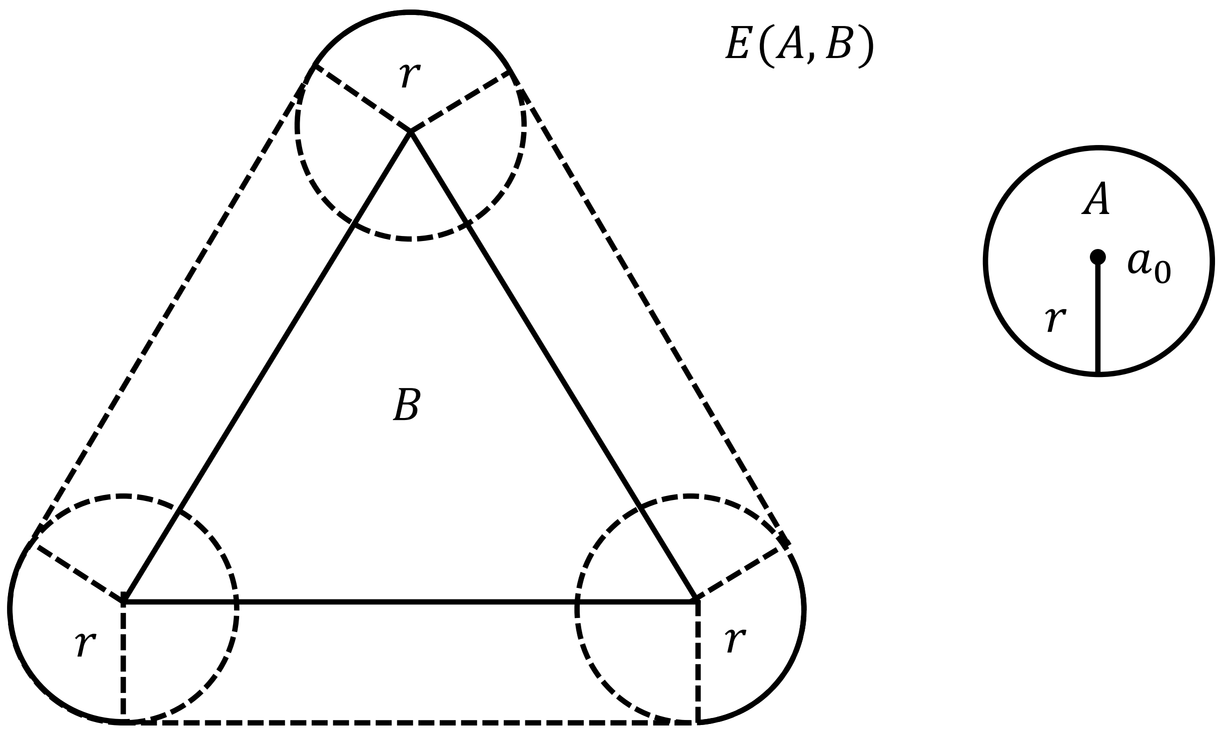
$$|\boldsymbol{a}_0, \partial\, E(A, B)| = \varepsilon.$$

Entrance block and distance



$$|A,B| = |\boldsymbol{a}_0, E(A,B)|$$

Entrance block and distance



$|A, B| = |\boldsymbol{a}_0, E(A, B)|$

2. Theorem of exit

$$int(A \cap B) \neq \emptyset$$

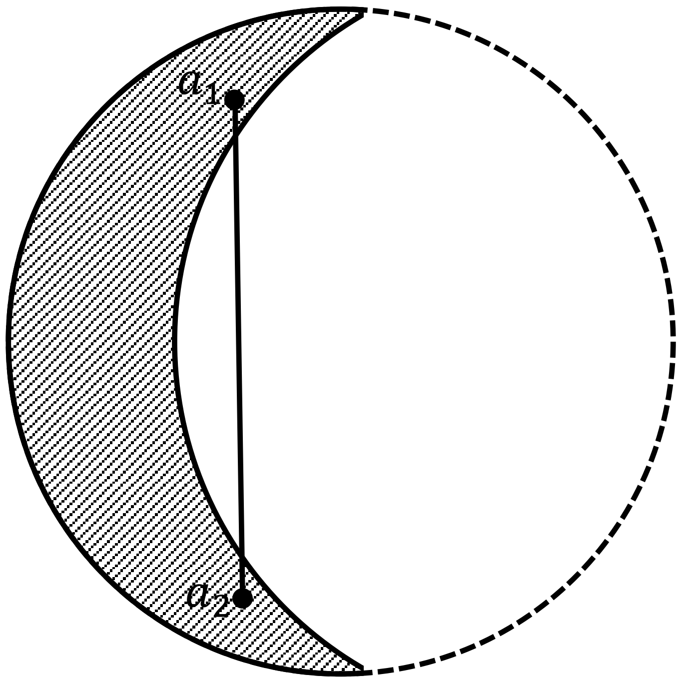
$$|\boldsymbol{A}, \boldsymbol{B}| = -\boldsymbol{\varepsilon} \Leftrightarrow -|\boldsymbol{a}_{\mathbf{0}}, \boldsymbol{\partial}\, \boldsymbol{E}(\boldsymbol{A}, \boldsymbol{B})| = -\boldsymbol{\varepsilon}.$$

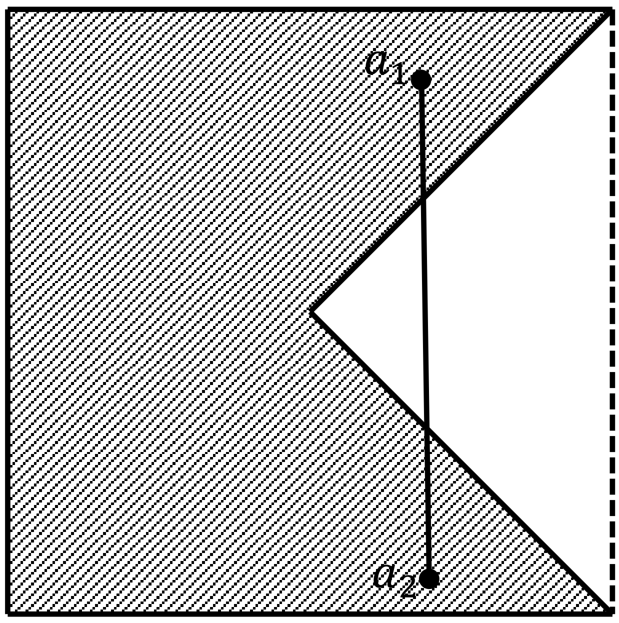
3. Theorem of convex block

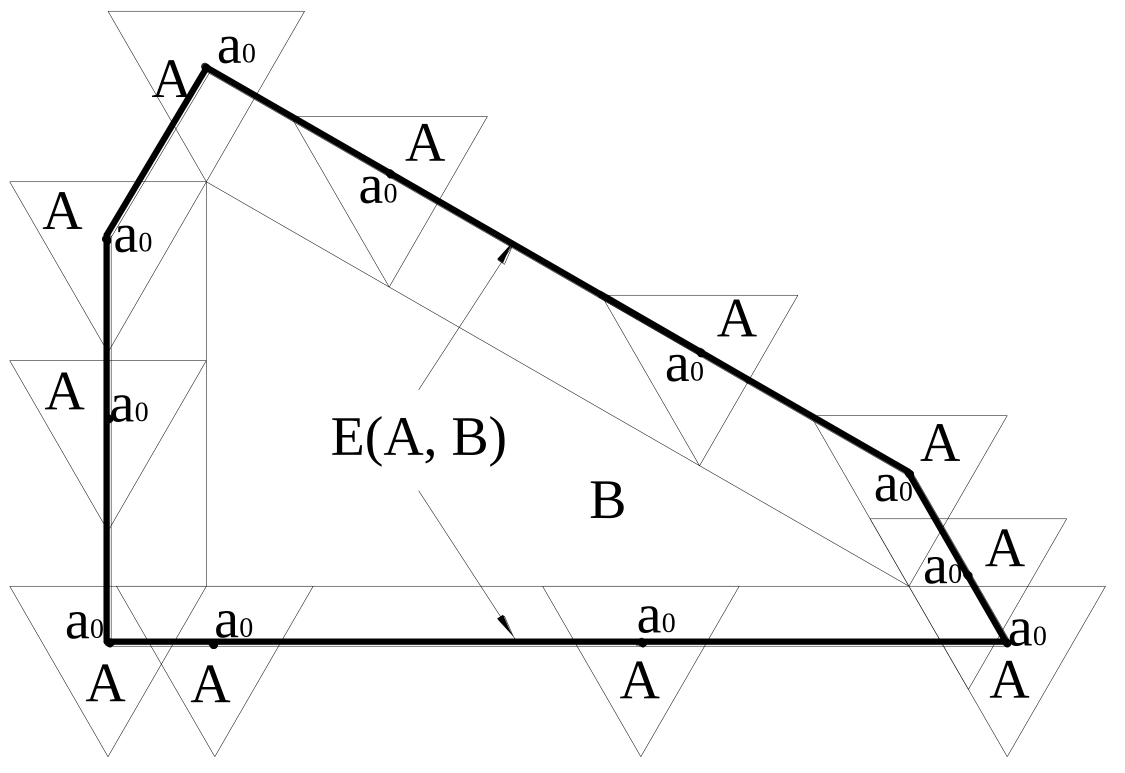
Block A is convex
Block B is convex
$\Rightarrow$

$E(A, B)$ is convex.

Concave blocks







a0
A
A
a0
a0
A
A
a0
A
a0
A
a0
E(A, B)
B
A
a0
A
a0
a0
a0
a0
a0
A
A
A
A

4. Theorem of finite covers of entrance blocks

A is n d block, n=1, 2, 3

B is m d block, m=1, 2, 3

$$n \geq m$$

$$E(A,B) = \cup_{k=0}^{m} E(A(n-k), B(k))$$

$$m \geq n$$

$$E(A,B) = \cup_{k=0}^{n} E(A(k), B(m-k))$$

5. Theorem of finite covers of entrance surface

A and B are n d blocks, n=2, 3

$$\partial E(A,B) \subset \cup_{k=0}^{n-1} E(A(n-1-k),B(k))$$

6. Theorem of entrance surface and solid angle

A and B are n d block, $n = 2,3$

$$\exists \ \boldsymbol{a} \in int(A_r(i)) = int(\measuredangle \boldsymbol{a}),$$

$$\exists \ \boldsymbol{b} \in int(B_s(j)) = int(\measuredangle \boldsymbol{b}),$$

$$E(\boldsymbol{a}, \boldsymbol{b}) \in \partial E(A, B),$$

$$i, j \leq n - 1$$

$$\Rightarrow int\big(\measuredangle A_r(i)\big) \cap int\big(\measuredangle B_s(j)\big) = \emptyset.$$

7. Theorem of solid angle contact

A and B are n d block, $n = 2,3$

$$i, j \leq n-1,\ i+j>0$$

$$int\left(\measuredangle A_r(i)\right) \cap int\left(\measuredangle B_s(j)\right) = \emptyset \Leftrightarrow$$

$$\measuredangle A_r(i)\ \cap \measuredangle B_s(j)$$

$$\subset (\partial \measuredangle A_r(i)) \cap (\partial \measuredangle B_s(j)) \Leftrightarrow$$

$$\partial E\left(\measuredangle A_r(i), \measuredangle B_s(j)\right) \neq \emptyset$$

$$i = j = 0$$

$$\partial E\left(\measuredangle A_r(0), \measuredangle B_s(0)\right) \neq \emptyset \Rightarrow$$

$$int\left(\measuredangle A_r(0)\right) \cap int\left(\measuredangle B_s(0)\right) = \emptyset$$

8. Theorem of contact of convex solid angle

A and B are solid angles,

A is convex,

$$int(A) \cap int(B) = \emptyset \Leftrightarrow$$

$$int\big(E(A, B)\big) \cap int(A) = \emptyset.$$

9. Theorem of entrance surface of convex blocks

A and B are convex

$$\boldsymbol{a} \in int(A_r(i)), \qquad \boldsymbol{b} \in int(B_s(j)).$$

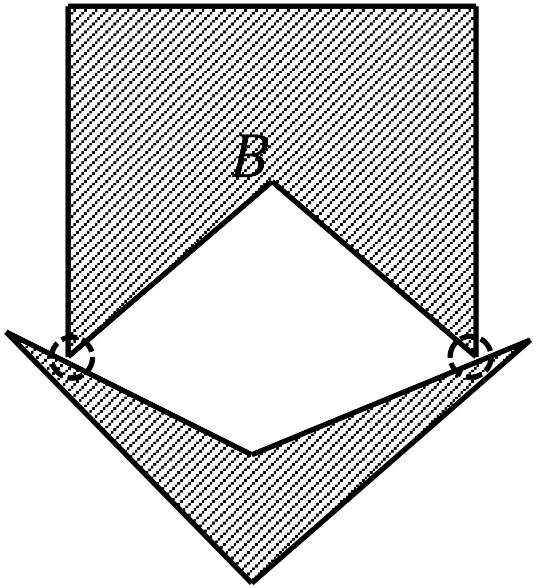
$$E(\boldsymbol{a}, \boldsymbol{b}) \in \partial E(A, B) \Leftrightarrow$$

$$int(\measuredangle \boldsymbol{a}) \cap int(\measuredangle \boldsymbol{b}) = \emptyset.$$

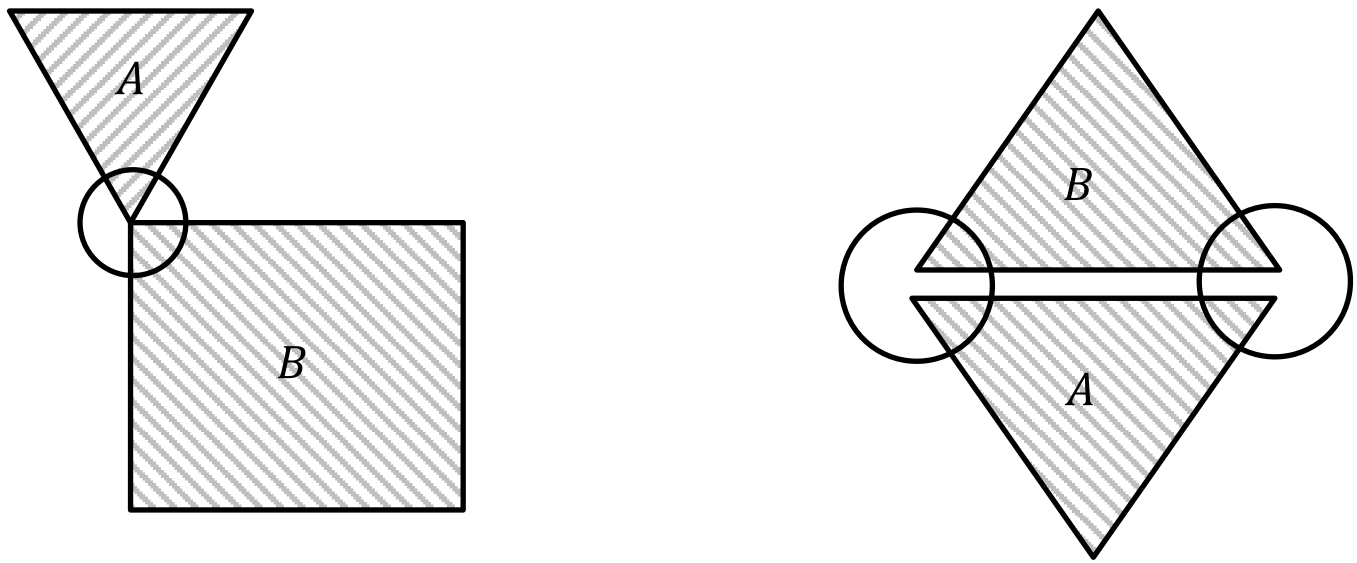
$E(A, B)$ is convex

2D Entrance Solid Angle of Solid Angles

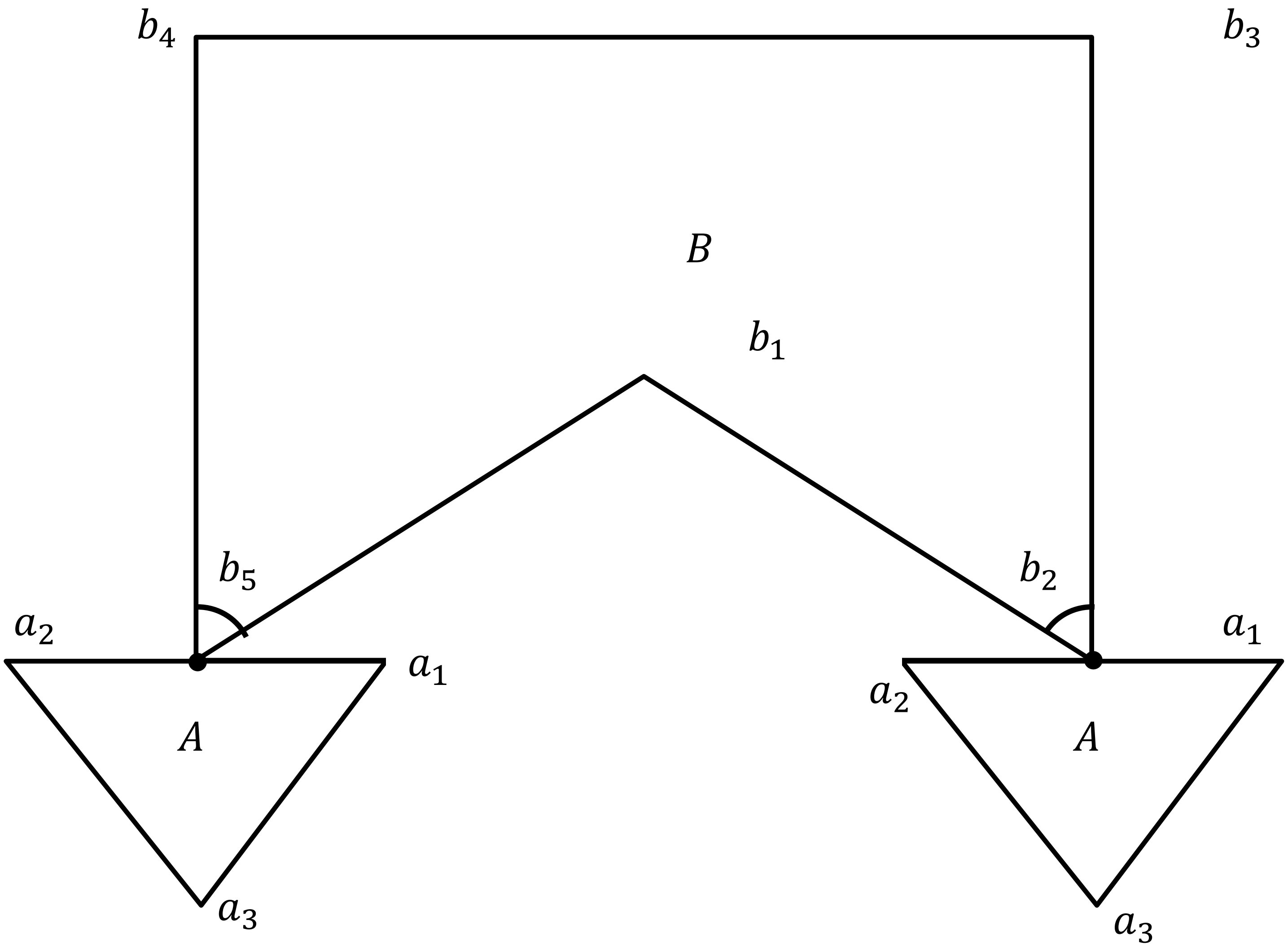
Entrance of local angles



Entrance of local angles



2D Vertex-Edge Contact



Boundary
of
2D entrance solid angle

$$\partial E(A,B) \subset E(\partial A, \partial B)$$

$$\partial E(A,B) \subset E\big(A(1), B(1)\big)$$

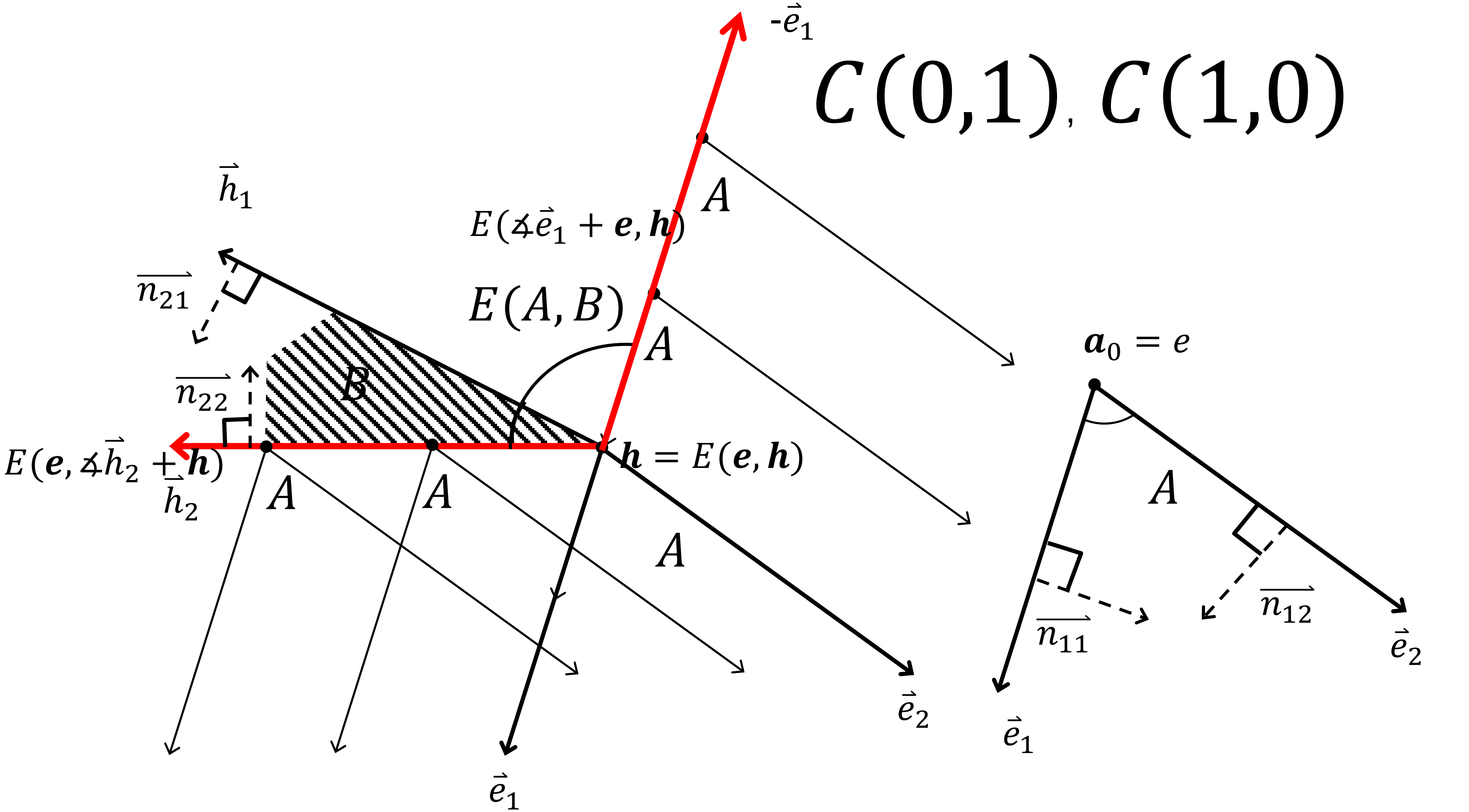
$$\partial E(A,B) \subset E\big(A(0), B(1)\big) \cup E\big(A(1), B(0)\big)$$

Contact 1D solid angles
of
2D convex solid angles

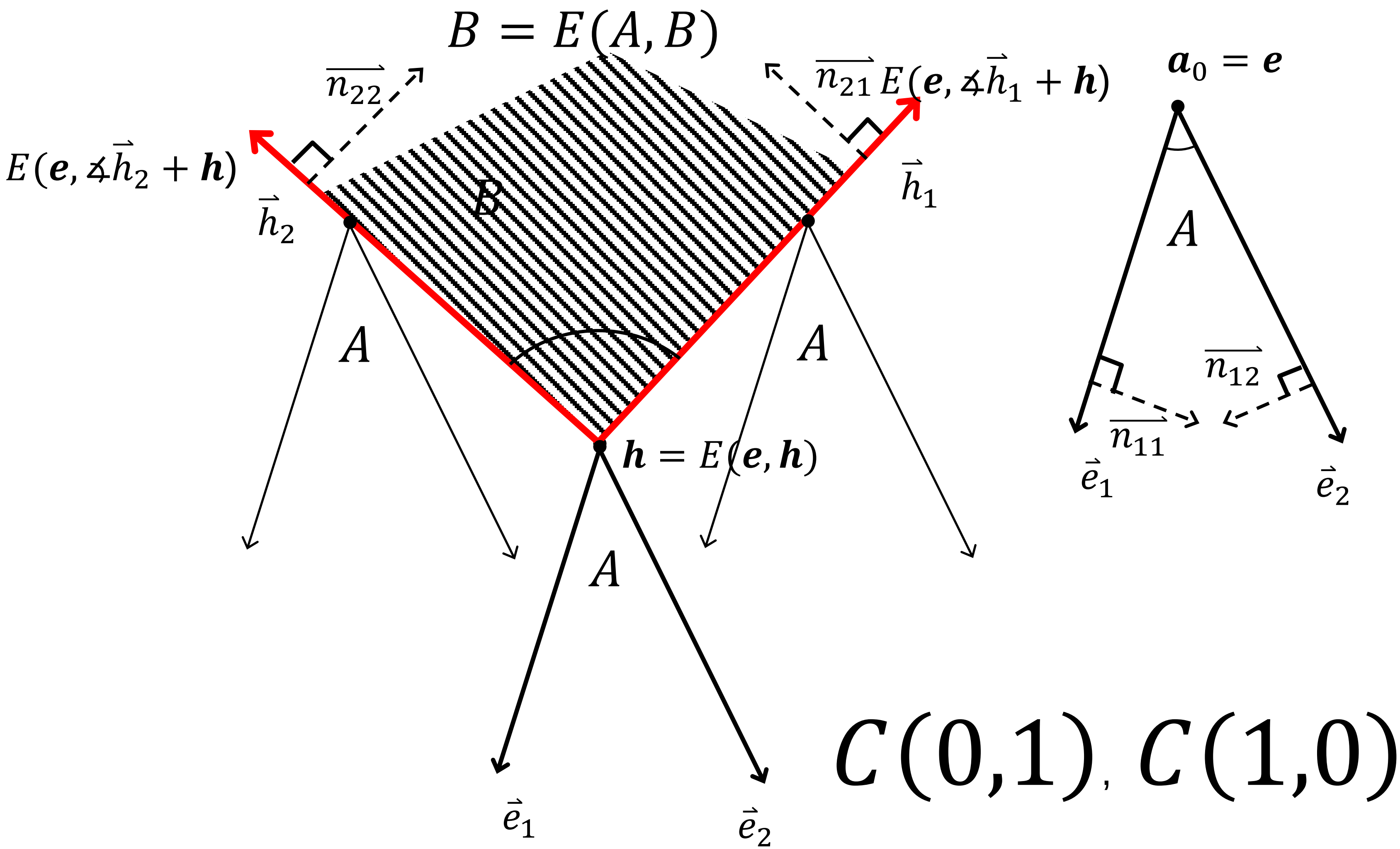
For convex 2D solid angle A, B

$$\partial E(A, B) \subset C(0,1) \cup C(1,0)$$

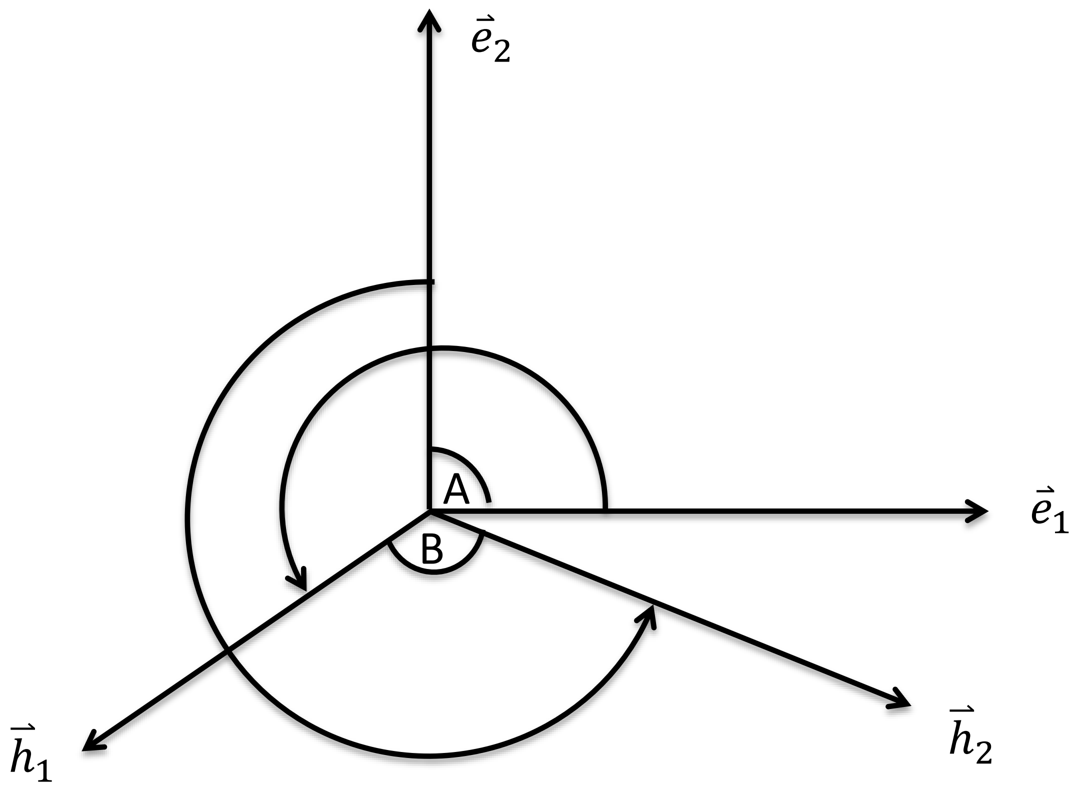
$$\partial E(A, B) \supset C(0,1) \cup C(1,0)$$



$C(0,1)$, $C(1,0)$
$-\vec{e}_1$
$\vec{h}_1$
$E(\measuredangle\vec{e}_1 + \boldsymbol{e}, \boldsymbol{h})$
$\overrightarrow{n_{21}}$
$E(A,B)$
A
A
$\boldsymbol{a}_0 = e$
$\overrightarrow{n_{22}}$
B
$E(\boldsymbol{e}, \measuredangle\vec{h}_2 + \boldsymbol{h})$
$\boldsymbol{h} = E(\boldsymbol{e}, \boldsymbol{h})$
$\vec{h}_2$
A
A
A
A
$\overrightarrow{n_{12}}$
$\overrightarrow{n_{11}}$
$\vec{e}_2$
$\vec{e}_2$
$\vec{e}_1$
$\vec{e}_1$



$B = E(A, B)$
$\overrightarrow{n_{22}}$
$\overrightarrow{n_{21}}$
$E(\boldsymbol{e}, \measuredangle\vec{h}_1 + \boldsymbol{h})$
$E(\boldsymbol{e}, \measuredangle\vec{h}_2 + \boldsymbol{h})$
$\vec{h}_2$
$\vec{h}_1$
B
A
A
$\boldsymbol{h} = E(\boldsymbol{e}, \boldsymbol{h})$
A
$\vec{e}_1$
$\vec{e}_2$
$\boldsymbol{a}_0 = \boldsymbol{e}$
A
$\overrightarrow{n_{12}}$
$\overrightarrow{n_{11}}$
$\vec{e}_1$
$\vec{e}_2$
$C(0,1), C(1,0)$

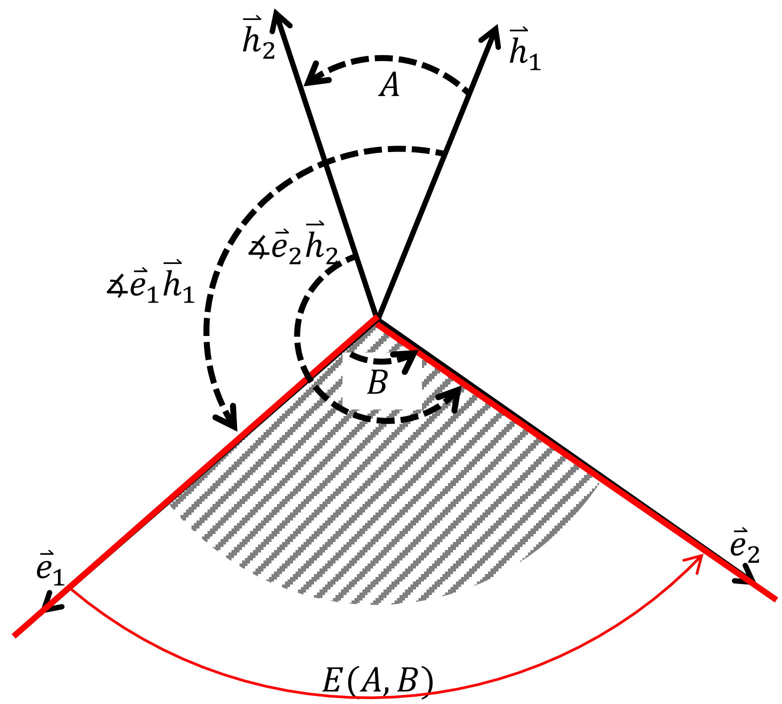


$\angle \vec{e}_1 \vec{h}_1 \leq 180° \quad : \vec{h}_1$ $\qquad$ $\angle \vec{e}_1 \vec{h}_1 \geq 180° \quad : \vec{e}_1$

$\angle \vec{e}_2 \vec{h}_2 \leq 180° \quad : \vec{e}_2$ $\qquad$ $\angle \vec{e}_2 \vec{h}_2 \geq 180° \quad : \vec{h}_2$

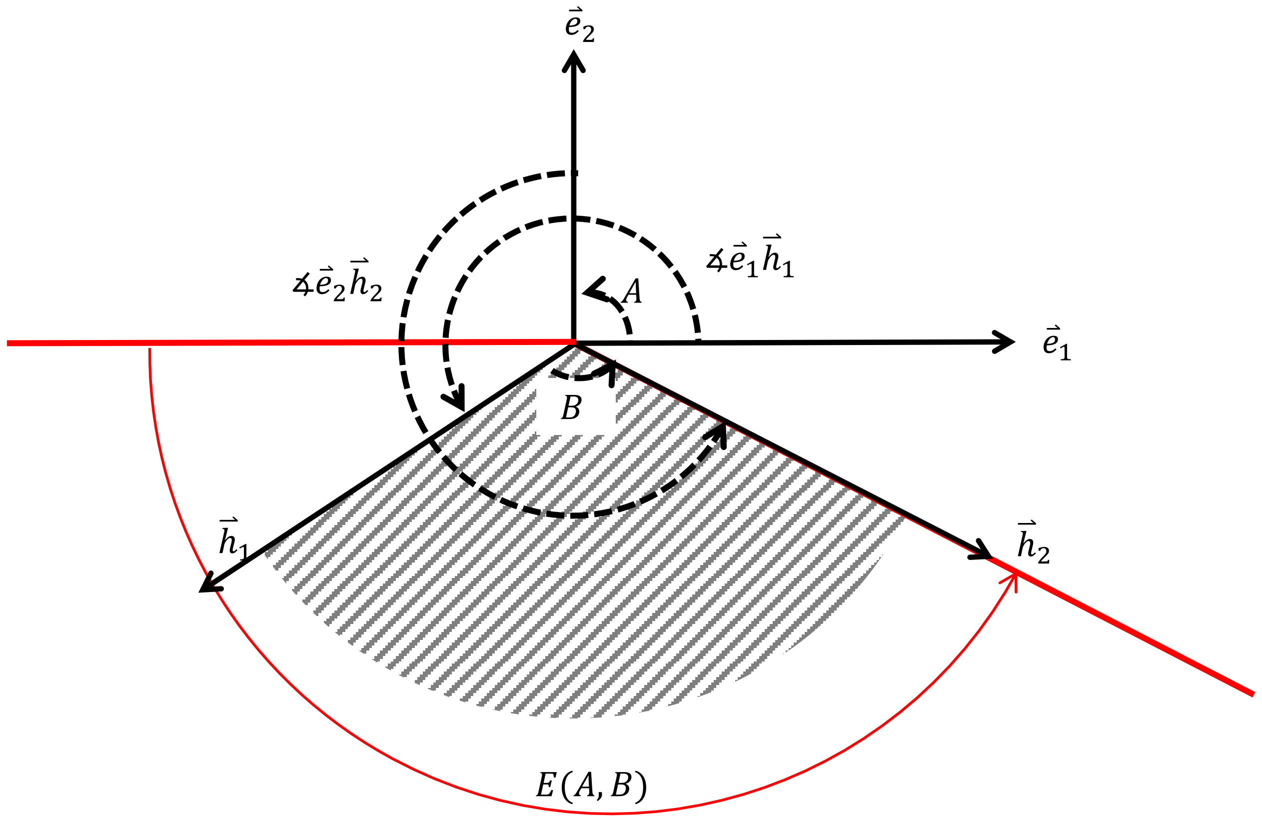
1. Angle method of 2D angle-angle entrance

$C(0,1)$, $C(1,0)$

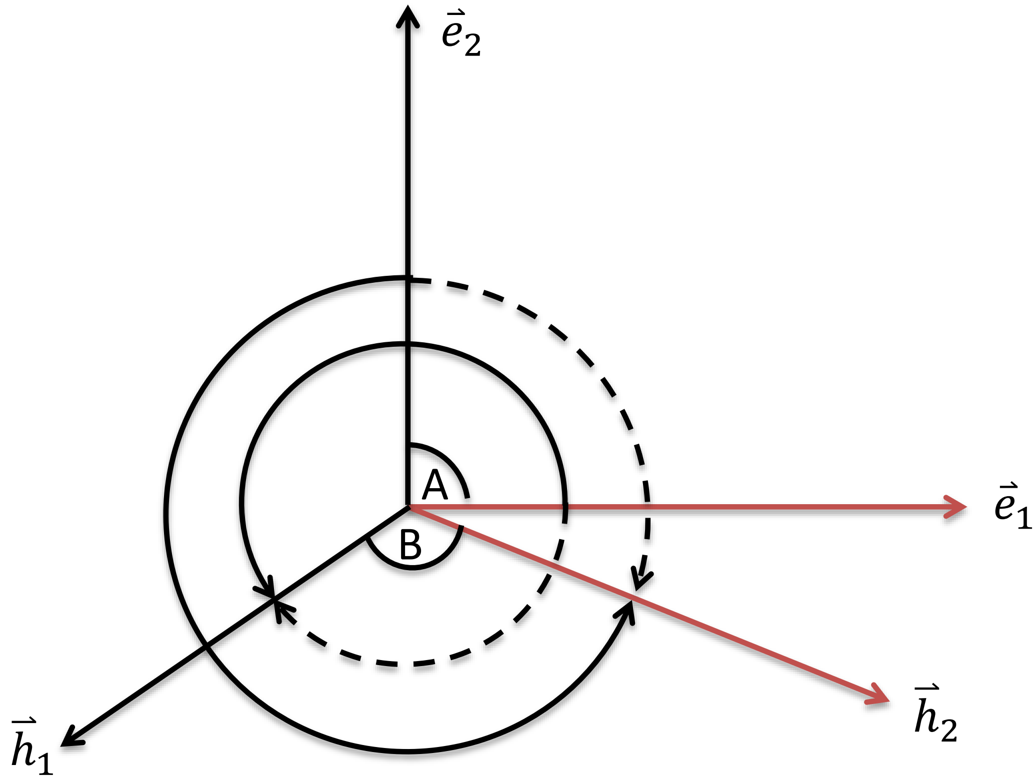


$\vec{h}_2$
$\vec{h}_1$
A
$\measuredangle \vec{e}_2 \vec{h}_2$
$\measuredangle \vec{e}_1 \vec{h}_1$
B
$\vec{e}_1$
$\vec{e}_2$
$E(A,B)$

$C(0,1)$, $C(1,0)$



$\vec{e}_2$
$\vec{e}_1$
$\measuredangle \vec{e}_2 \vec{h}_2$
$\measuredangle \vec{e}_1 \vec{h}_1$
A
B
$\vec{h}_1$
$\vec{h}_2$
$E(A,B)$



$\vec{e}_1 \times \vec{h}_1 \upuparrows (0,0,1) \quad : \vec{h}_1$

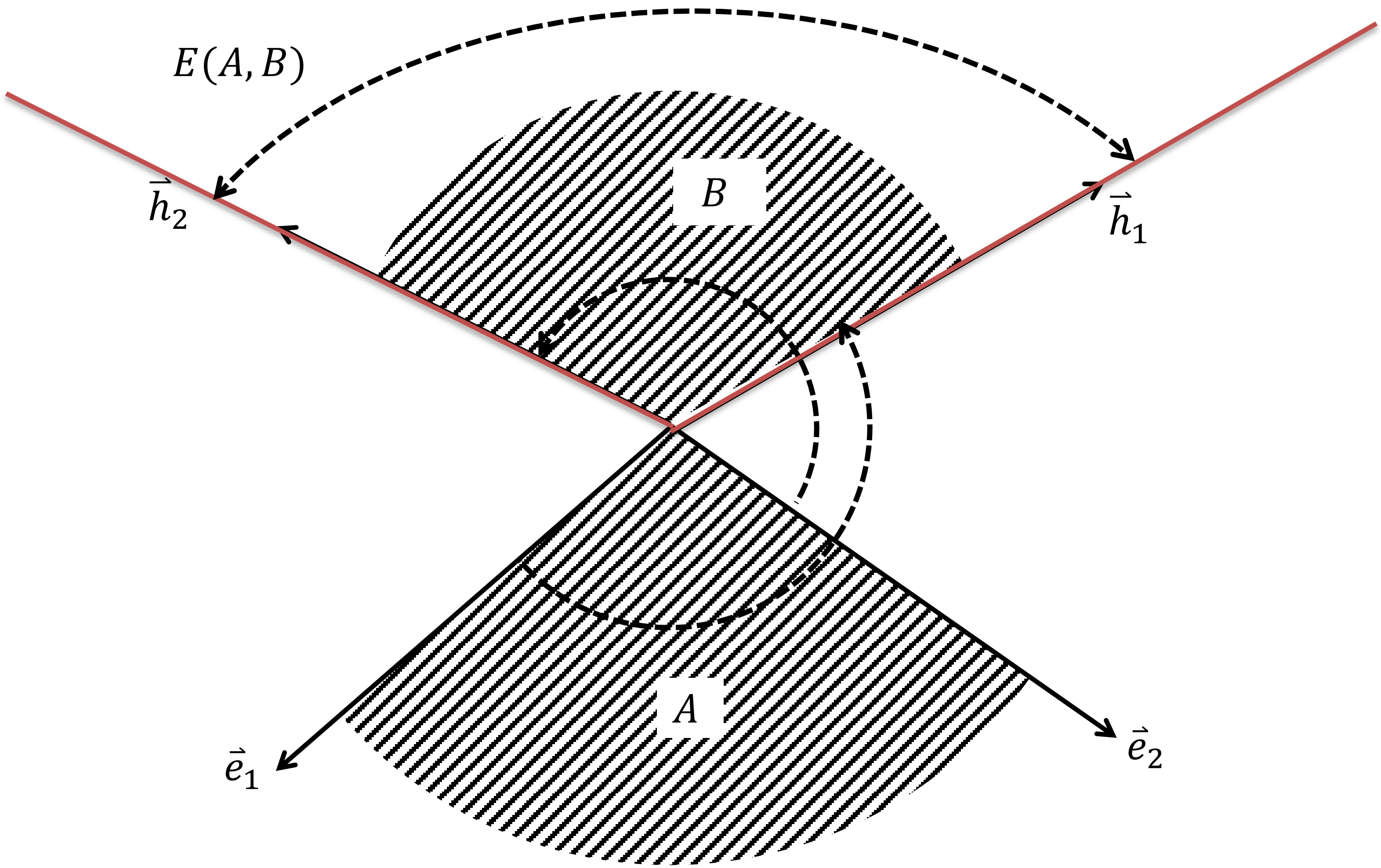
$\vec{e}_2 \times \vec{h}_2 \upuparrows (0,0,1) \quad : \vec{e}_2$

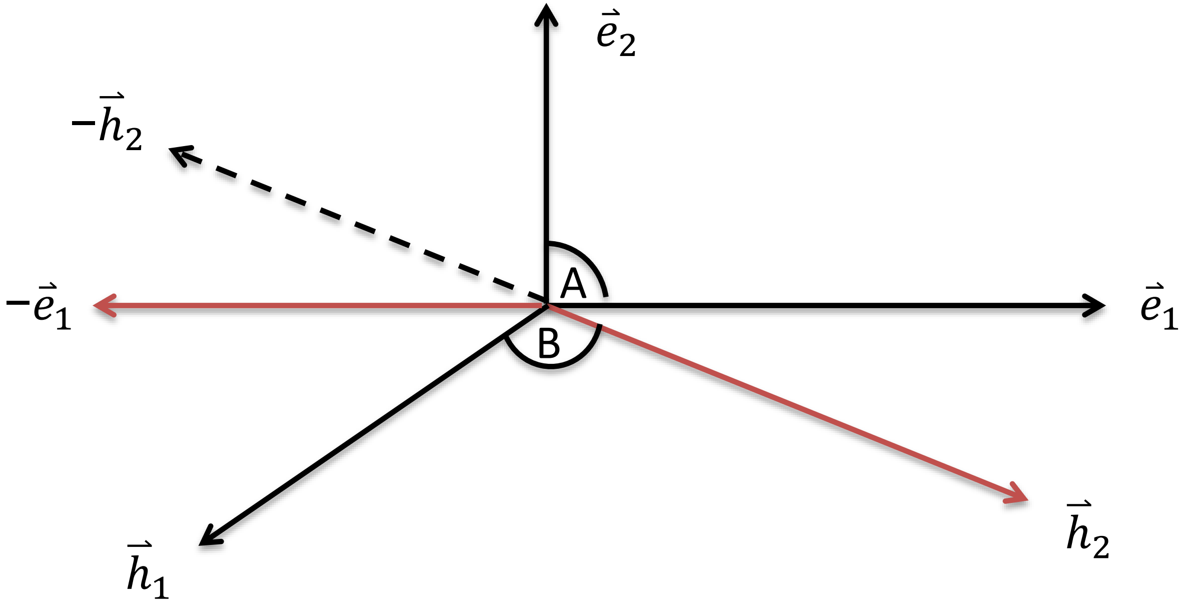
$\vec{e}_1 \times \vec{h}_1 \upuparrows (0,0,-1) \quad : \vec{e}_1$

$\vec{e}_2 \times \vec{h}_2 \upuparrows (0,0,-1) : \vec{h}_2$

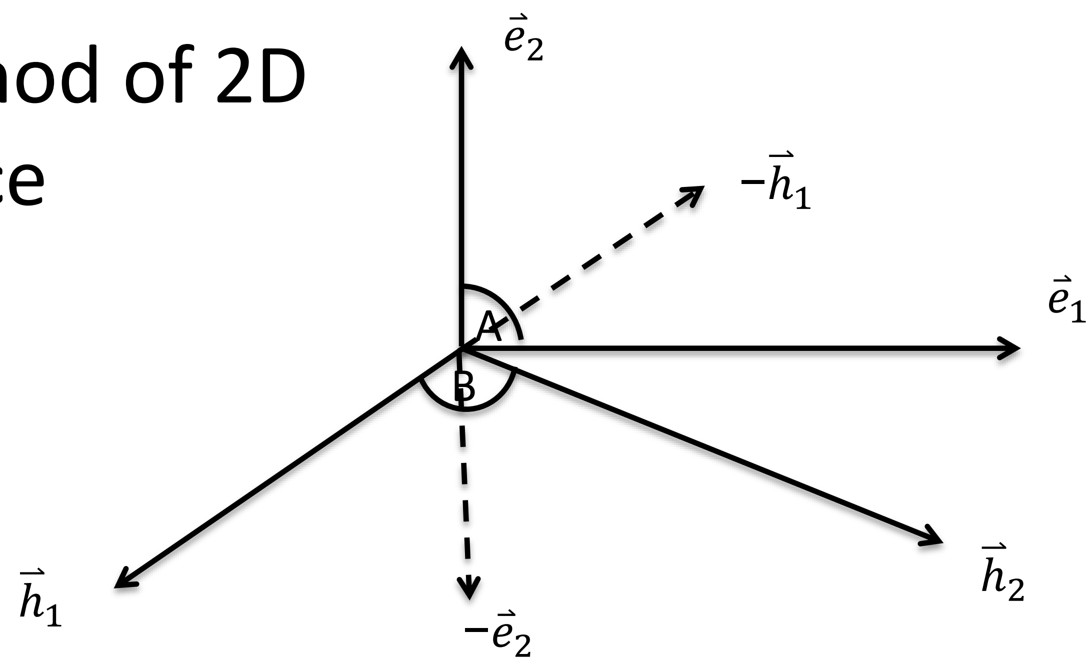
2. Rotation method of 2D angle-angle entrance

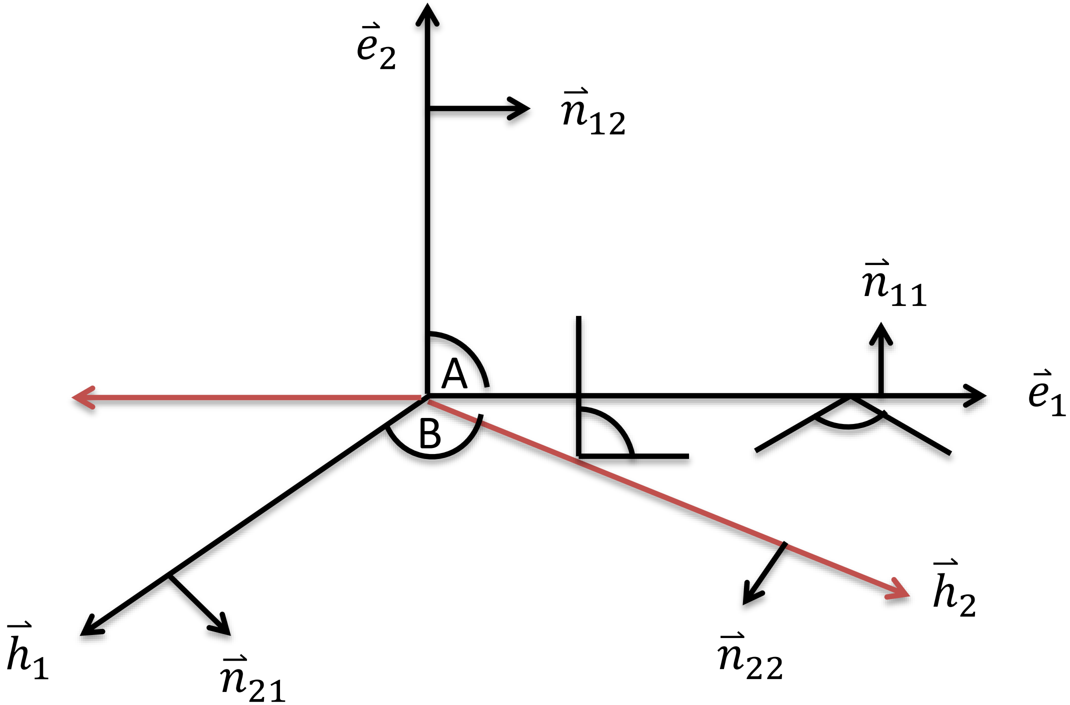
Entrance of Two 2D Angles



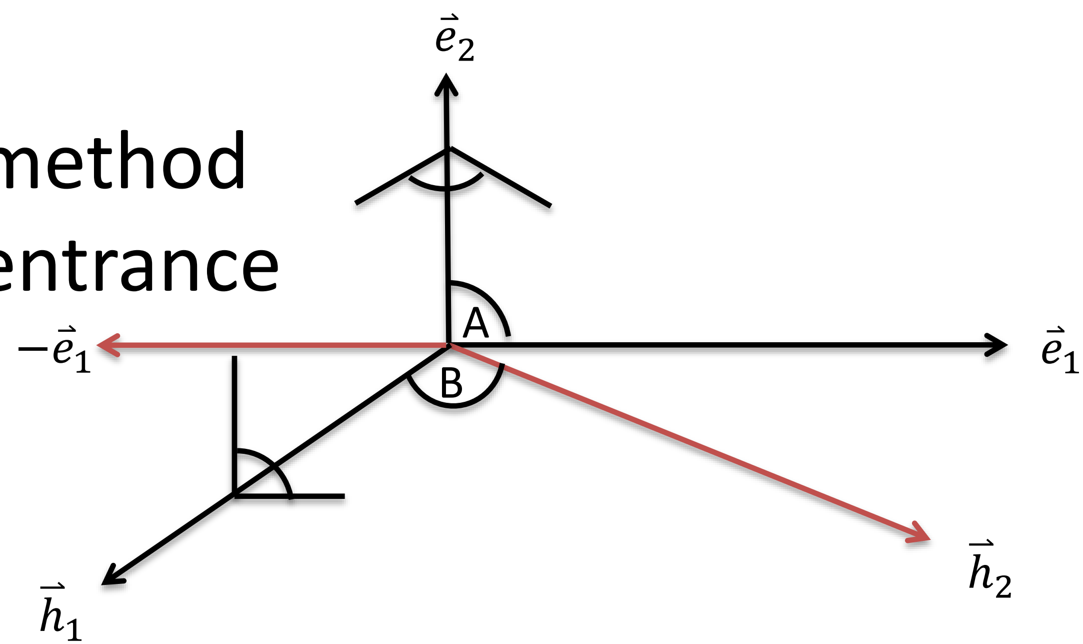


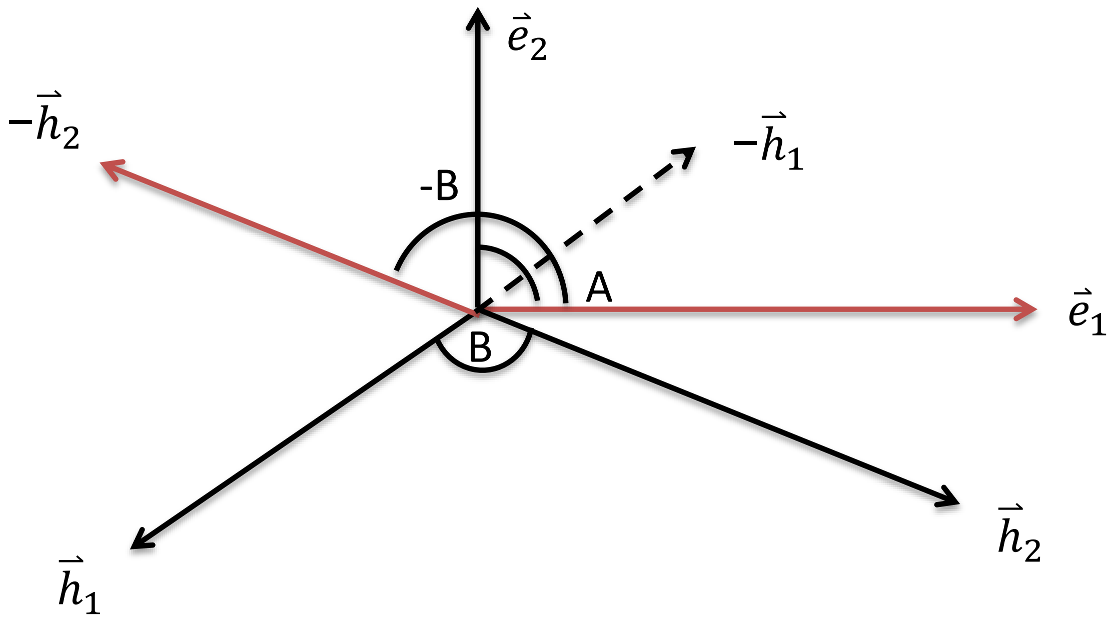
3. Dividing line method of 2D angle-angle entrance



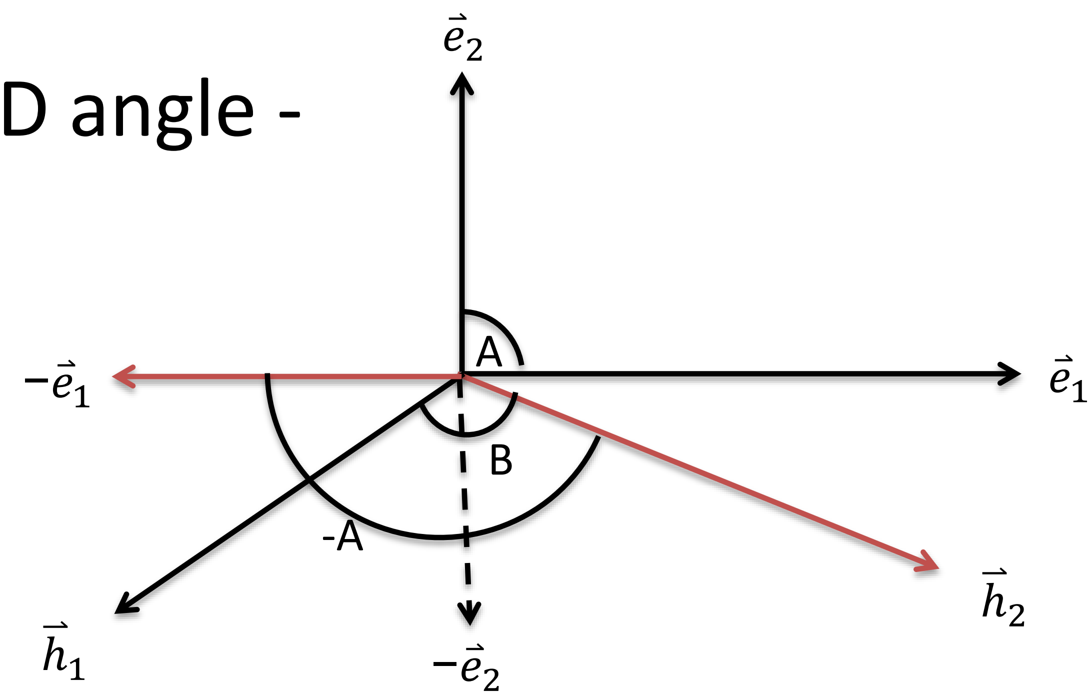


4. Contact vector method of 2D angle-angle entrance





5. B-A method of 2D angle - angle entrance



$$int(\measuredangle\vec{e}_1\vec{e}_2) \cap int(\measuredangle\vec{h}_1\vec{h}_2) = \emptyset$$

$$E(\measuredangle\vec{e}_1\vec{e}_2, \uparrow n_{21}) = \uparrow n_{21} + \boldsymbol{a}_0 \Rightarrow$$

$$E\left(\boldsymbol{e}, \measuredangle\vec{h}_1 + \boldsymbol{h}\right) \subset \partial E(A, B)$$

$$E\left(\uparrow n_{11}, \measuredangle\vec{h}_1\vec{h}_2\right) = -\uparrow n_{11} + \boldsymbol{a}_0 \Rightarrow$$

$$E(\measuredangle\vec{e}_1 + \boldsymbol{e}, \boldsymbol{h}) \subset \partial E(A, B)$$

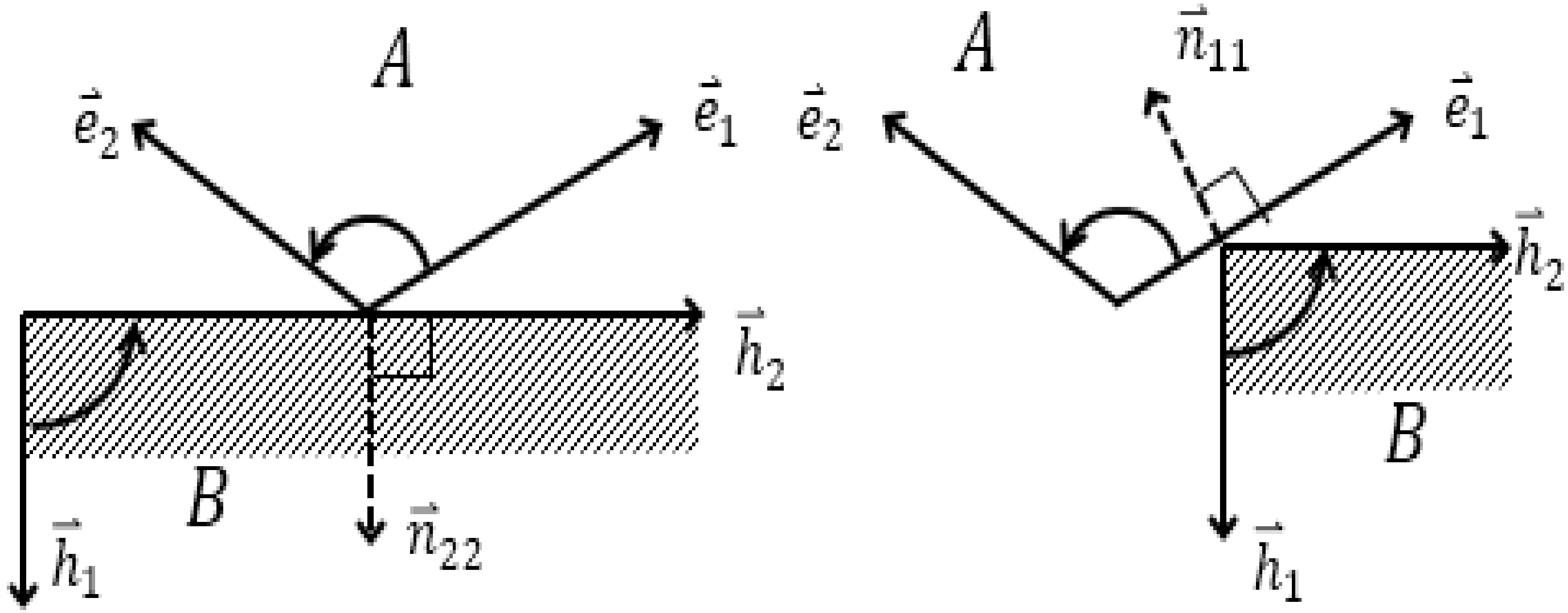
$$E\left(\uparrow n_{12}, \measuredangle\vec{h}_1\vec{h}_2\right) = -\uparrow n_{12} + \boldsymbol{a}_0 \Rightarrow$$

$$E(\measuredangle\vec{e}_2 + \boldsymbol{e}, \boldsymbol{h}) \subset \partial E(A, B)$$

$$E(\measuredangle\vec{e}_1\vec{e}_2, \uparrow n_{22}) = \uparrow n_{22} + \boldsymbol{a}_0 \Rightarrow$$

$$E\left(\boldsymbol{e}, \measuredangle\vec{h}_2 + \boldsymbol{h}\right) \subset \partial E(A, B)$$

6. Angle and half-plane entrance



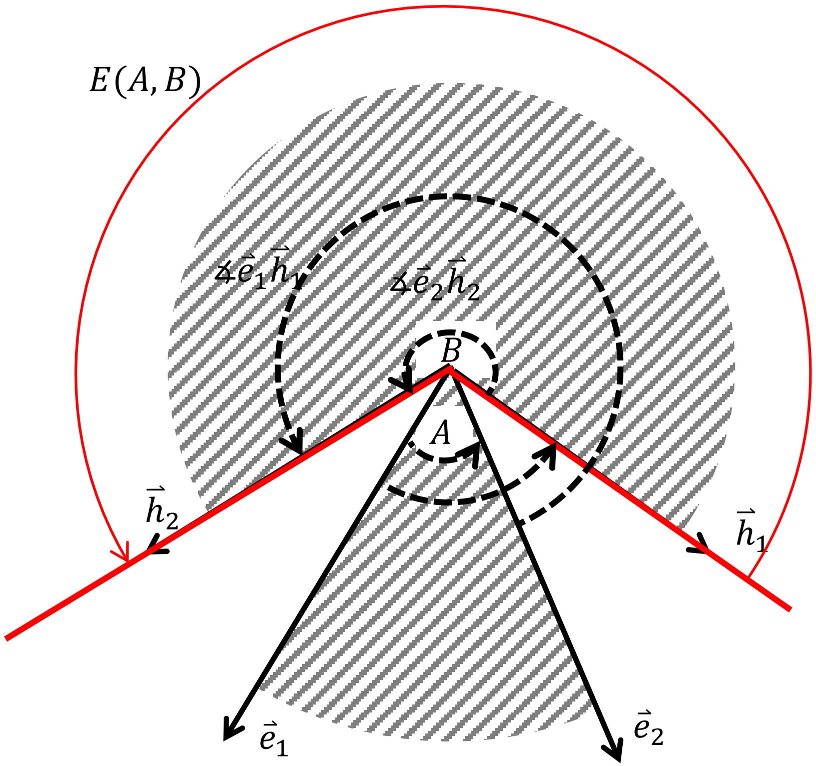
6. Angle and half-plane entrance

For general 2D solid angle A, B

$$\partial E(A,B) \subset C(0,1) \cup C(1,0)$$

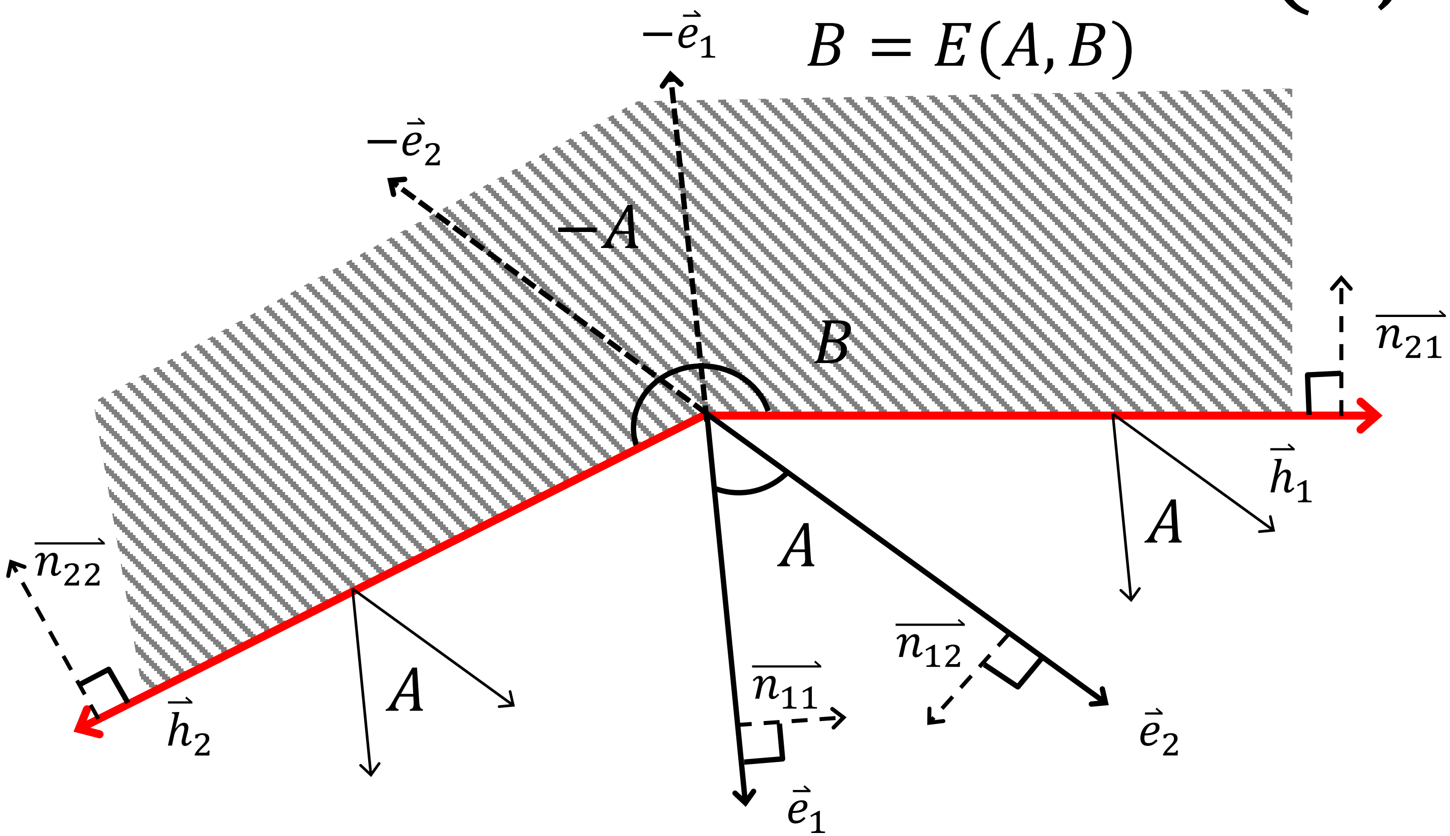
$$\partial E(A,B) \supset C(0,1) \cup C(1,0)$$

$C(0,0)$



$E(A,B)$
$\measuredangle \vec{e}_1\vec{h}_1$
$\measuredangle \vec{e}_2\vec{h}_2$
B
A
$\vec{h}_2$
$\vec{h}_1$
$\vec{e}_1$
$\vec{e}_2$

$C(0,0)$



Convex: $\vec{n}_{11} = \vec{e}_1 \times \vec{e}_2 \times \vec{e}_1$

$\vec{n}_{12} = \vec{e}_2 \times \vec{e}_1 \times \vec{e}_2$

Concave: $\vec{n}_{21} = \vec{h}_2 \times \vec{h}_1 \times \vec{h}_1$

$\vec{n}_{22} = \vec{h}_1 \times \vec{h}_2 \times \vec{h}_2$

Contact Vector
of
2D Parallel Vectors

$$int(\measuredangle \vec{e}_1 \vec{e}_2) \cap int\left(\measuredangle \overrightarrow{h}_1 \overrightarrow{h}_2\right) = \emptyset$$

$$\Rightarrow$$

$$\vec{e}_2 \parallel \overrightarrow{h}_1$$

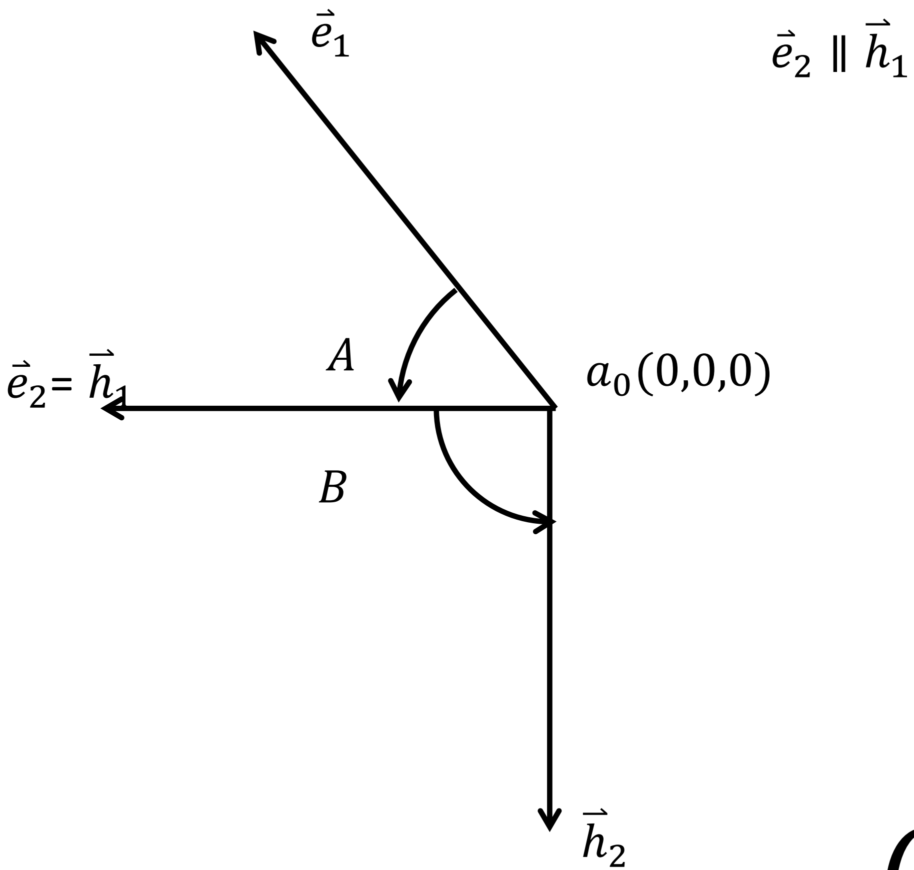
$$E\left(\measuredangle \vec{e}_2, \measuredangle \overrightarrow{h}_1\right) = (-\measuredangle \vec{e}_2) \cup \measuredangle \overrightarrow{h}_1$$

$$\vec{e}_2 \upuparrows \overrightarrow{h}_1$$

$$E\left(\measuredangle \vec{e}_2, \measuredangle \overrightarrow{h}_1\right) = (-\measuredangle \vec{e}_2) \cup \measuredangle \overrightarrow{h}_1$$

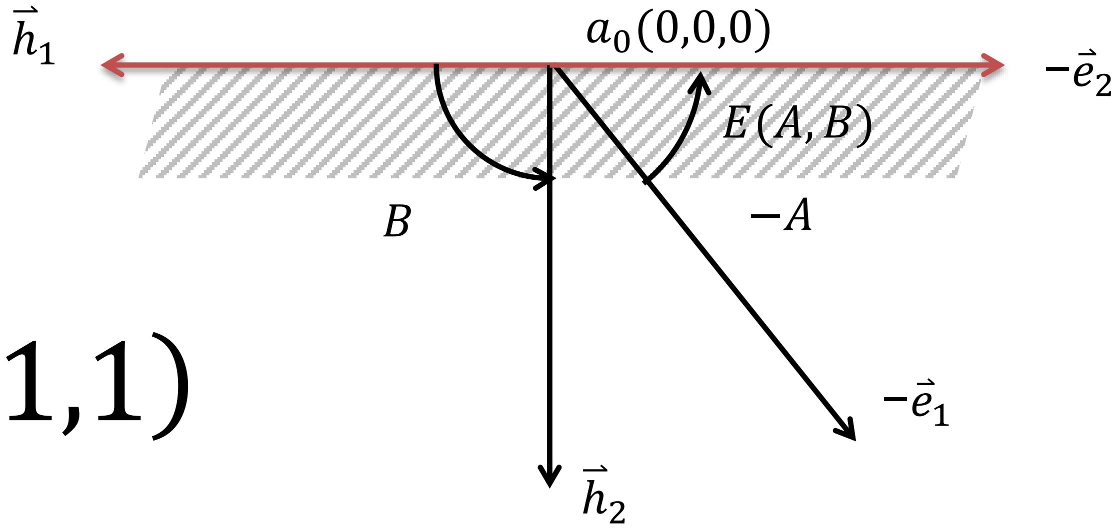
$$E\left(\measuredangle \vec{e}_2, \measuredangle \overrightarrow{h}_1\right) = \overleftrightarrow{e}_2 = \overleftrightarrow{h}_1$$

Entrance of two 2D solid angles with $\vec{e}_2 \parallel \vec{h}_1$



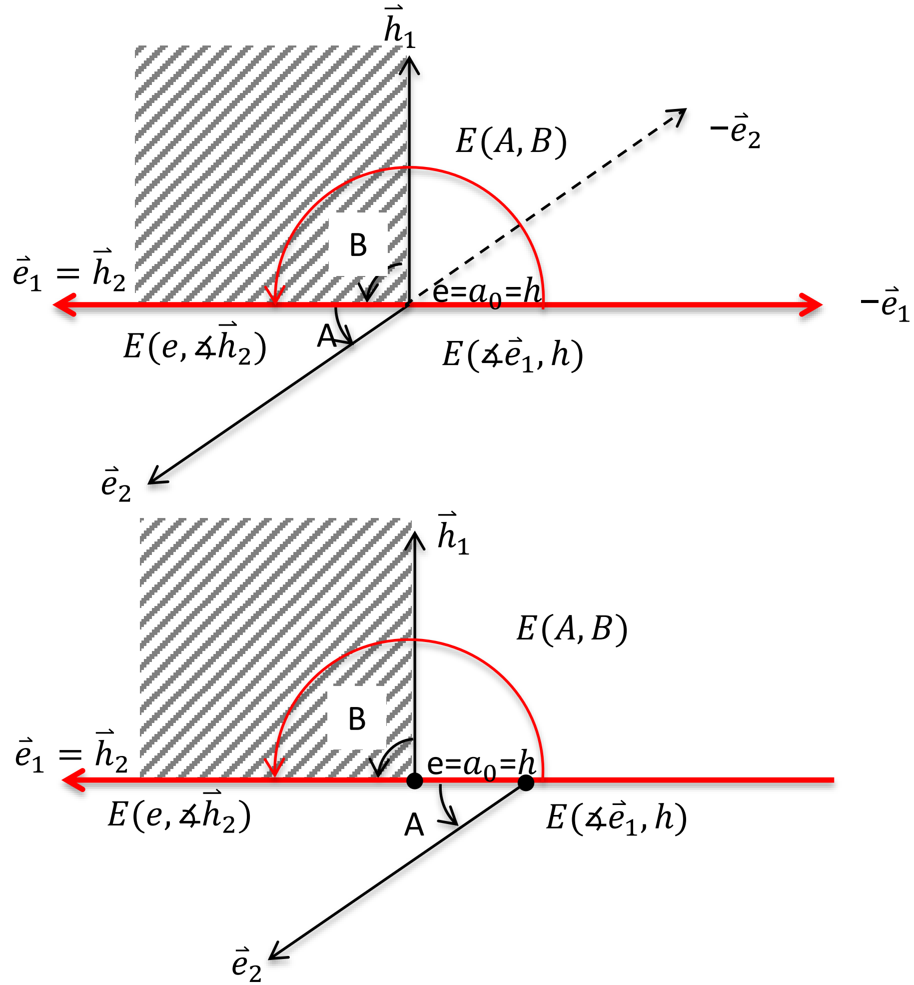
$C(1,1)$

Entrance of two 2D solid angles

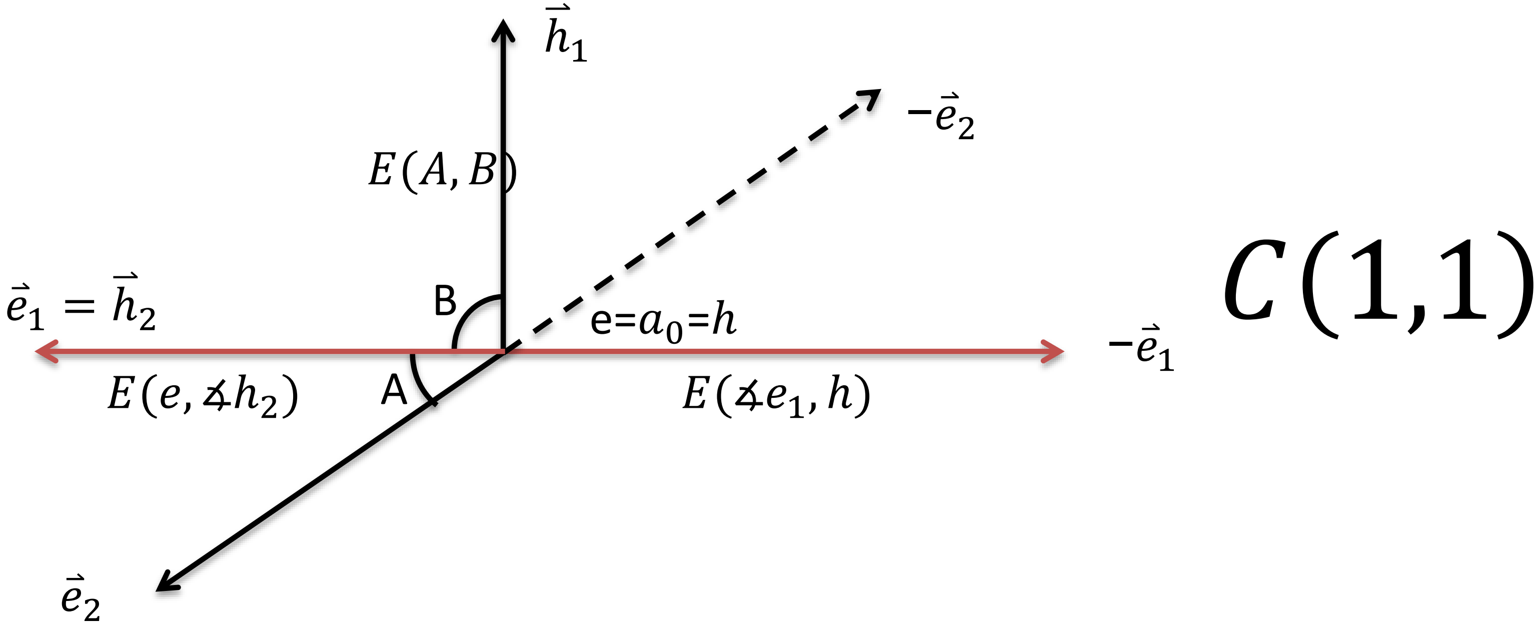


$$C(1,1)$$

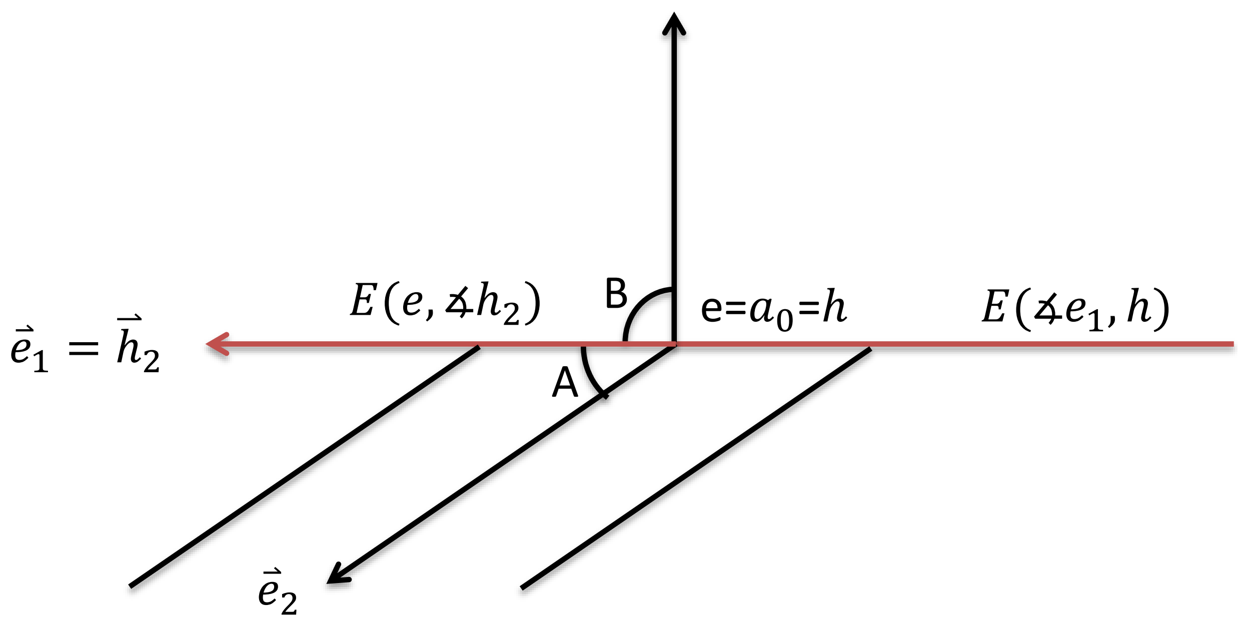
$C(1,1)$

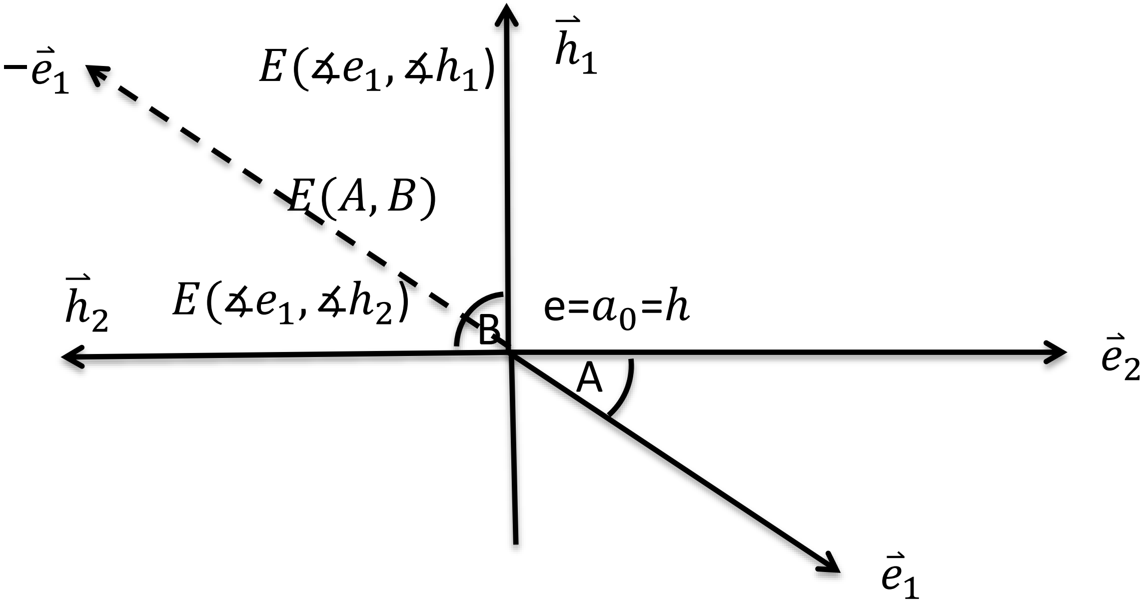


$\vec{h}_1$
$E(A,B)$
$-\vec{e}_2$
B
$\vec{e}_1 = \vec{h}_2$
e=a_0=h
$-\vec{e}_1$
$E(e,\measuredangle\vec{h}_2)$
A
$E(\measuredangle\vec{e}_1,h)$
$\vec{e}_2$
$\vec{h}_1$
$E(A,B)$
B
$\vec{e}_1 = \vec{h}_2$
e=a_0=h
$E(e,\measuredangle\vec{h}_2)$
A
$E(\measuredangle\vec{e}_1,h)$
$\vec{e}_2$



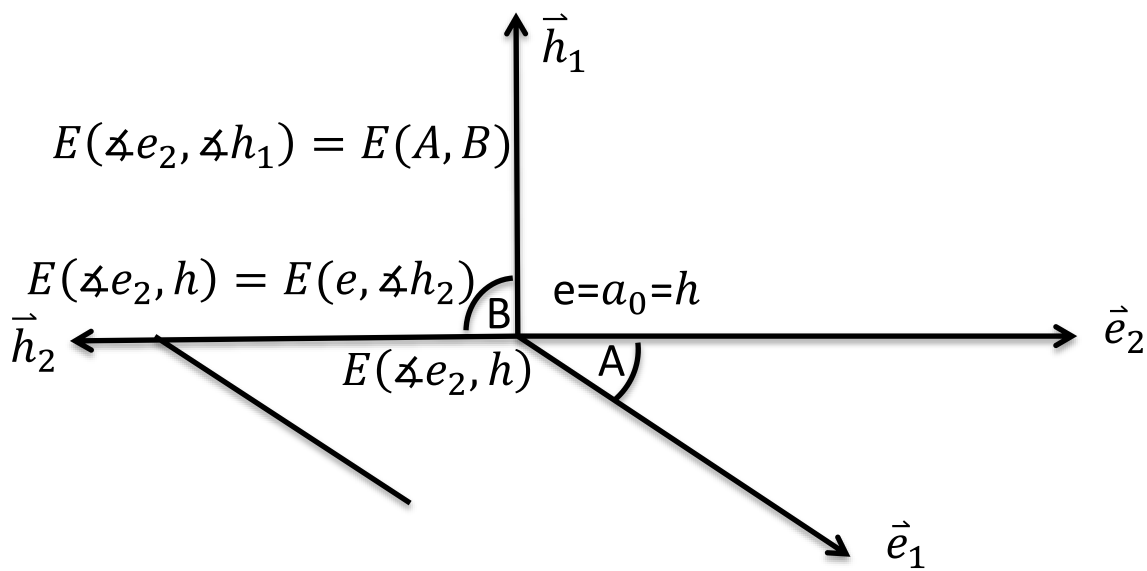
2D angle-angle entrance with parallel vector





$C(1,1)$

2D angle-angle entrance with parallel vector



$$int(\measuredangle\vec{e}_1\vec{e}_2) \cap int\left(\measuredangle\vec{h}_1\vec{h}_2\right) = \emptyset$$

$$\Rightarrow$$

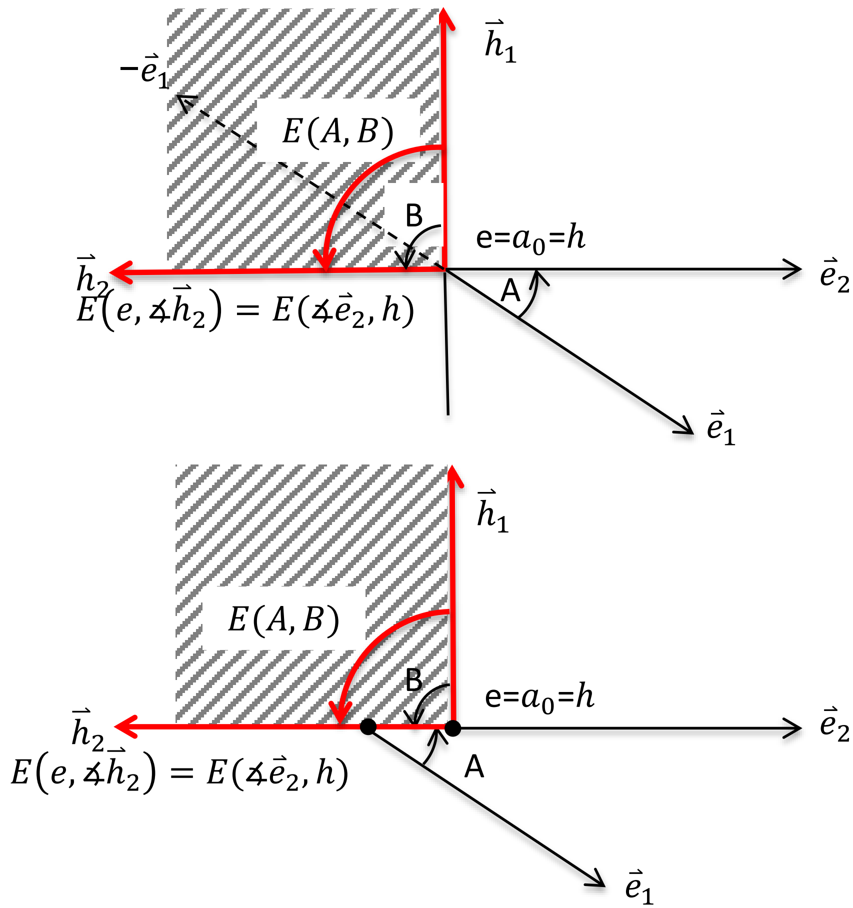
$$\vec{e}_2 \parallel \vec{h}_1$$

$$E\left(\measuredangle\vec{e}_2, \measuredangle\vec{h}_1\right) = (-\measuredangle\vec{e}_2) \cup \measuredangle\vec{h}_1$$

$$(-\vec{e}_2) \upuparrows \vec{h}_1$$

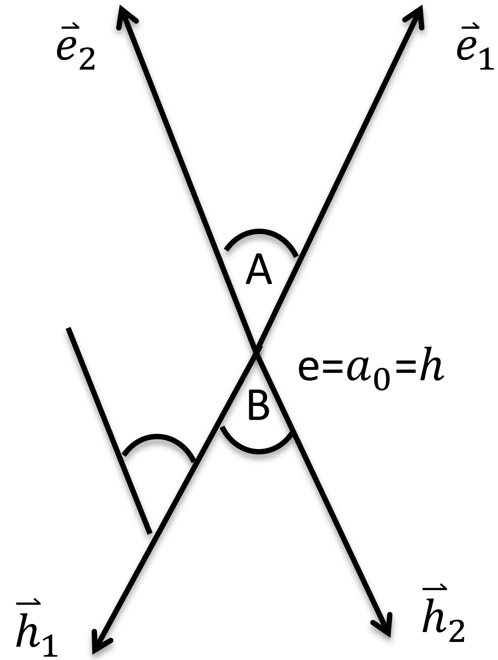
$$E\left(\measuredangle\vec{e}_2, \measuredangle\vec{h}_1\right) = (-\measuredangle\vec{e}_2) \cup (-\measuredangle\vec{e}_2)$$

$$E\left(\measuredangle\vec{e}_2, \measuredangle\vec{h}_1\right) = -\measuredangle\vec{e}_2$$


$\vec{h}_1$
$-\vec{e}_1$
$E(A,B)$
B
e=a_0=h
$\vec{e}_2$
$\vec{h}_2$
$E(e,\measuredangle\vec{h}_2) = E(\measuredangle\vec{e}_2,h)$
A
$\vec{e}_1$
$\vec{h}_1$
$E(A,B)$
B
e=a_0=h
$\vec{e}_2$
$\vec{h}_2$
$E(e,\measuredangle\vec{h}_2) = E(\measuredangle\vec{e}_2,h)$
A
$\vec{e}_1$

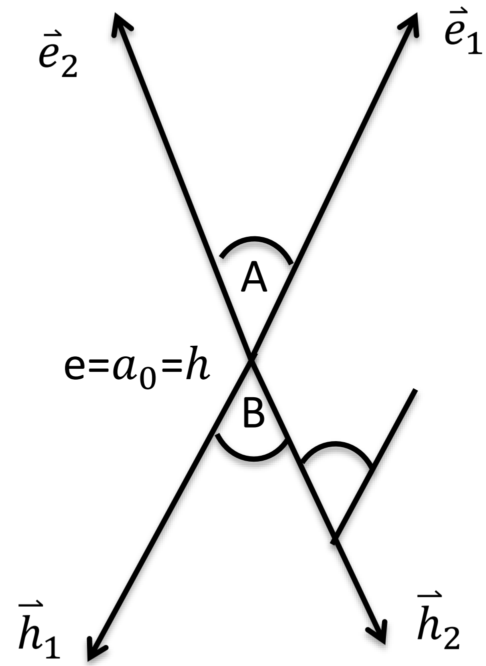
$C(1,1)$

Symmetric 2D angle-angle entrance



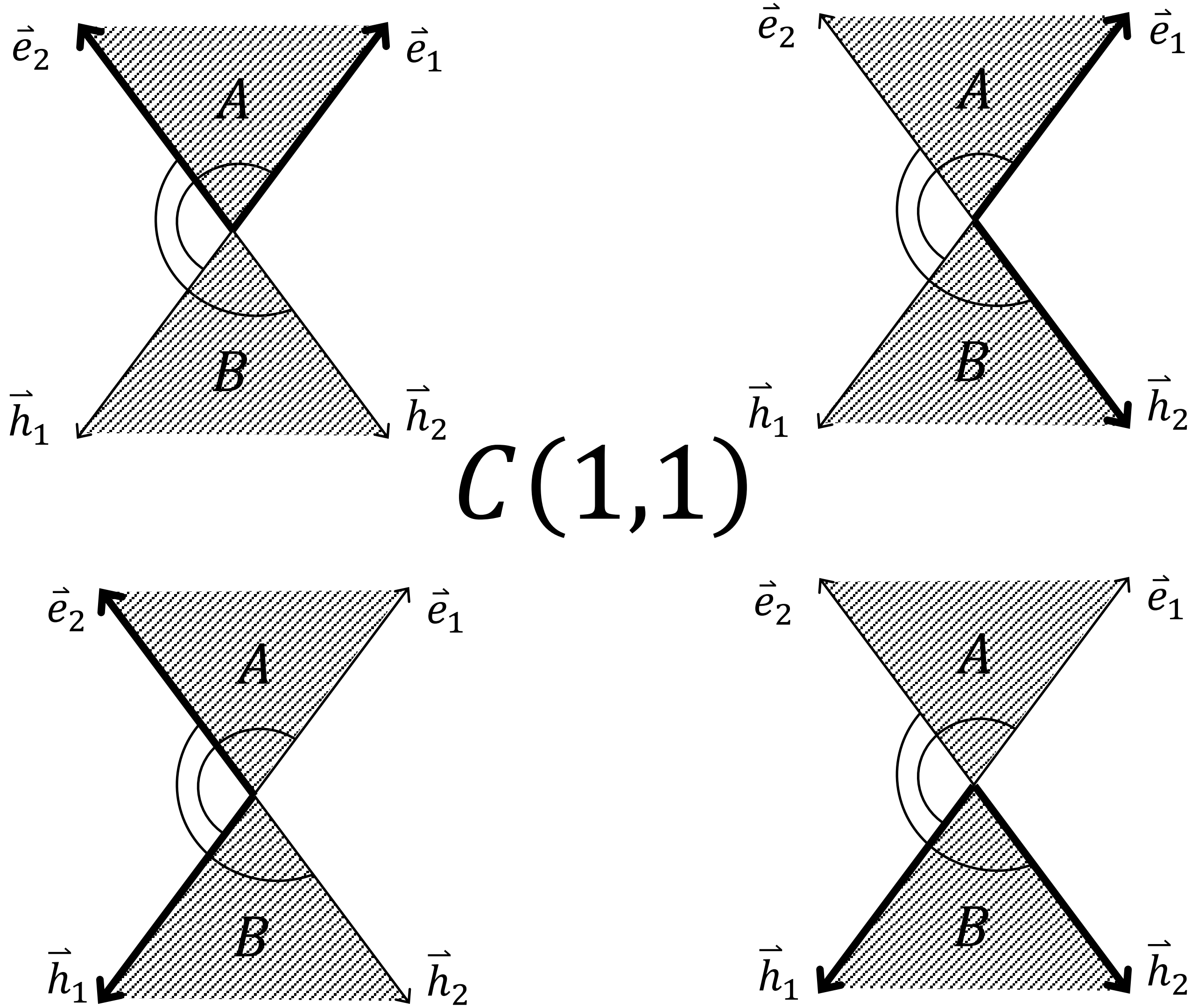
$E(e, \measuredangle h_1) = E(\measuredangle e_1, h)$

$C(1,1)$



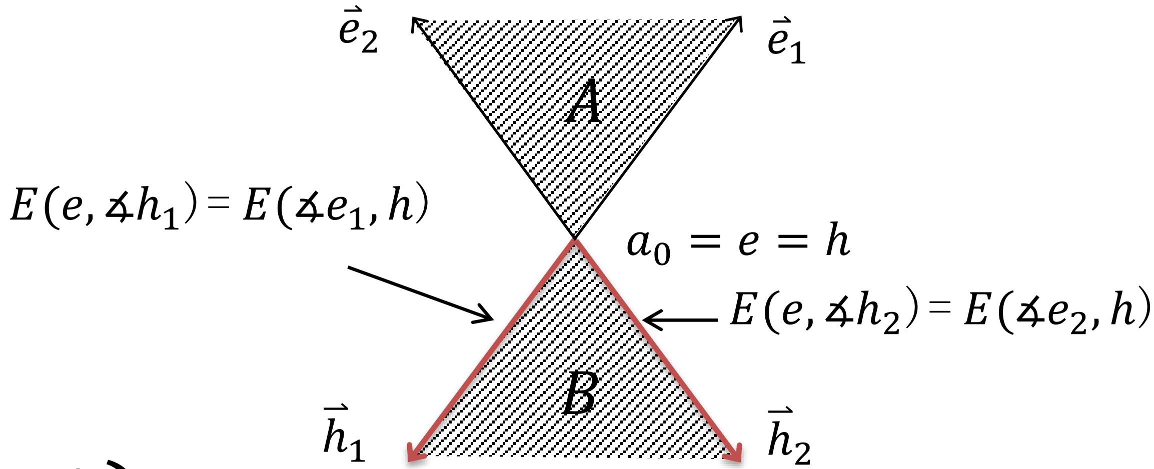
$E(e, \measuredangle h_2) = E(\measuredangle e_2, h)$

Reference lines of Two 2D Symmetric Angles

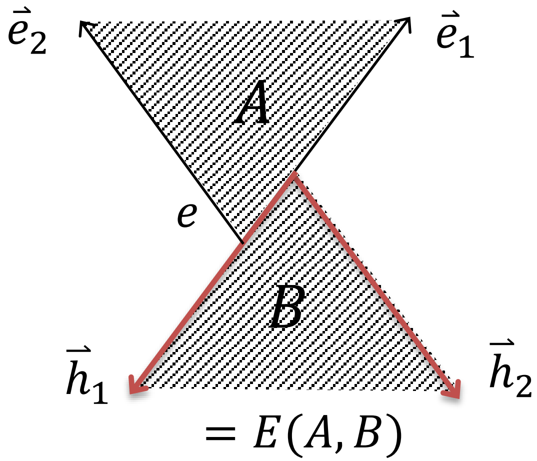


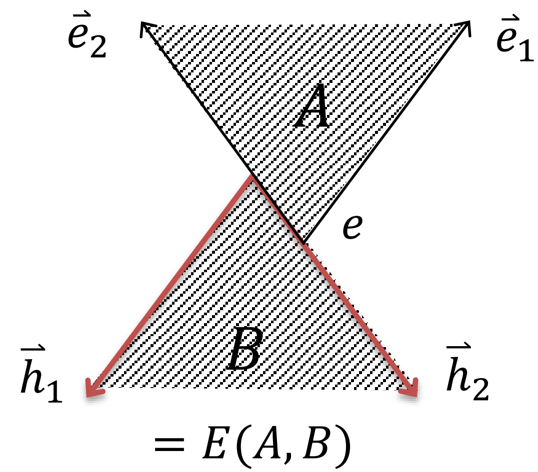
$C(1,1)$

Entrance of Two 2D Symmetric Angles

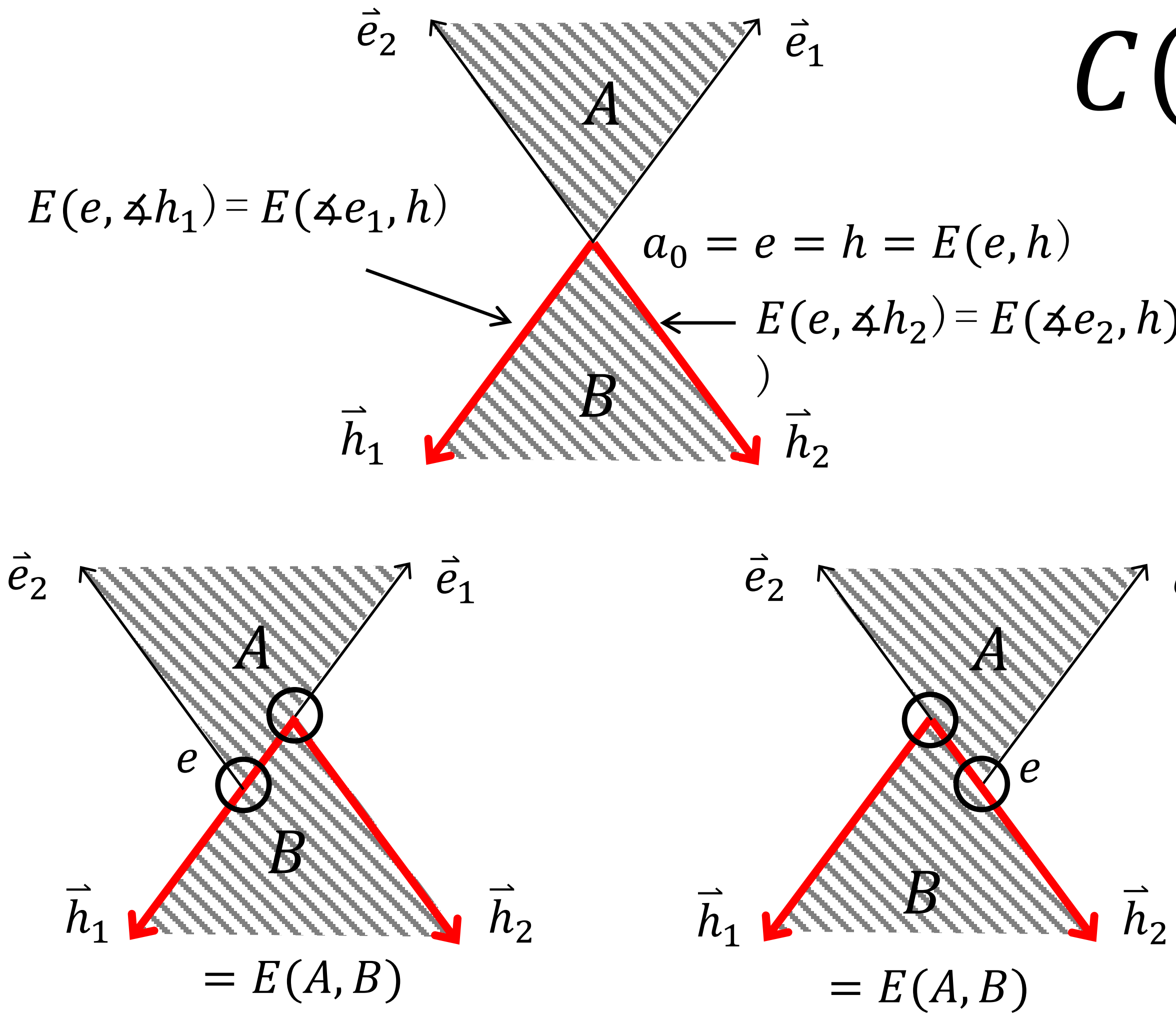


$$C(0,1)$$





$C(1,1)$



$\vec{e}_2$
$\vec{e}_1$
A
$E(e, \measuredangle h_1) = E(\measuredangle e_1, h)$
$a_0 = e = h = E(e, h)$
$E(e, \measuredangle h_2) = E(\measuredangle e_2, h)$
B
$\vec{h}_1$
$\vec{h}_2$
$\vec{e}_2$
$\vec{e}_1$
A
e
B
$\vec{h}_1$
$\vec{h}_2$
$= E(A, B)$
$\vec{e}_2$
$\vec{e}_1$
A
e
B
$\vec{h}_1$
$\vec{h}_2$
$= E(A, B)$

Entrance Round-Corner Angle of Two Round-Corner Angles

A_0:

$$(\boldsymbol{x} - \boldsymbol{e}) \cdot \vec{n}_{11} \geq 0$$
$$(\boldsymbol{x} - \boldsymbol{e}) \cdot \vec{n}_{12} \geq 0$$

$$A_0 = (\uparrow n_{11} + \boldsymbol{e}) \cap (\uparrow n_{12} + \boldsymbol{e})$$

$$\vec{n}_{11} \cdot \vec{n}_{11} = 1, \qquad \vec{n}_{12} \cdot \vec{n}_{12} = 1$$

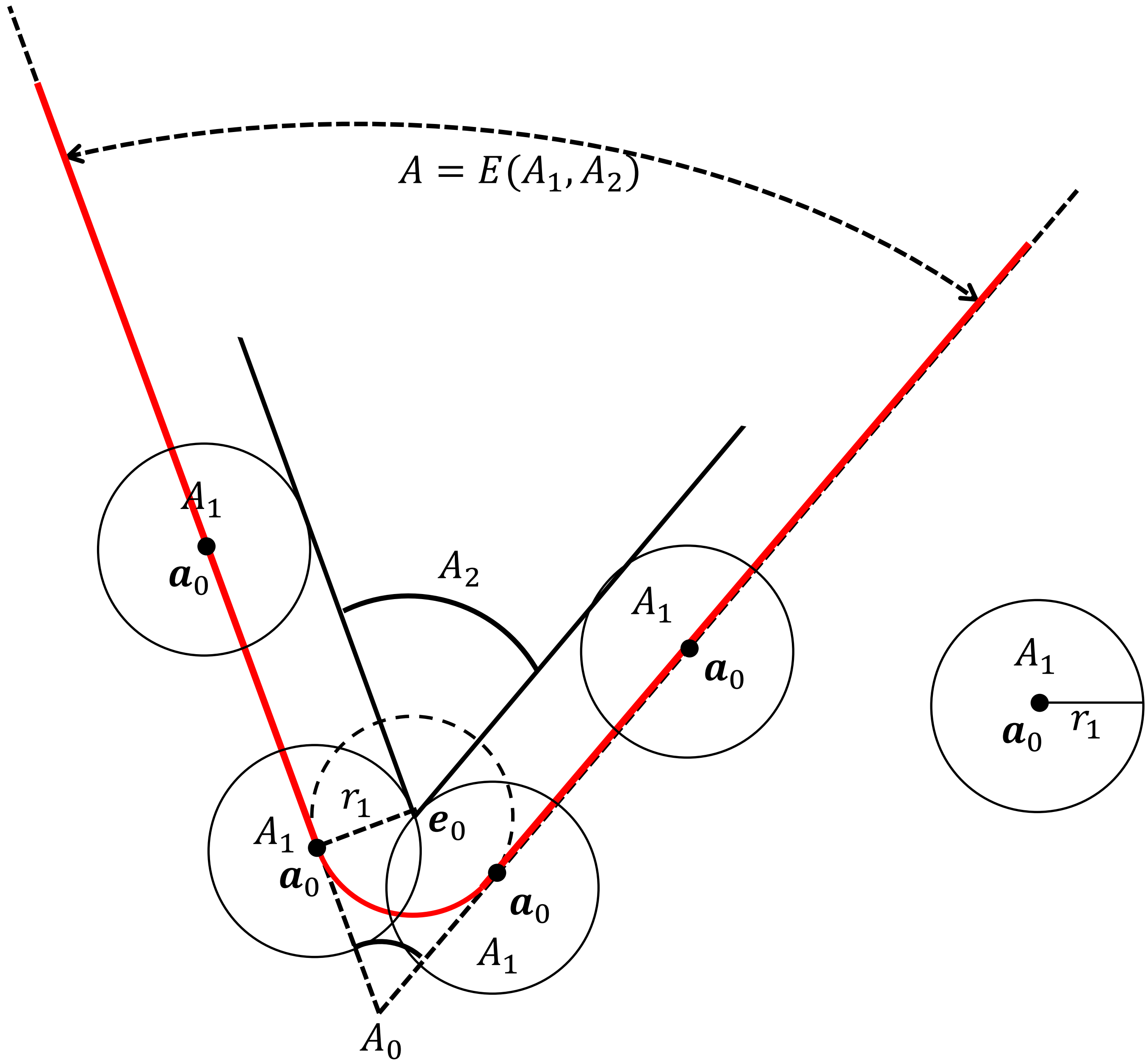
$$A_1 = \{|\boldsymbol{x} - \boldsymbol{a}_0| \leq r_1\},$$

Theorem of round corner solid angle

$$A = E(A_1, A_2)$$

$$A_2 = (\uparrow n_{11} + \boldsymbol{e}_0)$$
$$\cap (\uparrow n_{12} + \boldsymbol{e}_0)$$

$$\boldsymbol{e_0} = \boldsymbol{e} + r_1\,(\vec{n}_{11} + \vec{n}_{12})/(1 + \vec{n}_{11} \cdot \vec{n}_{12})$$



$A = E(A_1, A_2)$
A_1
$\boldsymbol{a}_0$
A_2
A_1
$\boldsymbol{a}_0$
A_1
$\boldsymbol{a}_0$
r_1
r_1
$\boldsymbol{e}_0$
A_1
$\boldsymbol{a}_0$
$\boldsymbol{a}_0$
A_1
A_0

Theorem of entrance of round corner solid angle and concave solid angle

$$B = (\uparrow n_{21} + \boldsymbol{h}) \cup (\uparrow n_{22} + \boldsymbol{h})$$

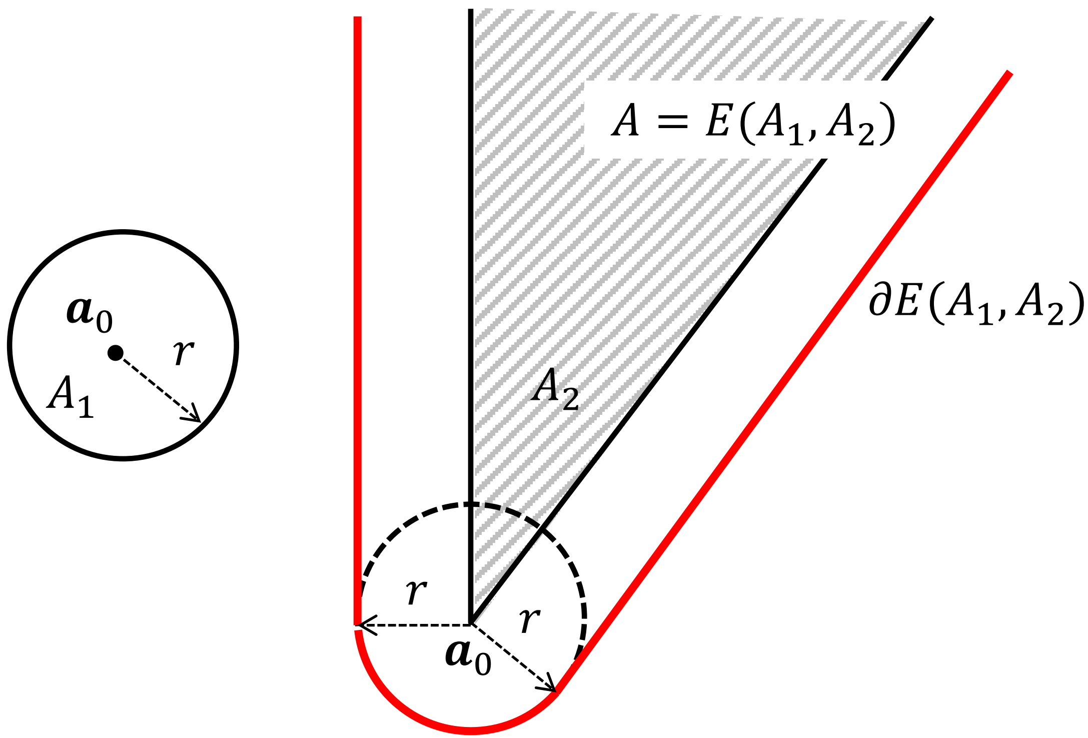
$$\vec{n}_{21} \cdot \vec{n}_{21} = 1, \qquad \vec{n}_{22} \cdot \vec{n}_{22} = 1$$

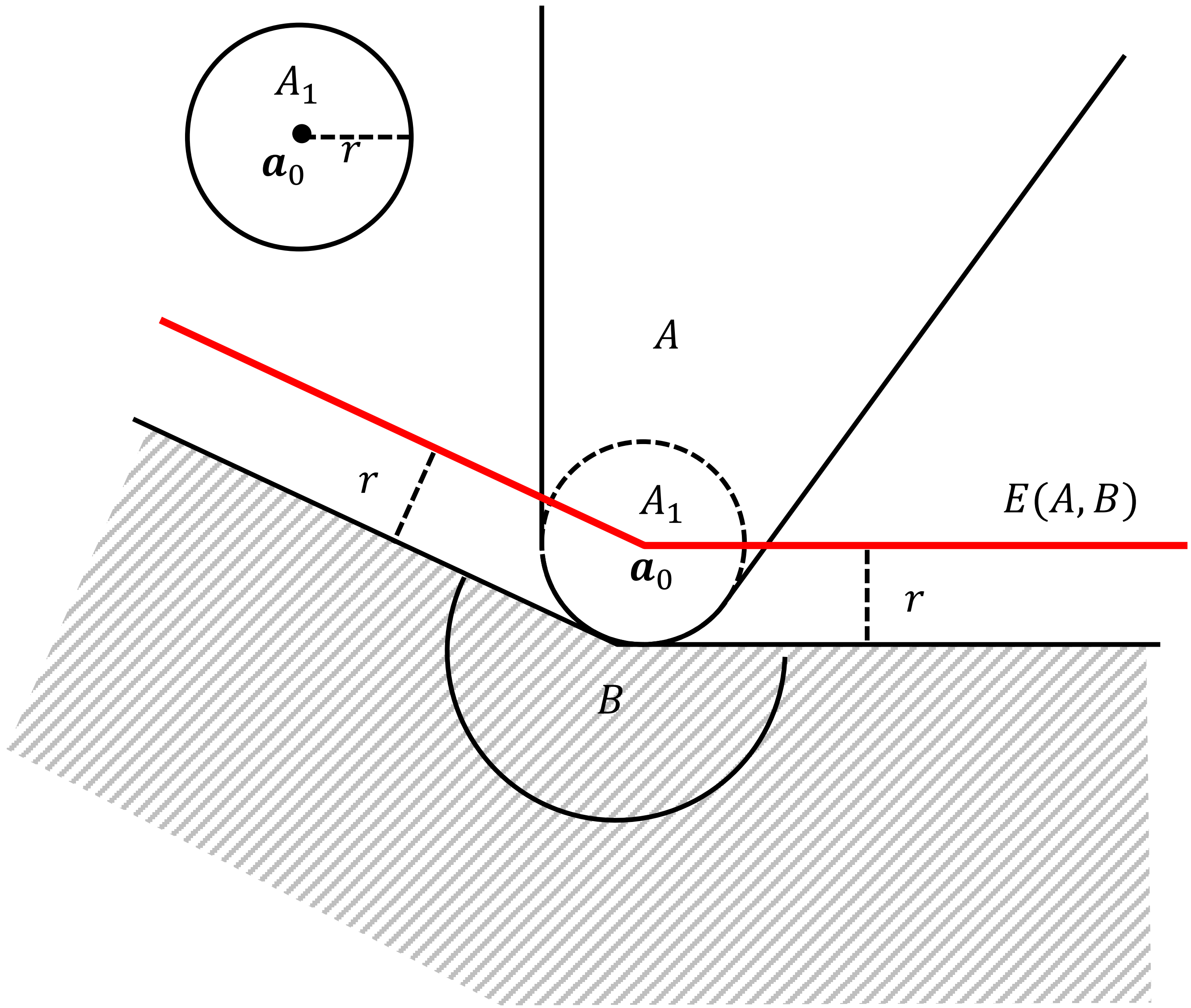
$$int(\uparrow n_{11} \cap \uparrow n_{12}) \cap int(\uparrow n_{21} \cup \uparrow n_{22}) = \emptyset$$

$$\Rightarrow$$

$$E(A, B) = (\uparrow n_{21} + \boldsymbol{h}_0) \cup (\uparrow n_{22} + \boldsymbol{h}_0)$$

$$\boldsymbol{h}_0 = \boldsymbol{h} - r\,(\vec{n}_{21} + \vec{n}_{22})/(1 + \vec{n}_{21} \cdot \vec{n}_{22})$$


$A = E(A_1, A_2)$
$\boldsymbol{a}_0$
r
A_1
A_2
$\partial E(A_1, A_2)$
r
r
$\boldsymbol{a}_0$



A_1
r
$\boldsymbol{a}_0$
A
r
A_1
$\boldsymbol{a}_0$
$E(A, B)$
r
B

B_0:

$$(\boldsymbol{x}-\boldsymbol{h})\cdot\vec{n}_{21}\geq 0$$
$$(\boldsymbol{x}-\boldsymbol{h})\cdot\vec{n}_{22}\geq 0$$

$$B_0=(\uparrow n_{21}+\boldsymbol{h})\ \cap(\uparrow n_{22}+\boldsymbol{h})$$

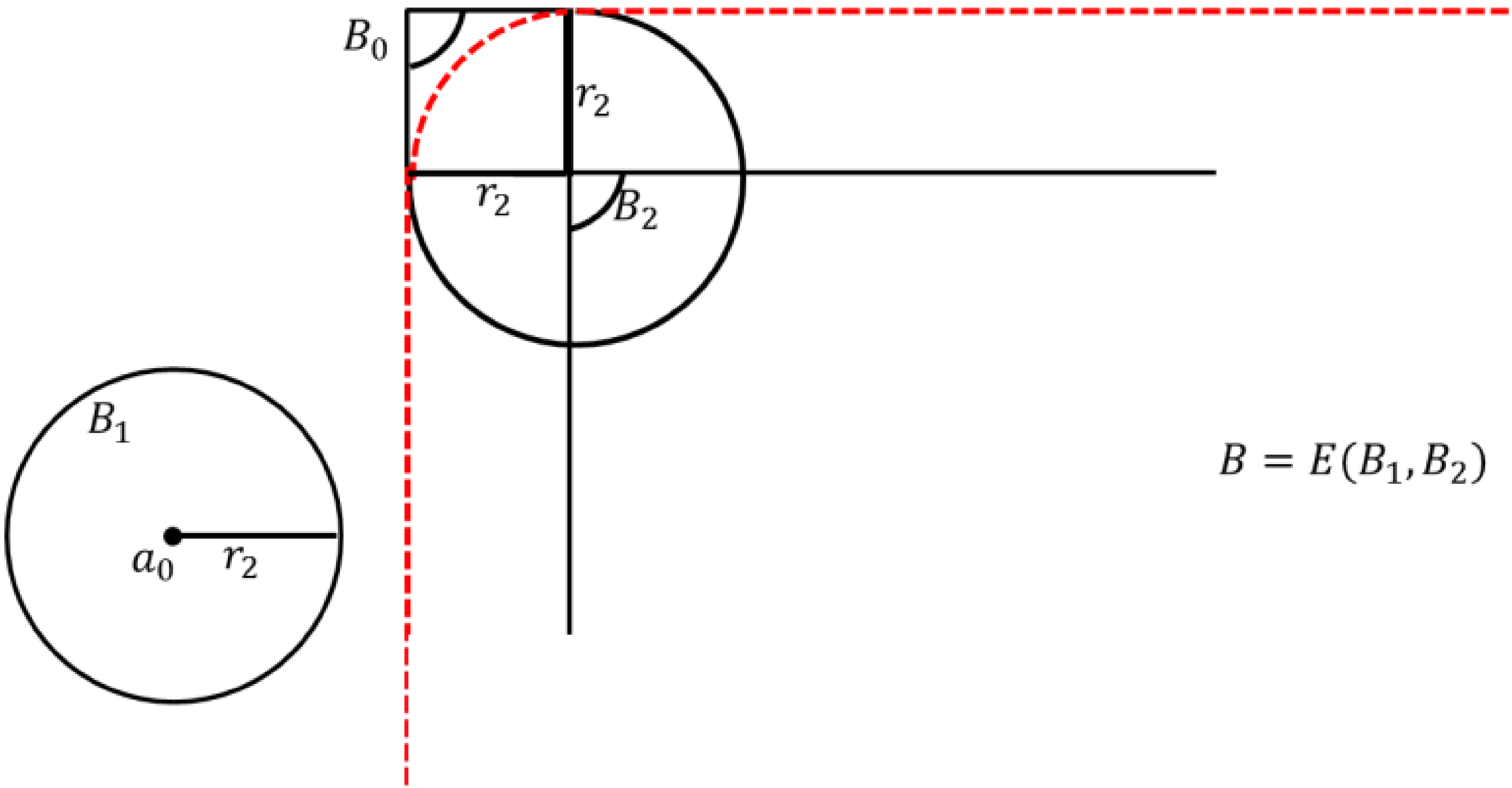
$$\vec{n}_{21}\cdot\vec{n}_{21}=1,\qquad \vec{n}_{22}\cdot\vec{n}_{22}=1$$

$$B_1=\{|\boldsymbol{x}-\boldsymbol{a}_0|\leq r_2\},$$

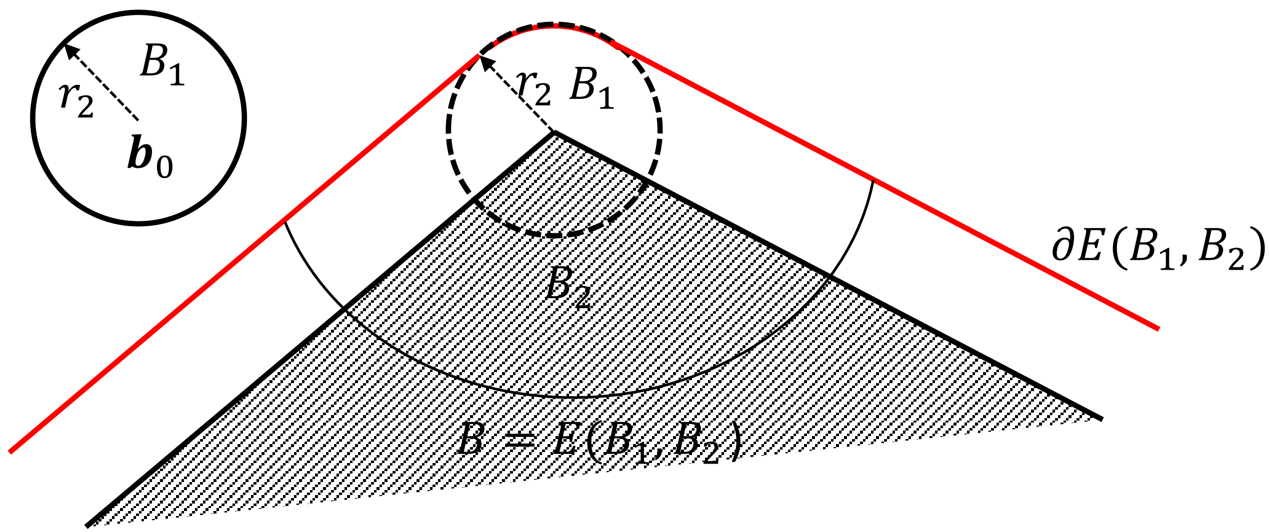
$$A_1 = \quad \{|\boldsymbol{x} - \boldsymbol{a}_0| \le r_1\}$$
$$B_1 = \quad \{|\boldsymbol{x} - \boldsymbol{b}_0| \le r_2\}$$

$$\boldsymbol{e}_0 = \boldsymbol{e} + r_1\,(\vec{n}_{11} + \vec{n}_{12})/(1 + \vec{n}_{11} \cdot \vec{n}_{12})$$
$$\boldsymbol{h}_0 = \boldsymbol{h} + r_2\,(\vec{n}_{21} + \vec{n}_{22})/(1 + \vec{n}_{21} \cdot \vec{n}_{22})$$

$$A_2 = \{(\boldsymbol{x} - \boldsymbol{e}_0) \cdot \vec{n}_{11} \ge 0\}$$
$$\cap \{(\boldsymbol{x} - \boldsymbol{e}_0) \cdot \vec{n}_{12} \ge 0\}$$
$$B_2 = \{(\boldsymbol{x} - \boldsymbol{h}_0) \cdot \vec{n}_{21} \ge 0\}$$
$$\cap \{(\boldsymbol{x} - \boldsymbol{h}_0) \cdot \vec{n}_{22} \ge 0\}$$



B_0
r_2
r_2
B_2
B_1
a_0
r_2
$B = E(B_1, B_2)$


B_1
r_2
$\boldsymbol{b}_0$
r_2 B_1
$\partial E(B_1, B_2)$
B_2
$B = E(B_1, B_2)$

A_2:

$$(\boldsymbol{x} - \boldsymbol{e}_0) \cdot \vec{n}_{11} \geq 0,$$
$$(\boldsymbol{x} - \boldsymbol{e}_0) \cdot \vec{n}_{12} \geq 0.$$

B_2:

$$(\boldsymbol{x} - \boldsymbol{h}_0) \cdot \vec{n}_{21} \geq 0,$$
$$(\boldsymbol{x} - \boldsymbol{h}_0) \cdot \vec{n}_{22} \geq 0.$$

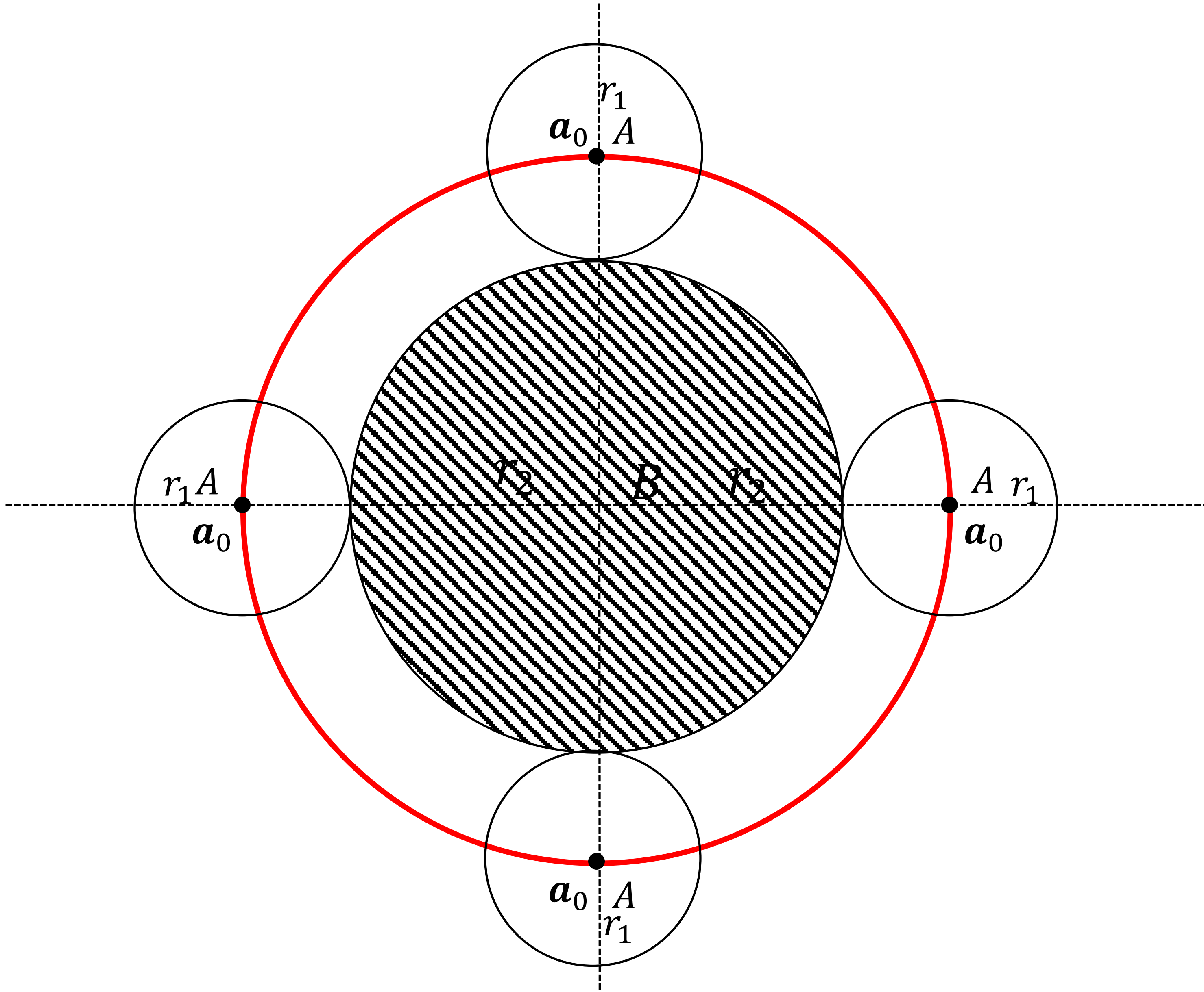
$$A = E(A_1, A_2) = A_2 + \boldsymbol{a_0} - A_1,$$

$$B = E(B_1, B_2) = B_2 + \boldsymbol{b_0} - B_1.$$

$E(A,B)$ is the entrance block of a disk and an angle.

$$E(A_1,B_1)=\{|\boldsymbol{x}-\boldsymbol{a}_0|\leq r_1+r_2\}$$

$$E(A,B)=E\big(E(A_1,B_1),E(A_2,B_2)\big).$$



r_1
$\boldsymbol{a}_0$
A
r_1 A
$\boldsymbol{a}_0$
r_2
B
r_2
A r_1
$\boldsymbol{a}_0$
$\boldsymbol{a}_0$
A
r_1

$$E(A, B) = E\big(E(A_1, A_2), E(B_1, B_2)\big)$$

$$= E(B_1, B_2) - E(A_1, A_2) + \boldsymbol{e}_0$$

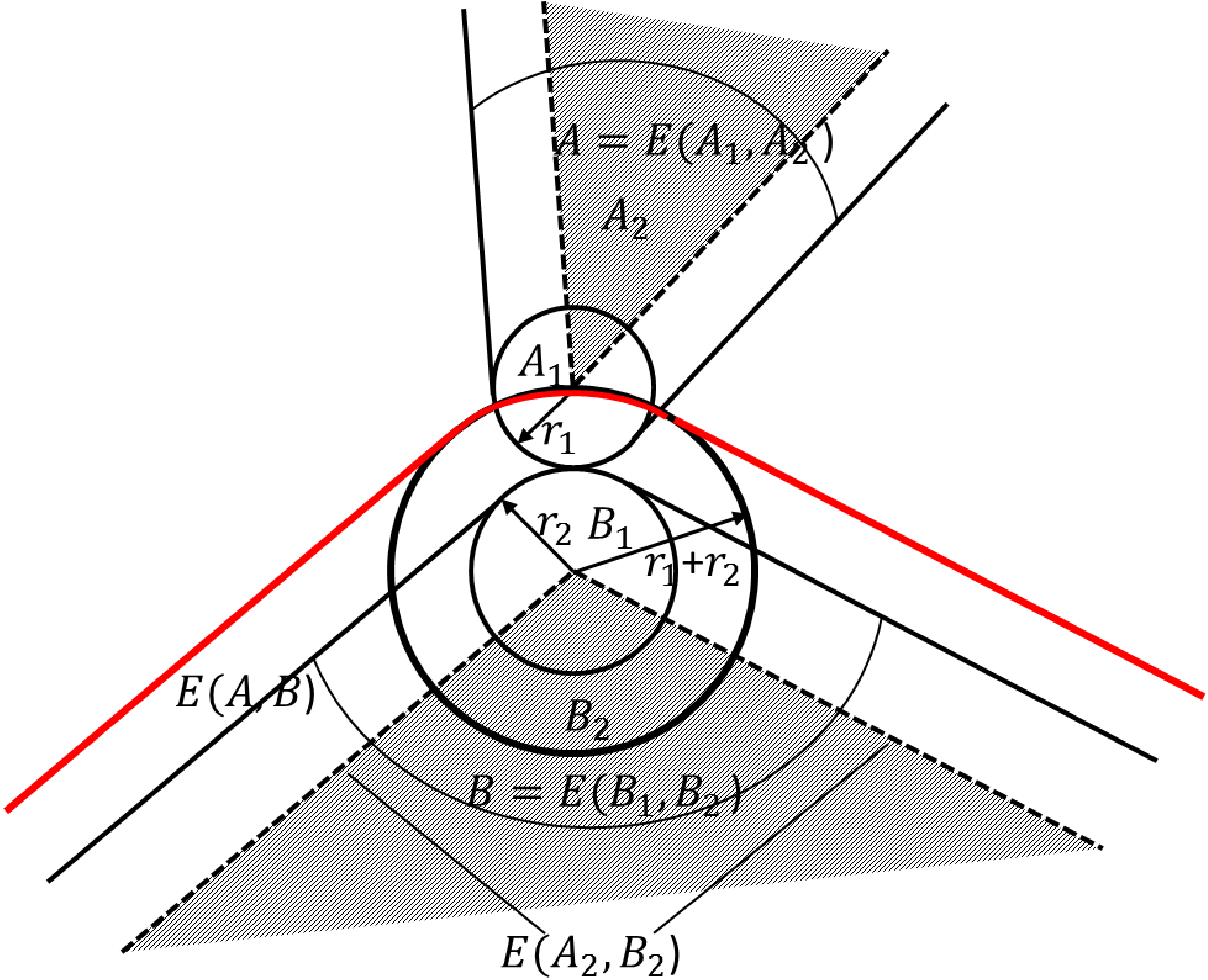
$$= (B_2 - B_1 + \boldsymbol{b}_0) + \boldsymbol{e}_0 - (A_2 - A_1 + \boldsymbol{a}_0)$$

$$= (B_2 - A_2 + \boldsymbol{e}_0) + (\boldsymbol{b}_0 - B_1) + (A_1 - \boldsymbol{a}_0)$$

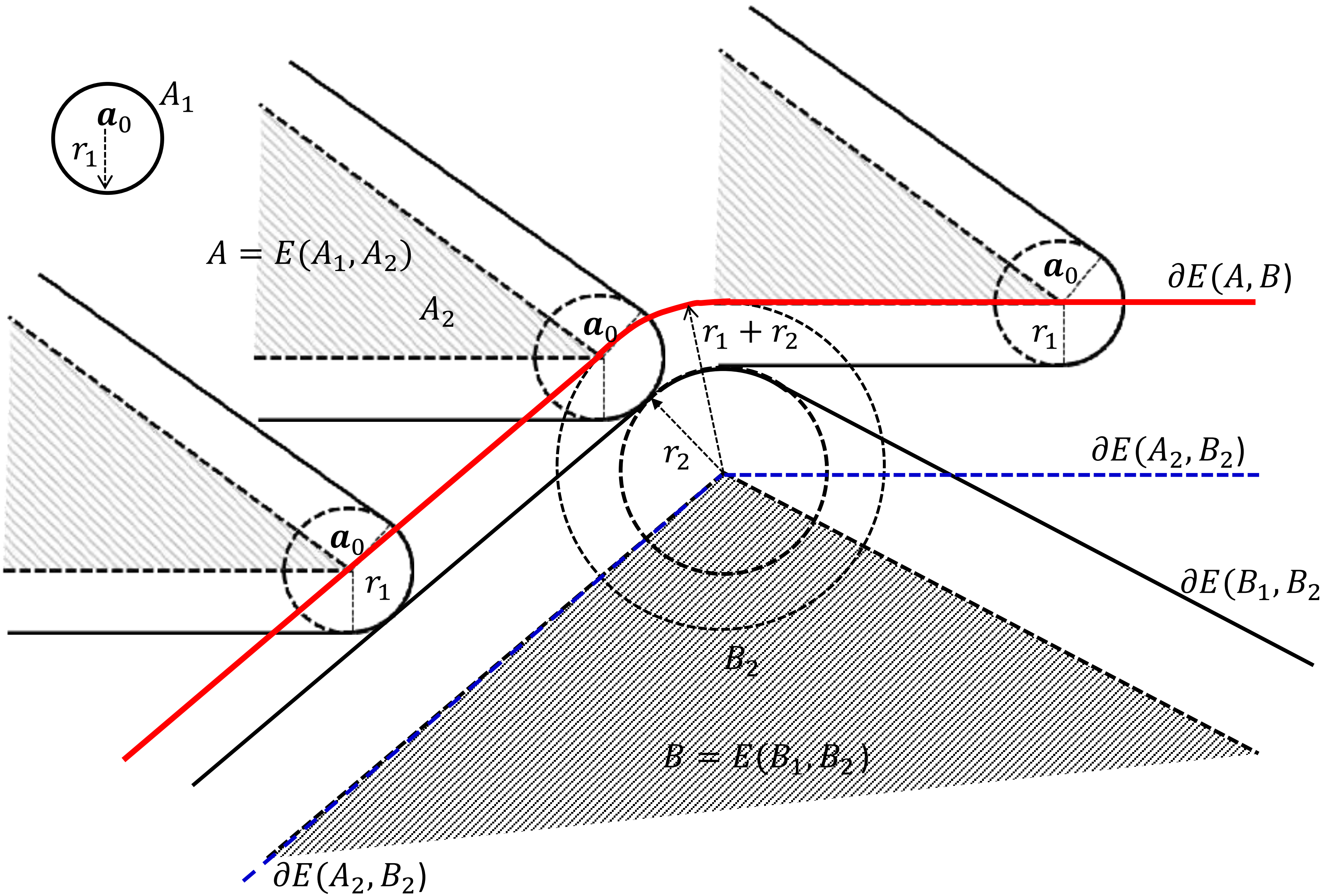
$$= E(A_2, B_2) - B_1 + \boldsymbol{b}_0 + A_1 - \boldsymbol{a}_0$$

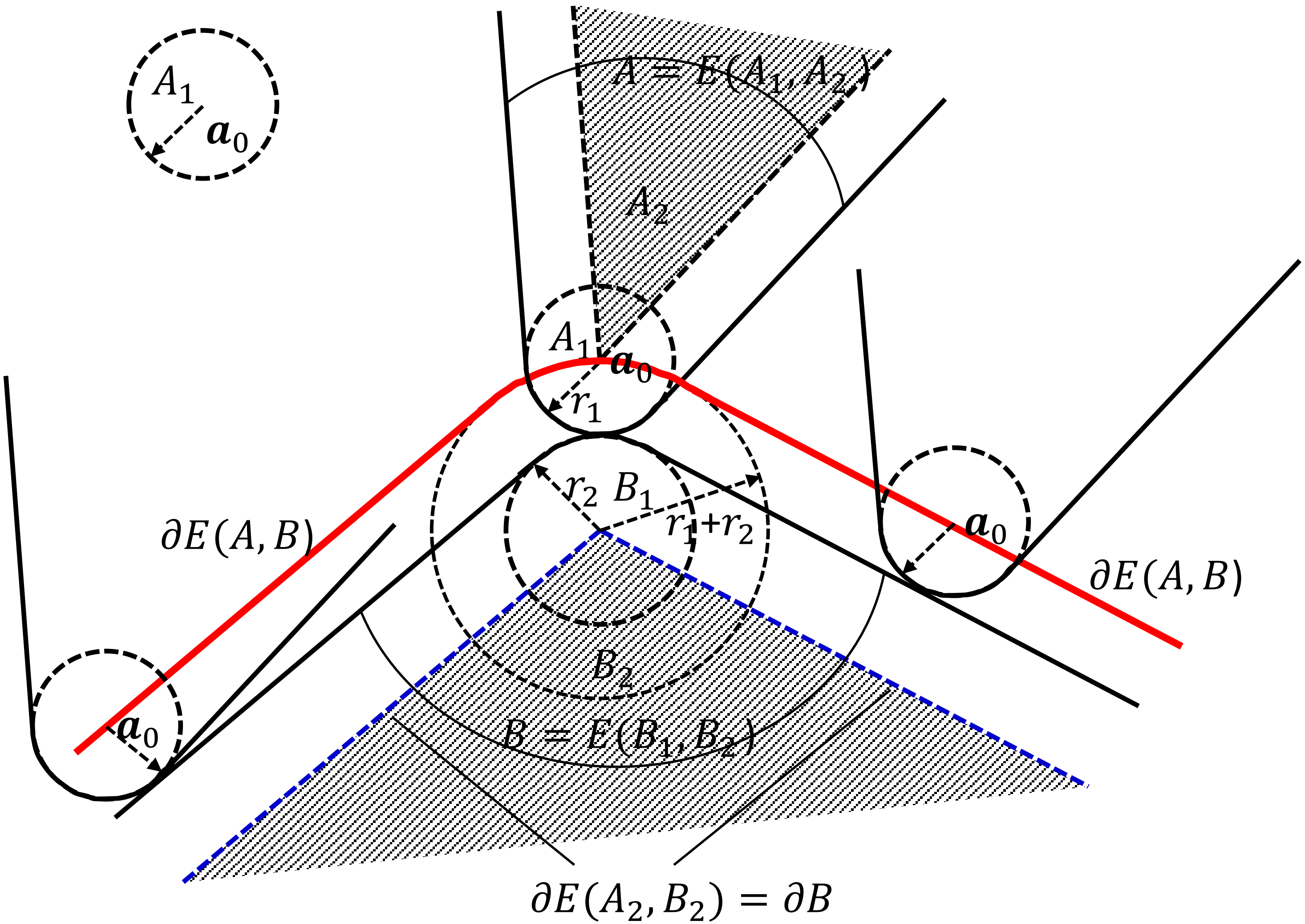
$$= E(A_2, B_2) - E(A_1, B_1) + \boldsymbol{b}_0$$

$$= E\big(E(A_1, B_1), E(A_2, B_2)\big)$$



$A = E(A_1, A_2)$
A_2
A_1
r_1
r_2
B_1
$r_1 + r_2$
$E(A, B)$
B_2
$B = E(B_1, B_2)$
$E(A_2, B_2)$


A_1
a_0
r_1
$A = E(A_1, A_2)$
A_2
a_0
a_0
r_1
a_0
r_1
$\partial E(A, B)$
$r_1 + r_2$
r_2
$\partial E(A_2, B_2)$
$\partial E(B_1, B_2)$
B_2
$B = E(B_1, B_2)$
$\partial E(A_2, B_2)$



A_1
$\boldsymbol{a}_0$
$A = E(A_1, A_2)$
A_2
A_1
$\boldsymbol{a}_0$
r_1
r_2 B_1
$r_1 + r_2$
$\partial E(A, B)$
$\boldsymbol{a}_0$
$\partial E(A, B)$
$\boldsymbol{a}_0$
B_2
$B = E(B_1, B_2)$
$\partial E(A_2, B_2) = \partial B$

2D Entrance Block of Blocks

Boundary
of
2D Entrance Block

$$\partial E(A,B) \subset E(\partial A, \partial B)$$

$$\partial E(A,B) \subset E\big(A(1), B(1)\big)$$

$$\partial E(A,B) \subset E\big(A(0), B(1)\big) \cup E\big(A(1), B(0)\big)$$

$$A = \boldsymbol{a}_1 \boldsymbol{a}_2 \cdots \boldsymbol{a}_{p-1} \boldsymbol{a}_p, \ \boldsymbol{a}_{p+1} = \boldsymbol{a}_1,$$
$$B = \boldsymbol{b}_1 \boldsymbol{b}_2 \cdots \boldsymbol{b}_{q-1} \boldsymbol{b}_q, \boldsymbol{b}_{q+1} = \boldsymbol{b}_1.$$

$$\partial E(A, B) \subset E\big(A(0), B(1)\big) \cup E\big(A(1), B(0)\big)$$

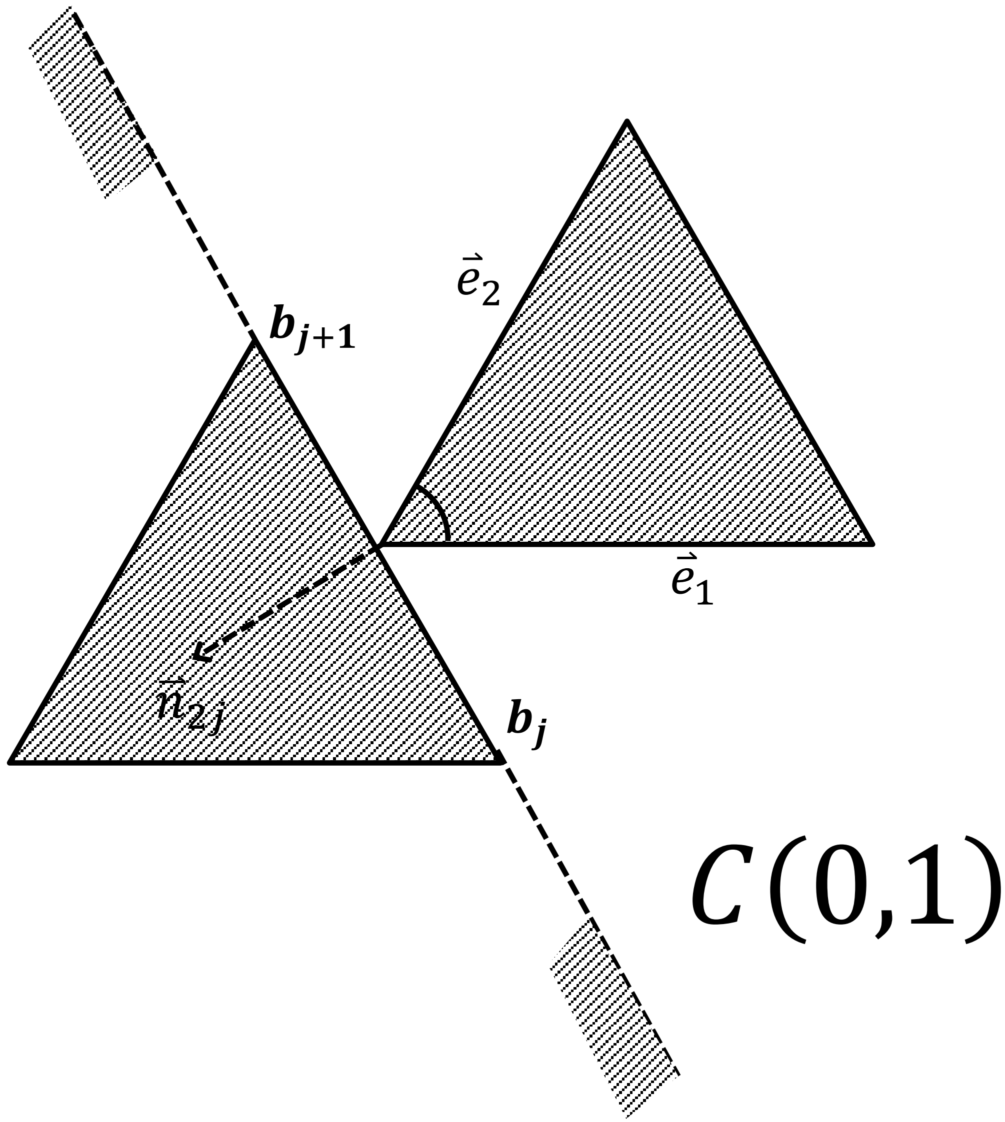
Contact Edge
of
2D Vertex and Edge

Theorem of 2D vertex-vertex contact

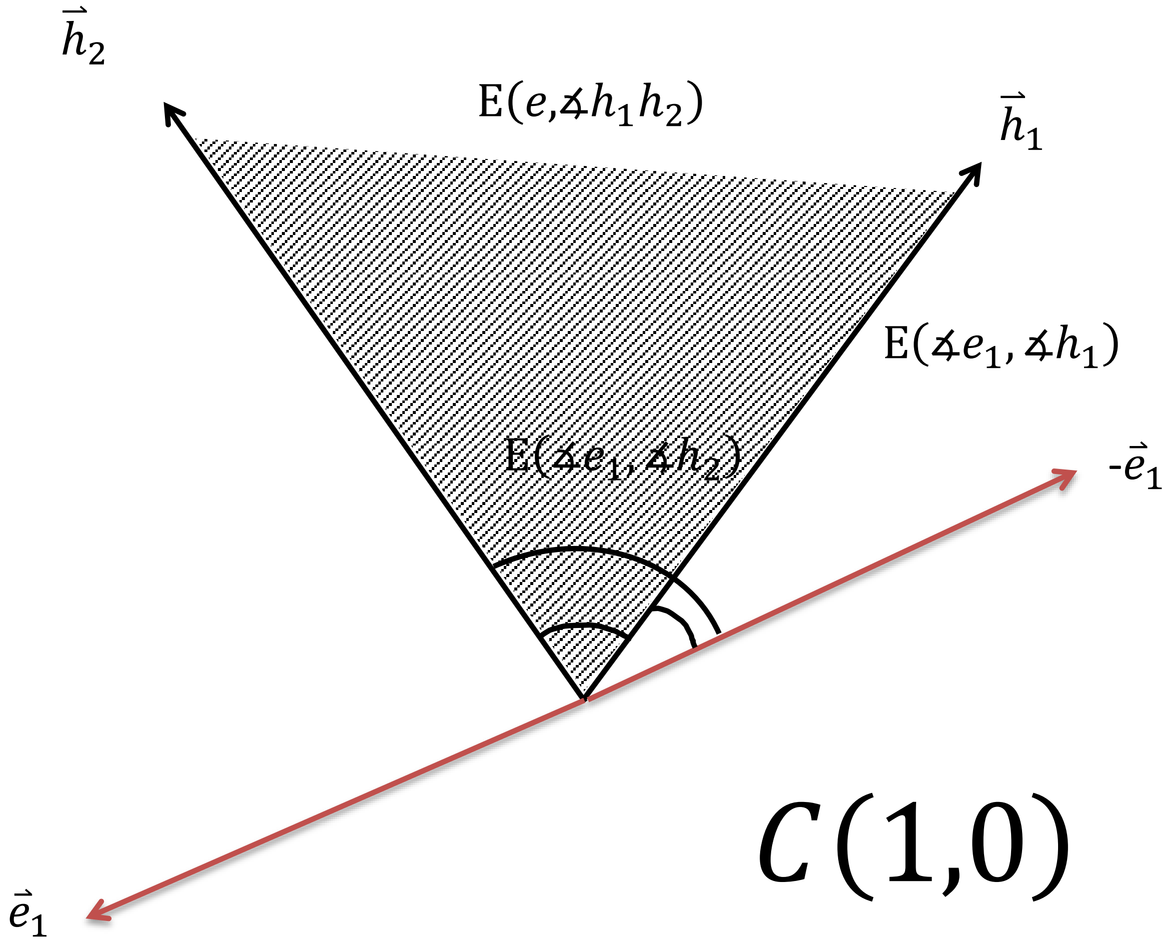
$$E(\boldsymbol{e},\boldsymbol{h}) \in \partial E(A,B)$$

$$\Rightarrow int(\measuredangle\boldsymbol{e}) \cap int(\measuredangle\boldsymbol{h}) = \emptyset$$

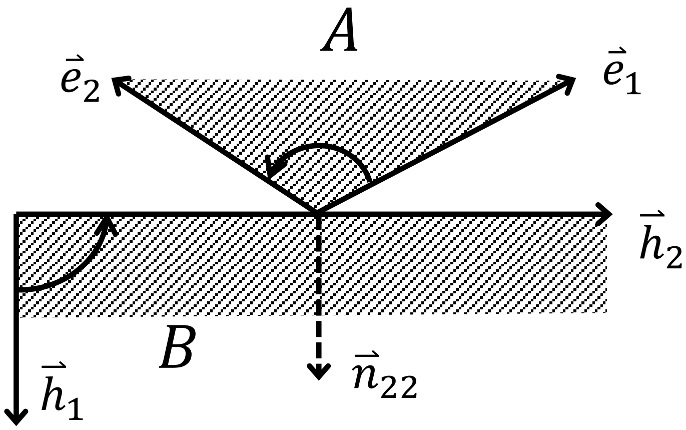
Contact Edge 2D Vertex-Edge Contact



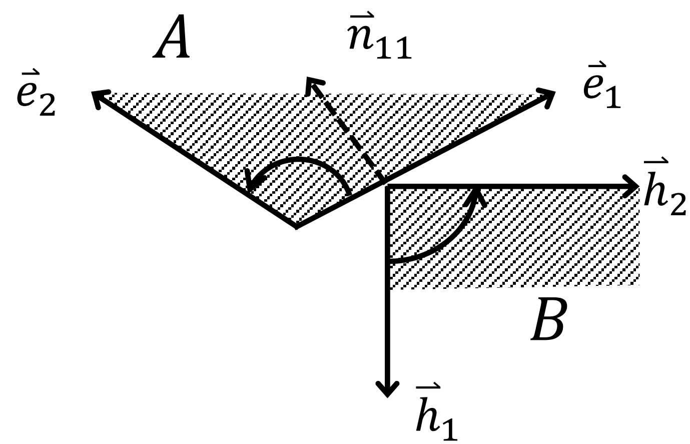
Contact of Edge and Vertex



Entrance of Two 2D Angles

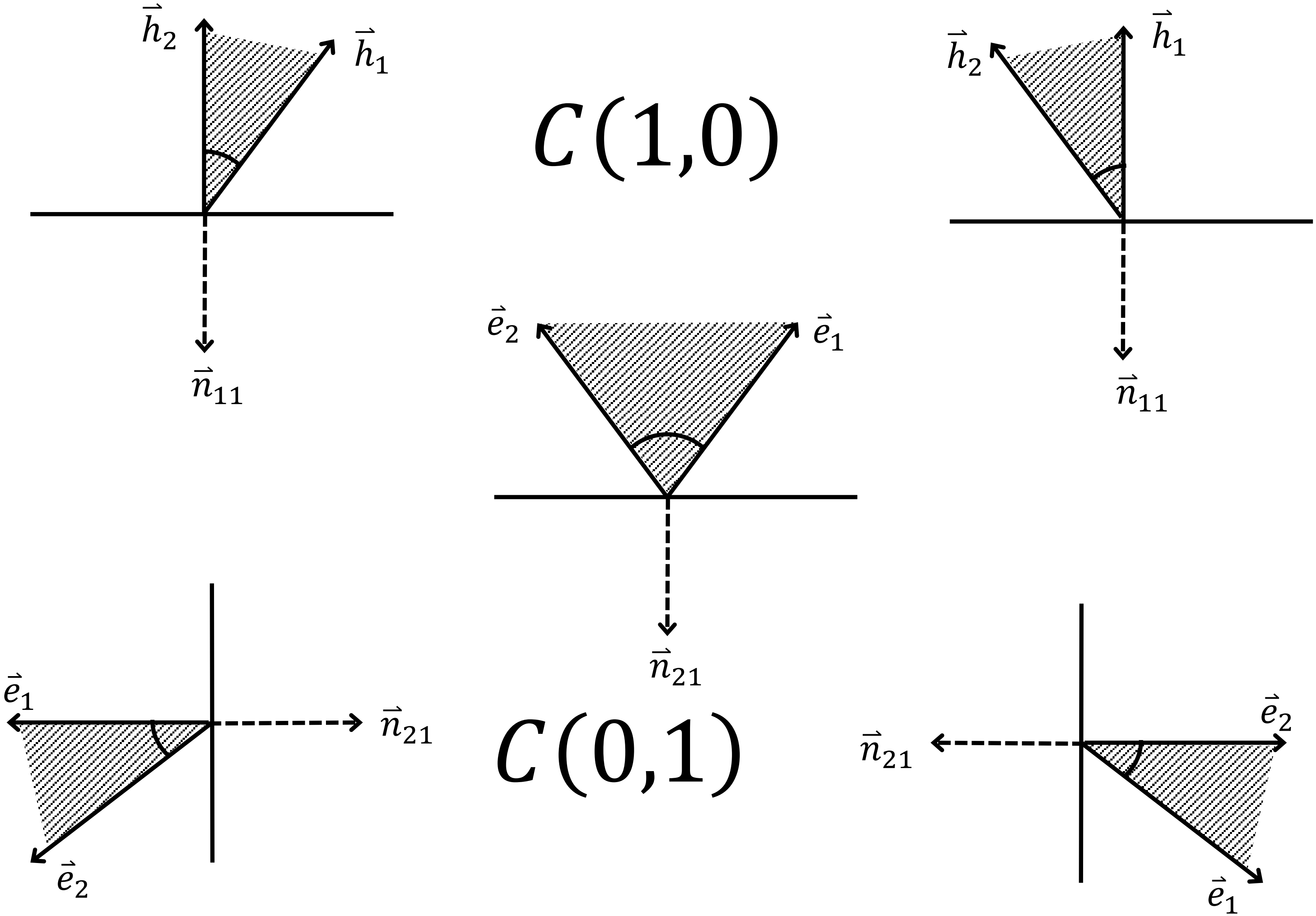


$$C(0,1)$$



$$C(1,0)$$

2D Vertex-Edge Contact



Contact Edges
of
2D Parallel Edges

Theorem of parallel edge contact

$$(\boldsymbol{a}_i \boldsymbol{a}_{i+1}) \parallel (\boldsymbol{b}_j \boldsymbol{b}_{j+1})$$

$$\exists \boldsymbol{a} \in int(\boldsymbol{a}_i \boldsymbol{a}_{i+1})$$

$$\exists \boldsymbol{b} \in int(\boldsymbol{b}_j \boldsymbol{b}_{j+1})$$

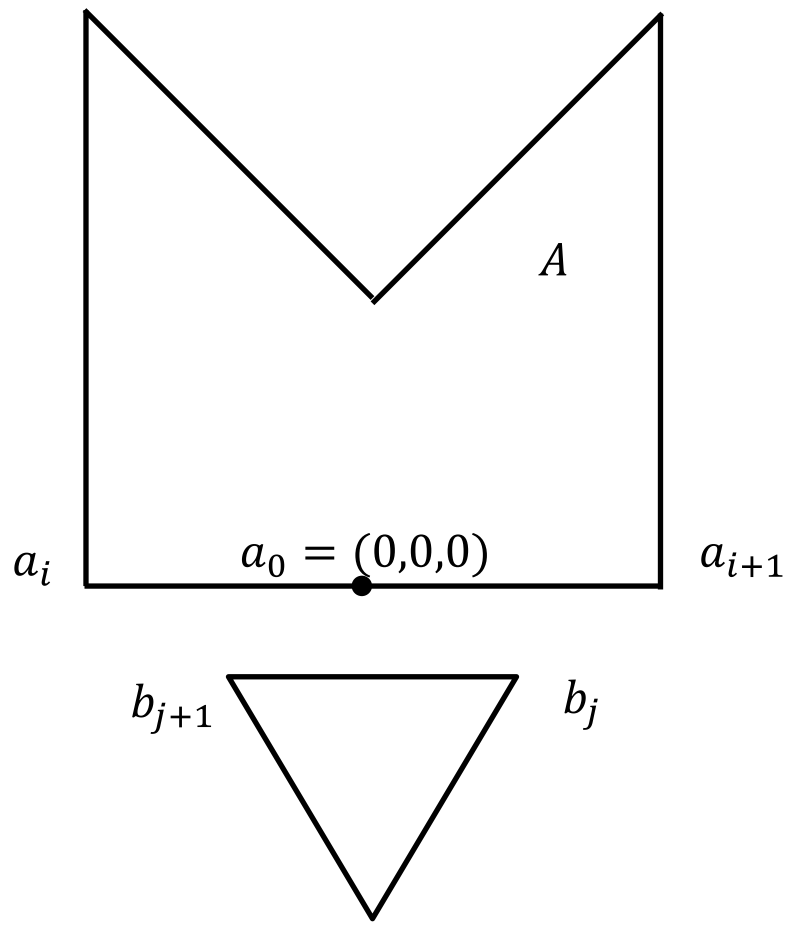
$$E(\boldsymbol{a}, \boldsymbol{b}) \in \partial E(A, B) \Rightarrow \overrightarrow{m}_{1i} \upuparrows -\overrightarrow{m}_{2j}$$

Theorem of parallel edge covers

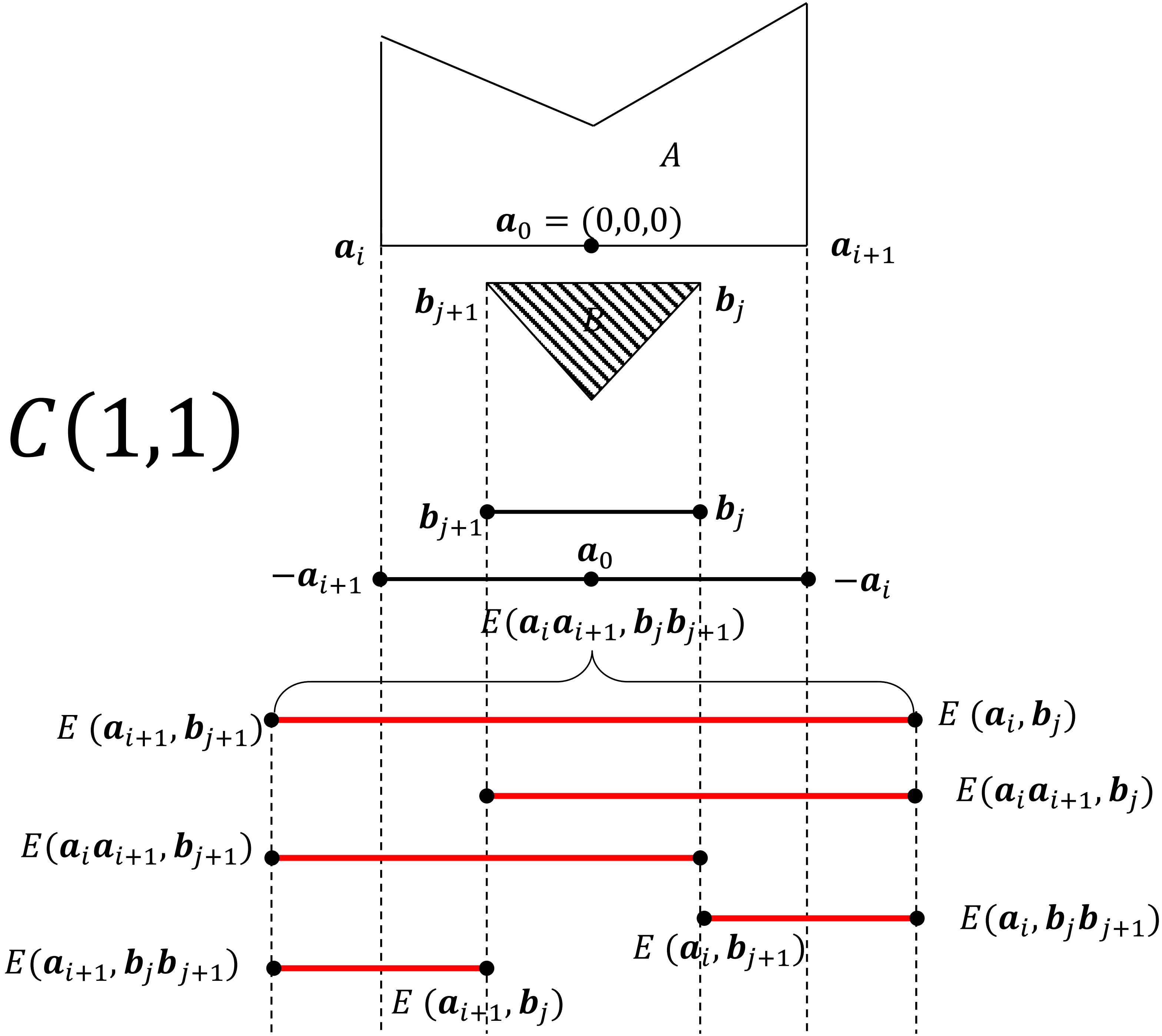
$$(\boldsymbol{a}_i\boldsymbol{a}_{i+1}) \parallel (\boldsymbol{b}_j\boldsymbol{b}_{j+1})$$

$$E(\boldsymbol{a}_i\boldsymbol{a}_{i+1}, \boldsymbol{b}_j\boldsymbol{b}_{j+1}) =$$
$$E(\boldsymbol{a}_i, \boldsymbol{b}_j\boldsymbol{b}_{j+1}) \cup$$
$$E(\boldsymbol{a}_{i+1}, \boldsymbol{b}_j\boldsymbol{b}_{j+1}) \cup$$
$$E(\boldsymbol{a}_i\boldsymbol{a}_{i+1}, \boldsymbol{b}_j) \cup$$
$$E(\boldsymbol{a}_i\boldsymbol{a}_{i+1}, \boldsymbol{b}_{j+1})$$

Contact Edges of Two Parallel Edges



$C(1,1)$



A
$a_0 = (0,0,0)$
a_i
a_{i+1}
b_{j+1}
B
b_j
C(1,1)
b_{j+1}
b_j
a_0
$-a_{i+1}$
$-a_i$
$E(a_i a_{i+1}, b_j b_{j+1})$
$E\ (a_{i+1}, b_{j+1})$
$E\ (a_i, b_j)$
$E(a_i a_{i+1}, b_j)$
$E(a_i a_{i+1}, b_{j+1})$
$E(a_i, b_j b_{j+1})$
$E\ (a_i, b_{j+1})$
$E(a_{i+1}, b_j b_{j+1})$
$E\ (a_{i+1}, b_j)$

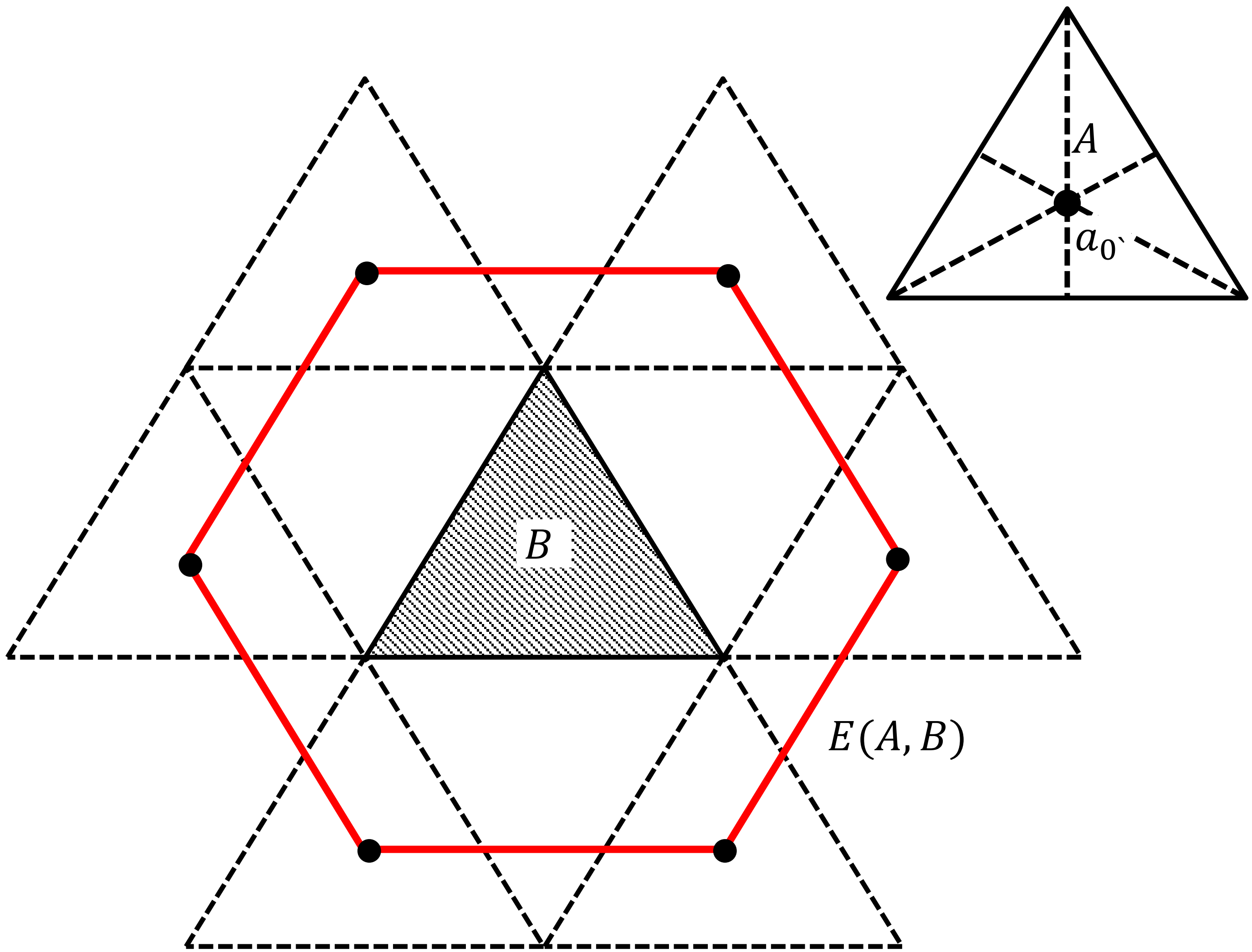
Entrance Block
of
2D Convex Blocks

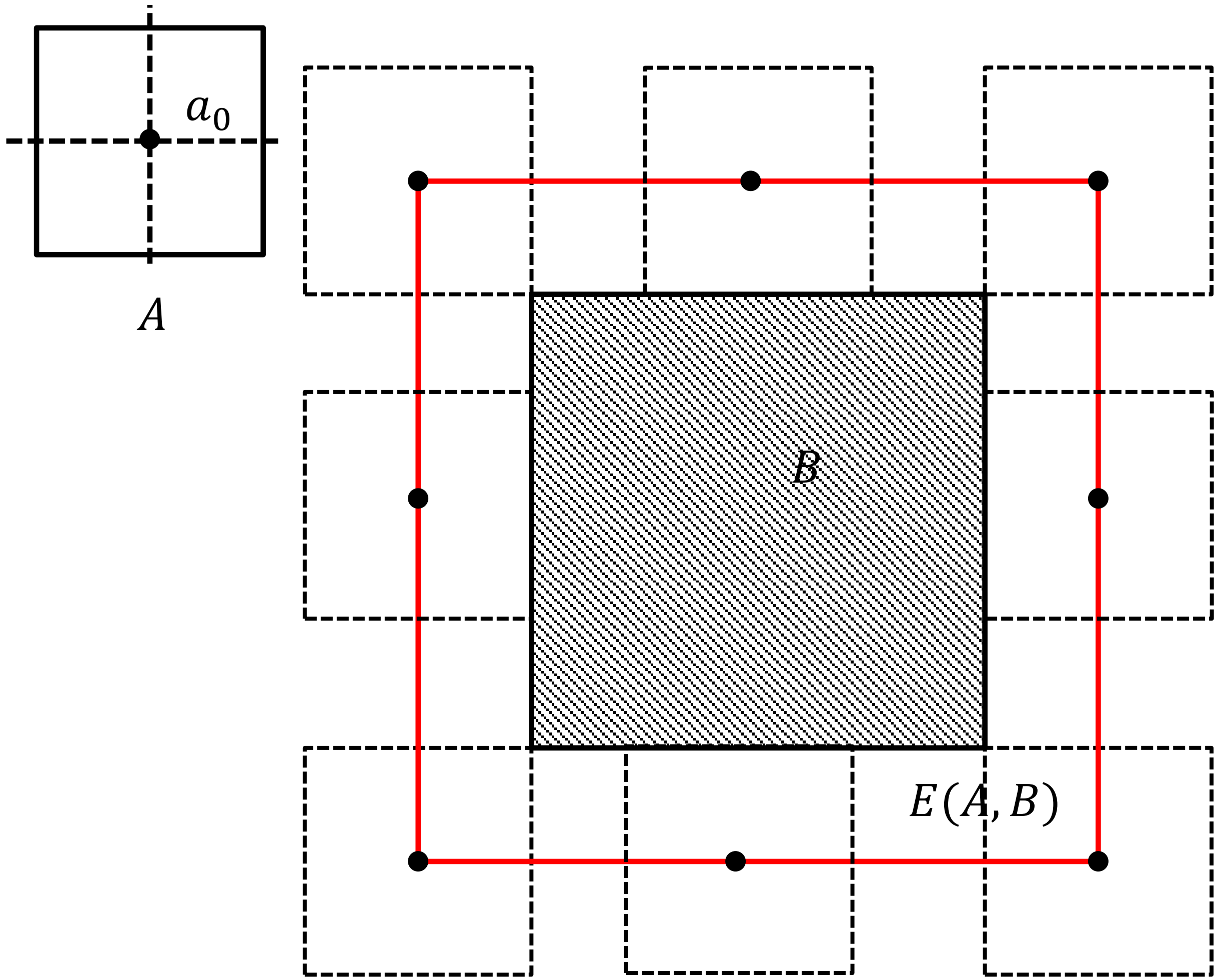
$$\partial E(A,B) \subset C(0,1) \cup C(1,0)$$

If A, B are convex

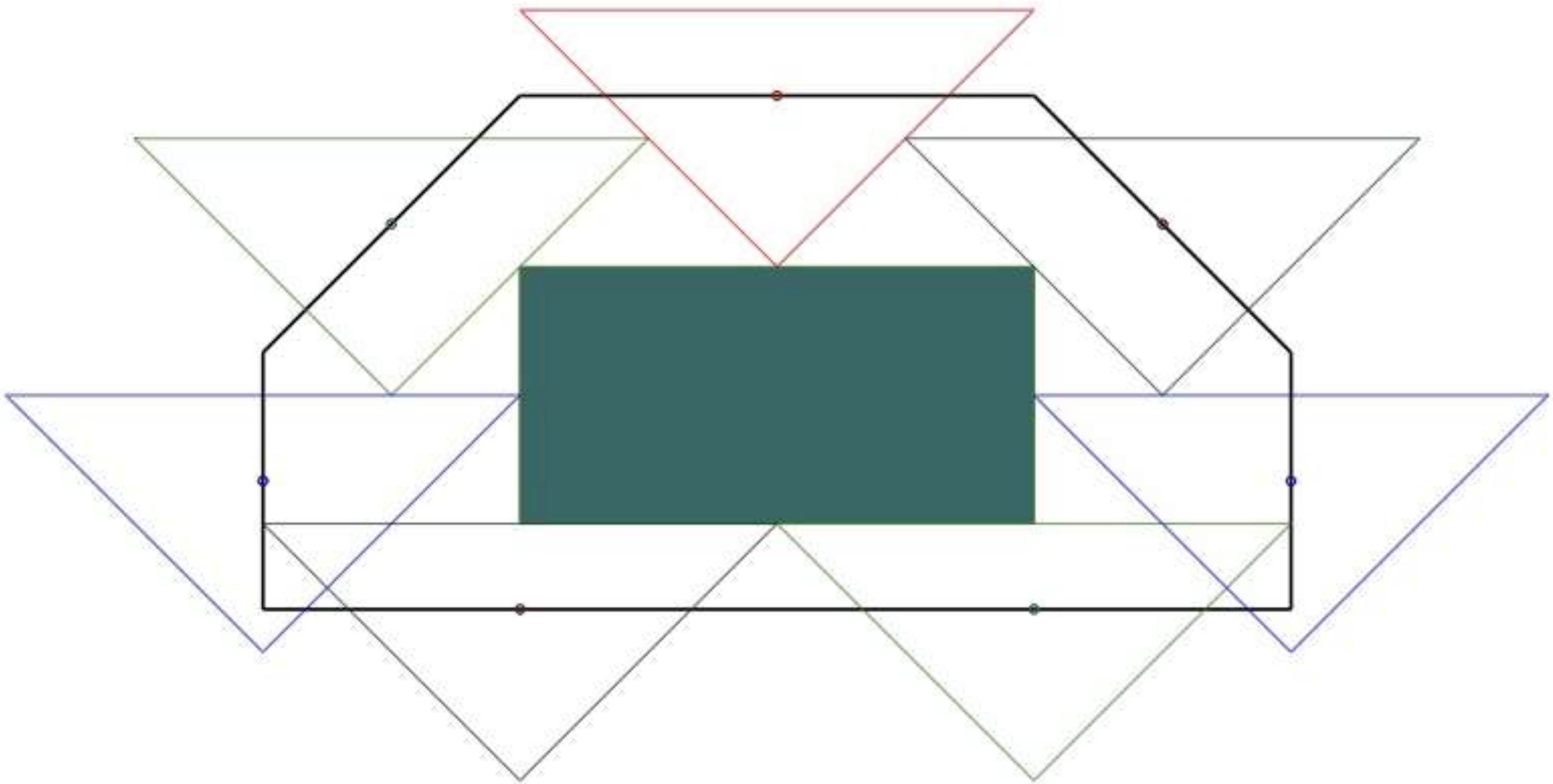
$$\partial E(A,B) \supset C(0,1) \cup C(1,0)$$

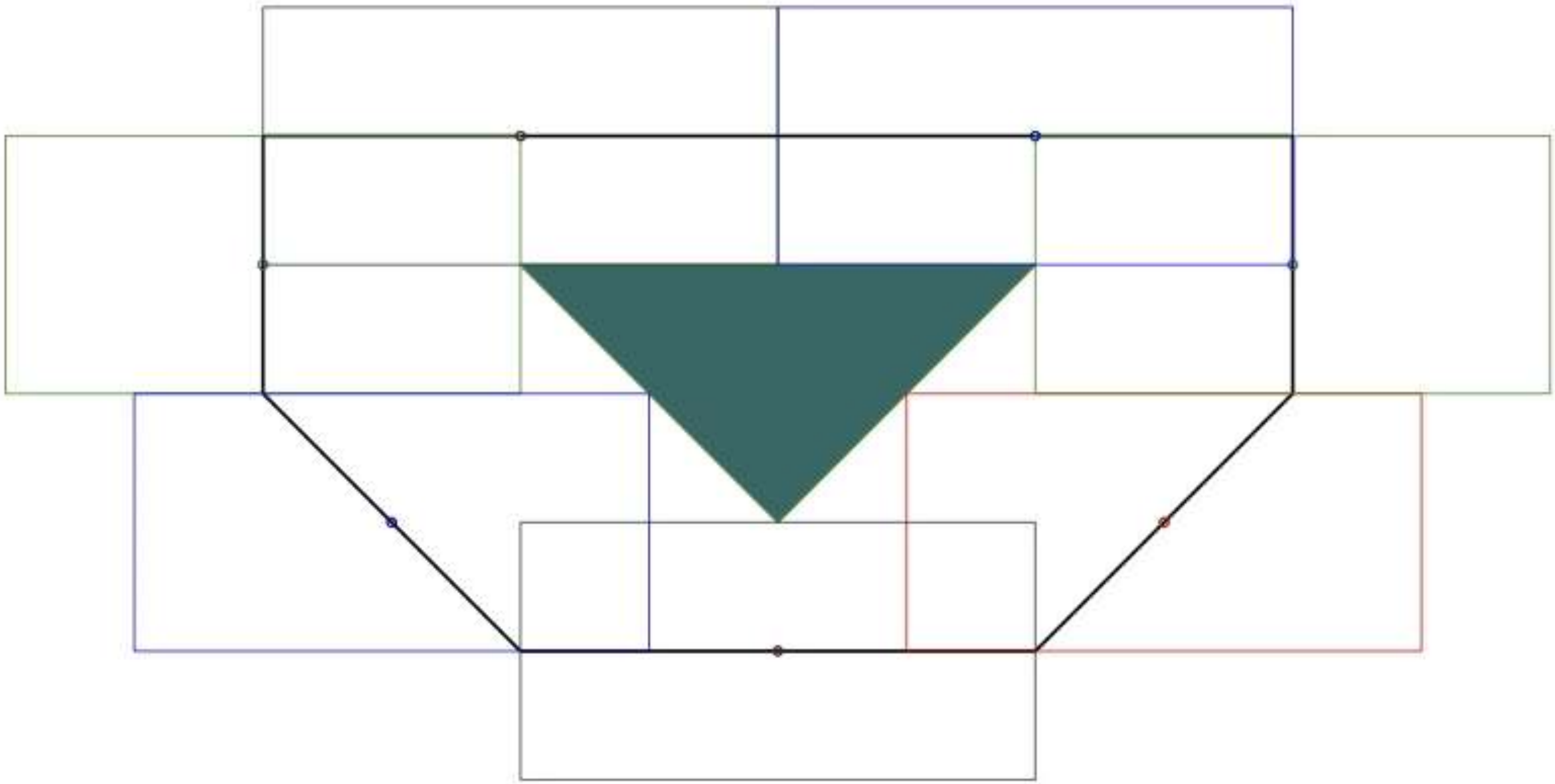
$$\partial E(A,B) = C(0,1) \cup C(1,0)$$

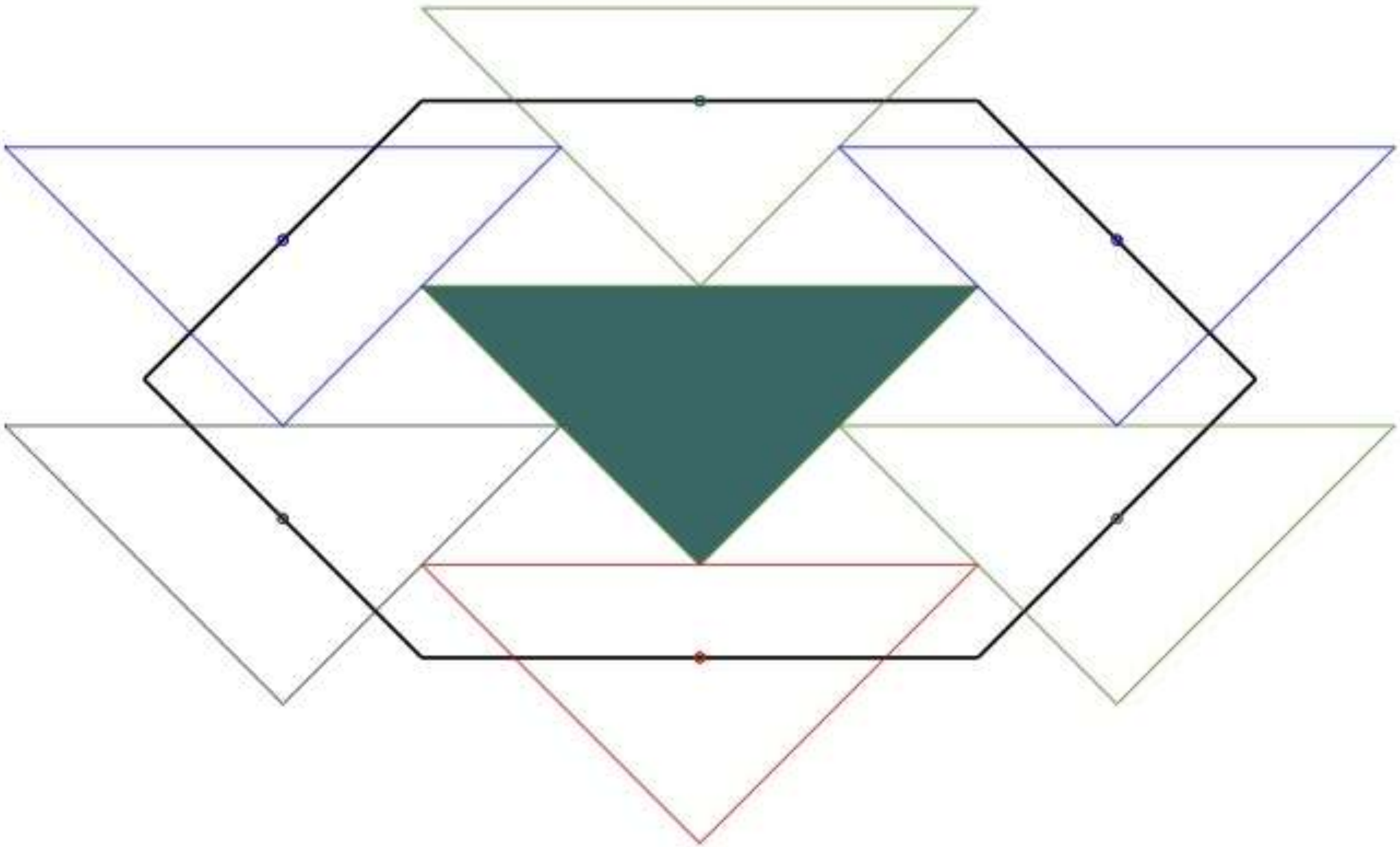

A
a_0
B
$E(A,B)$

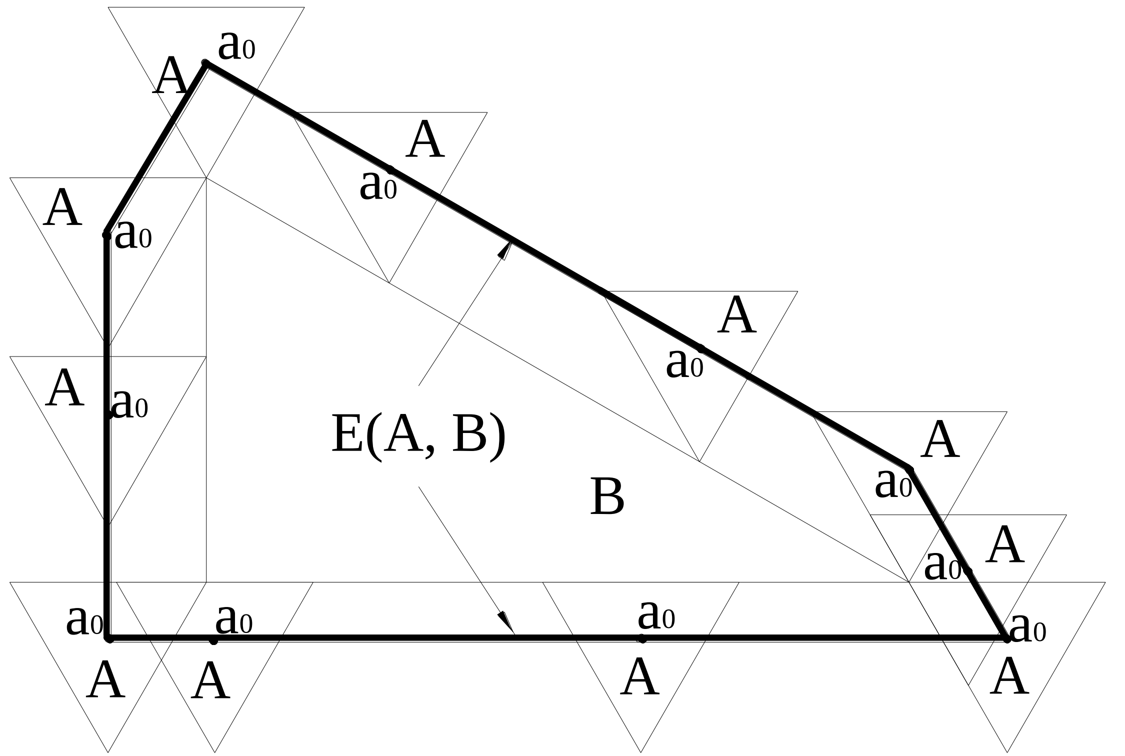


a_0
A
B
$E(A, B)$









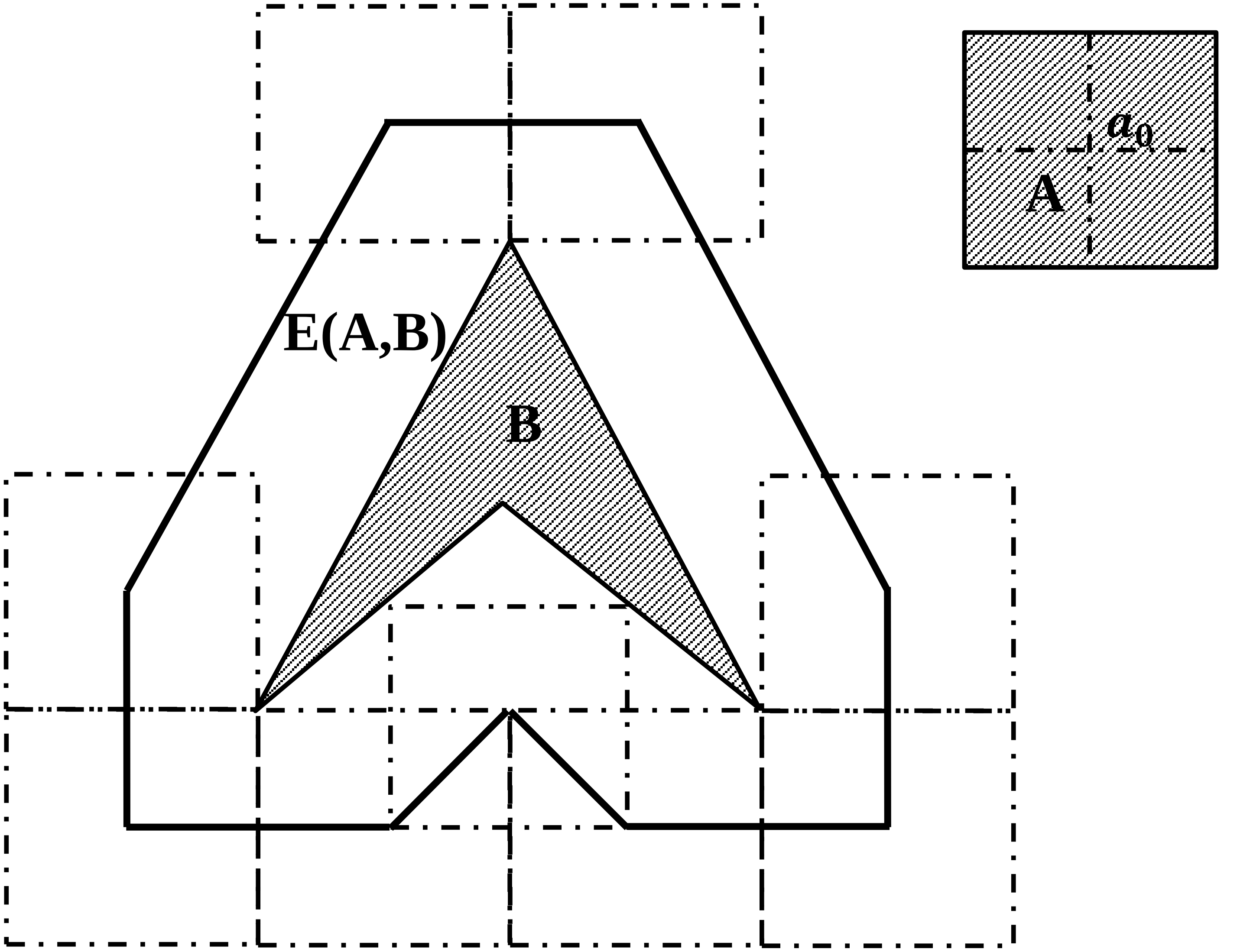
A
a_0
E(A, B)
B

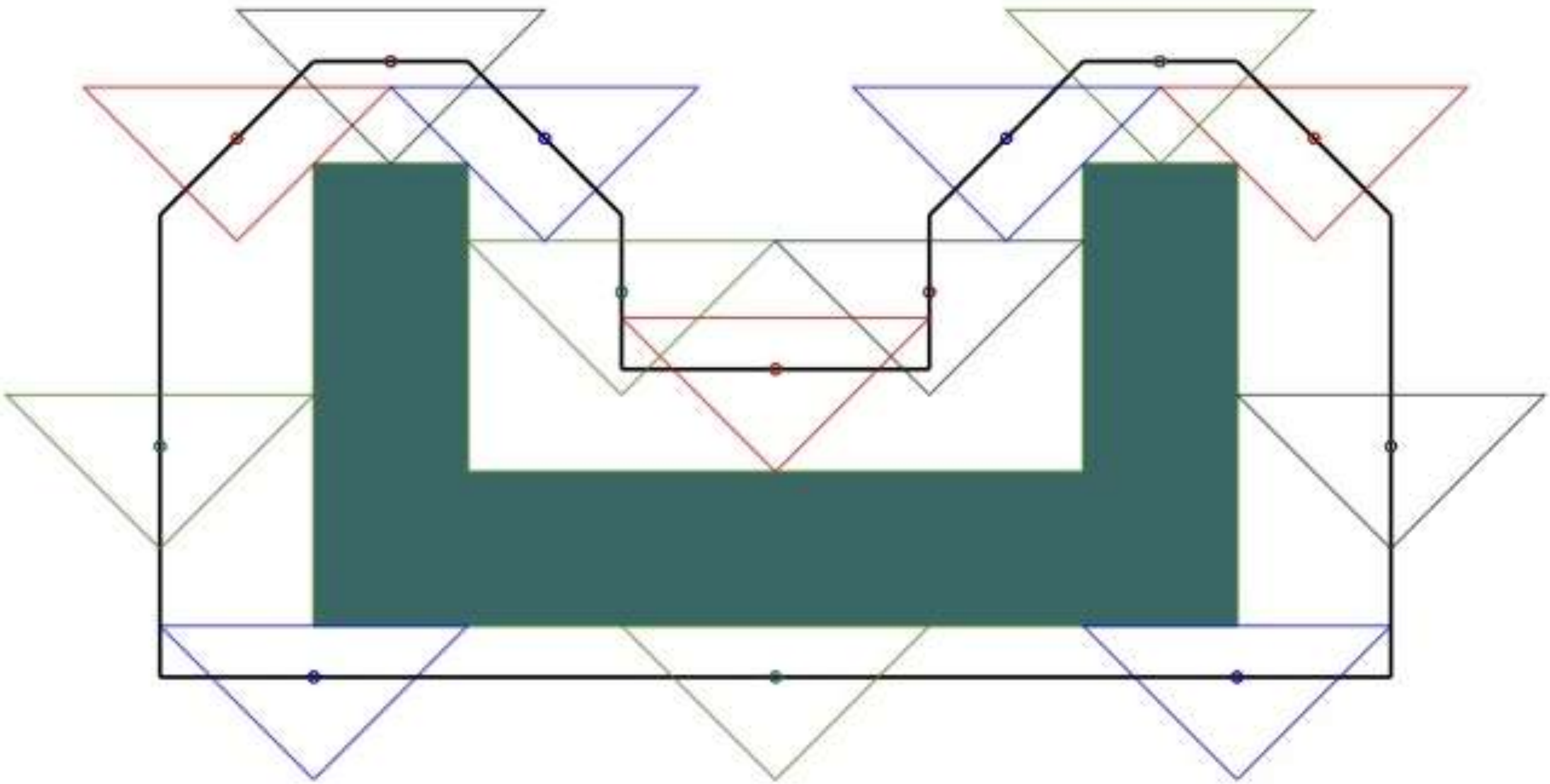
Entrance Block
of
2D General Blocks

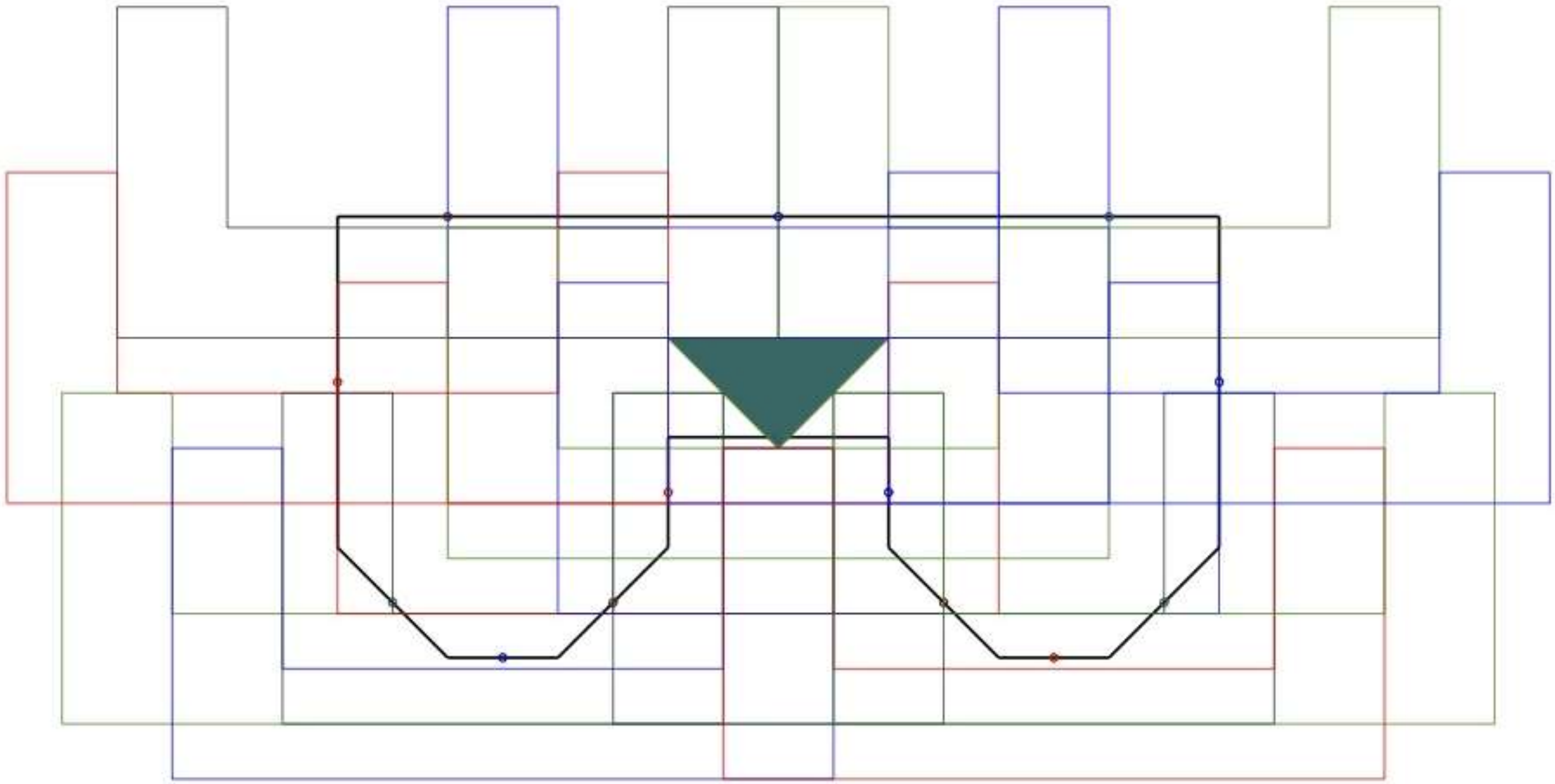
A, B General blocks

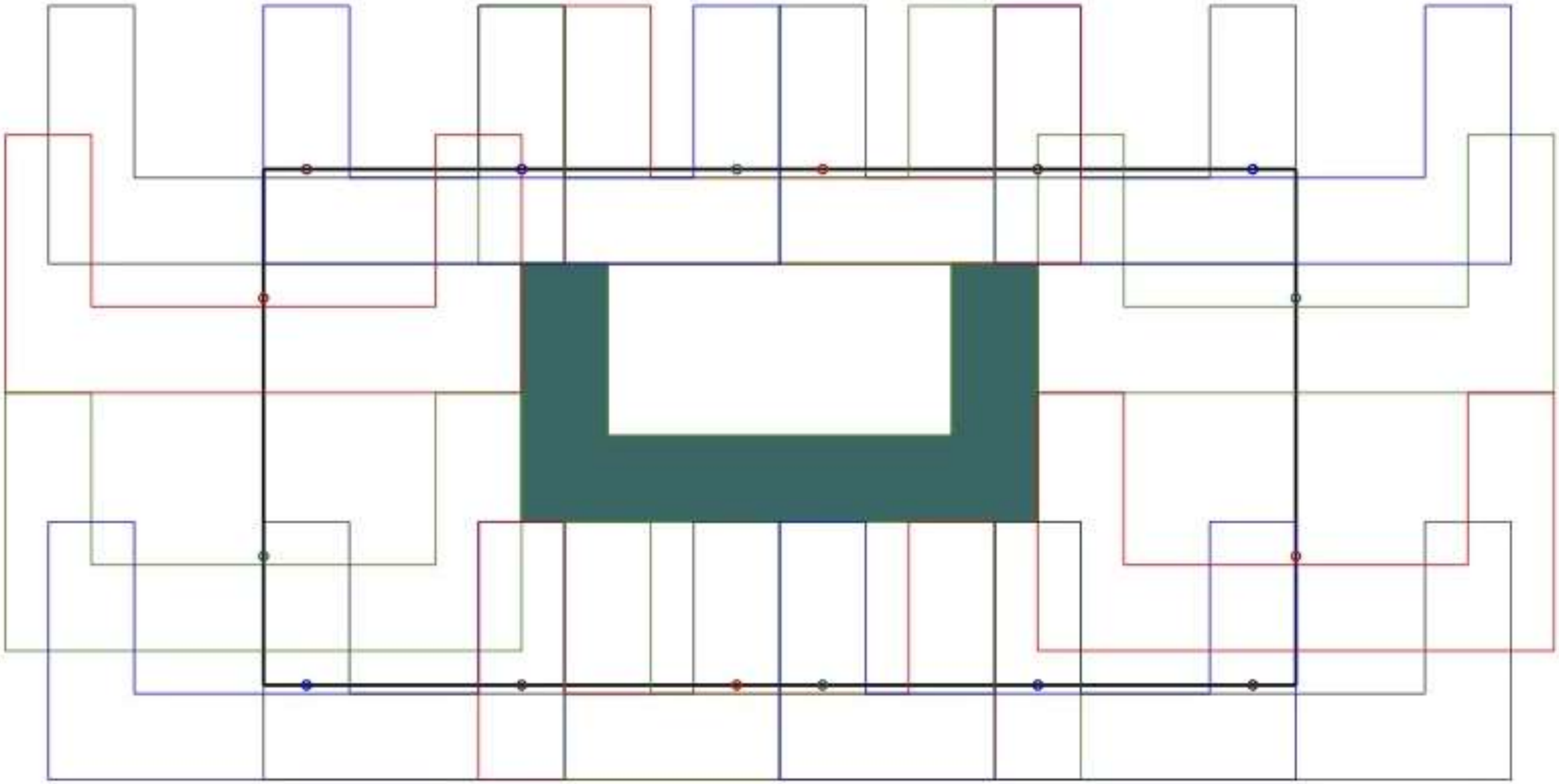
$$\partial E(A, B) \subset C(0,1) \cup C(1,0)$$

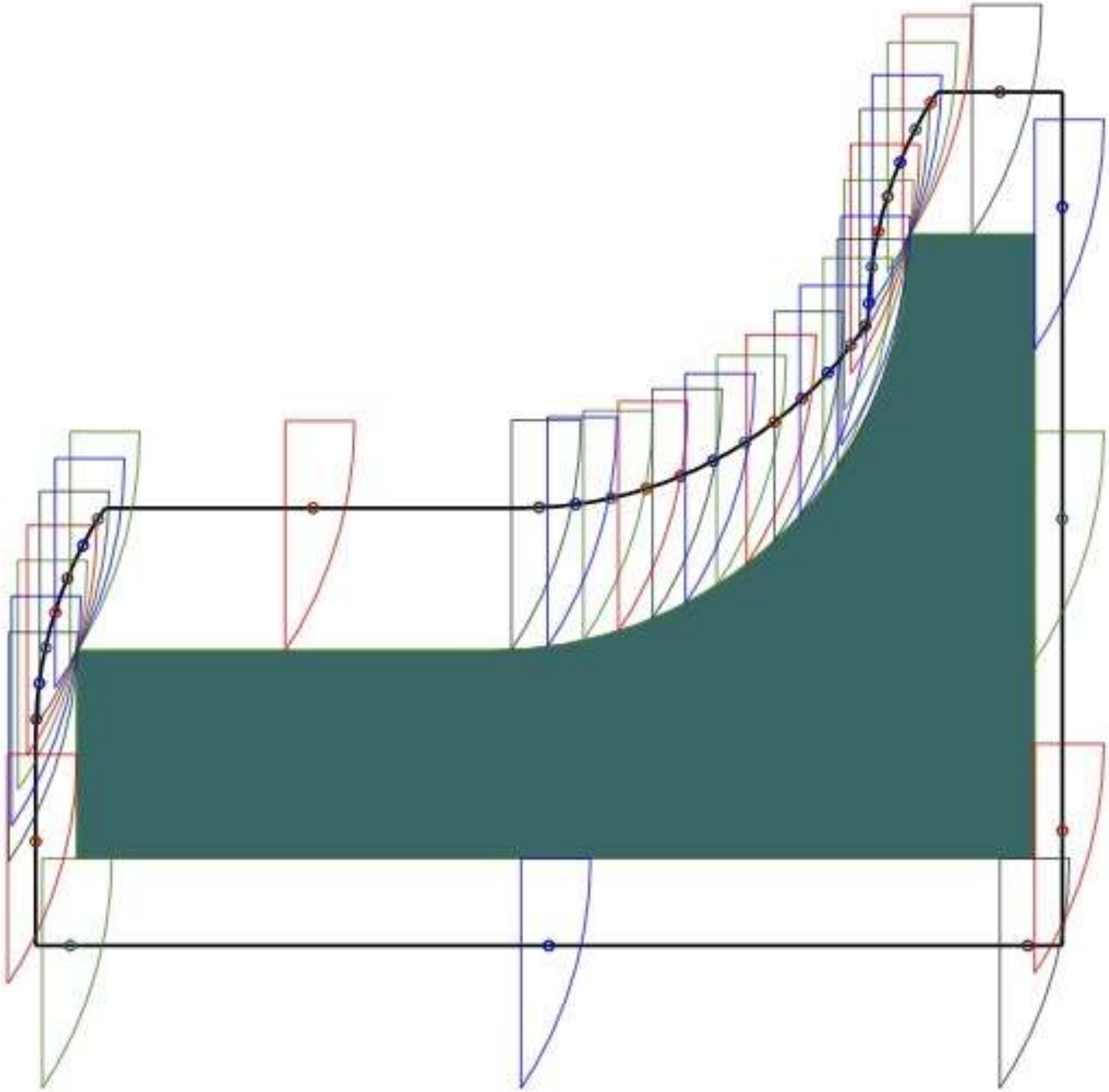
2D Concave Entrance Block

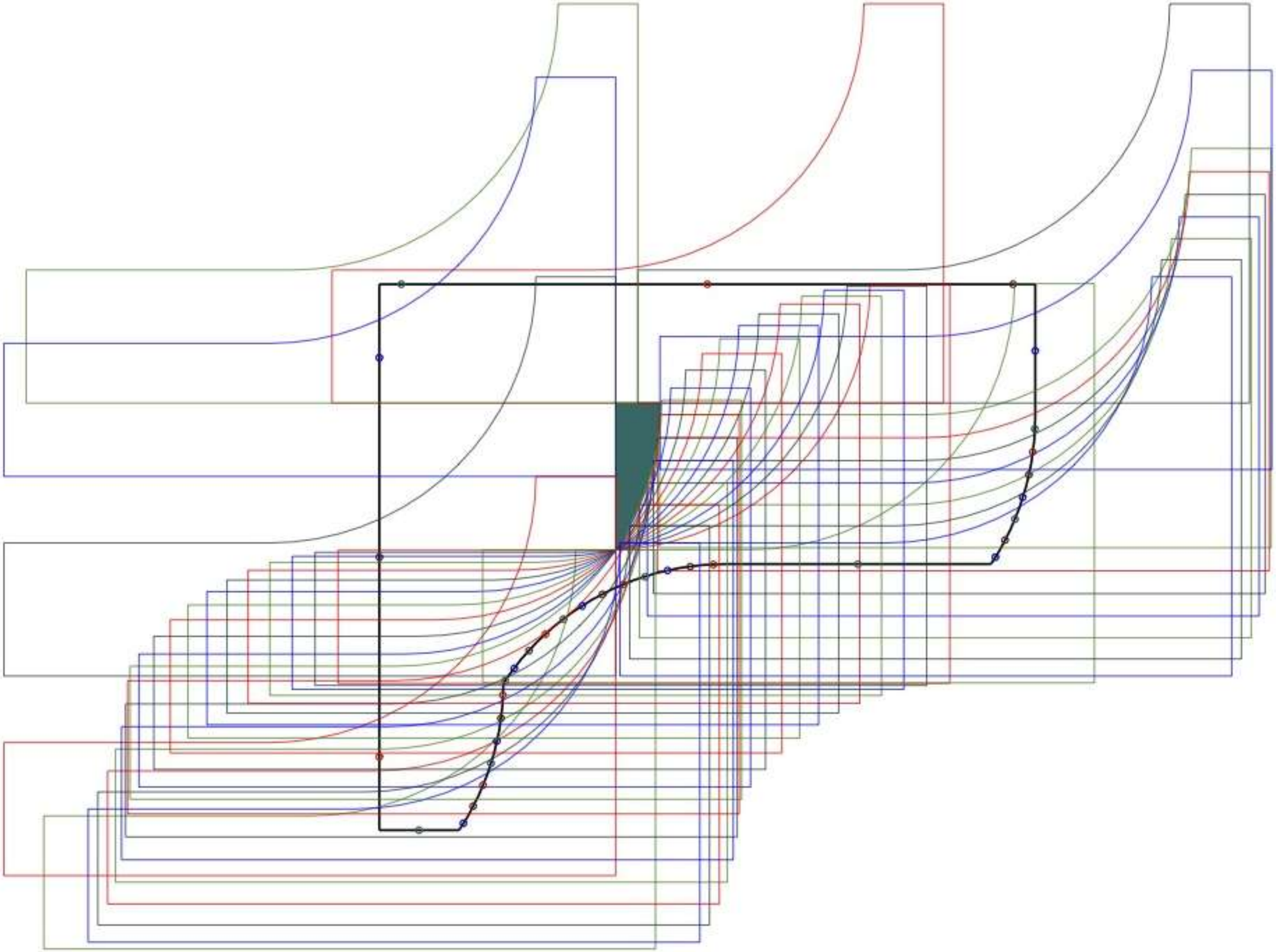


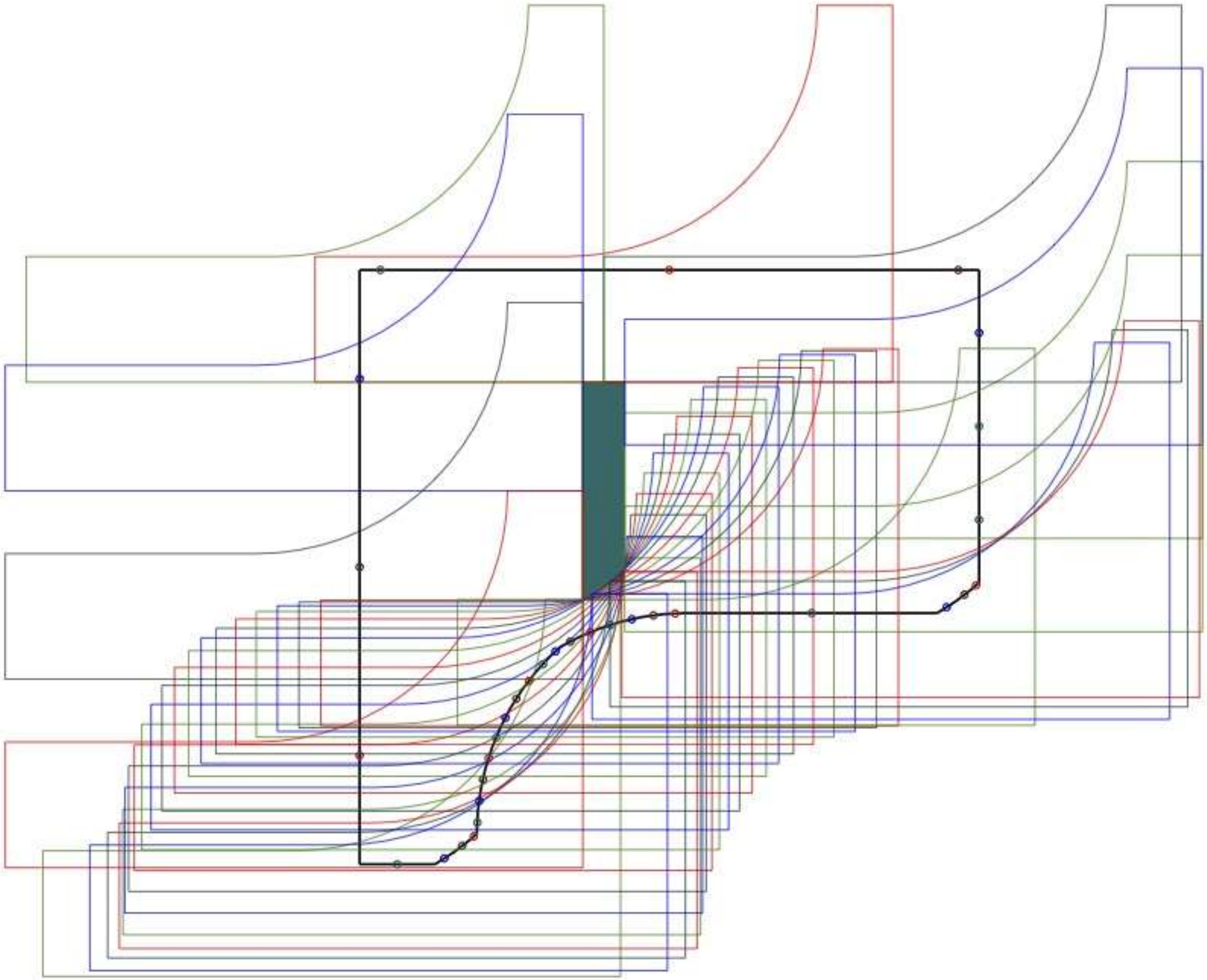


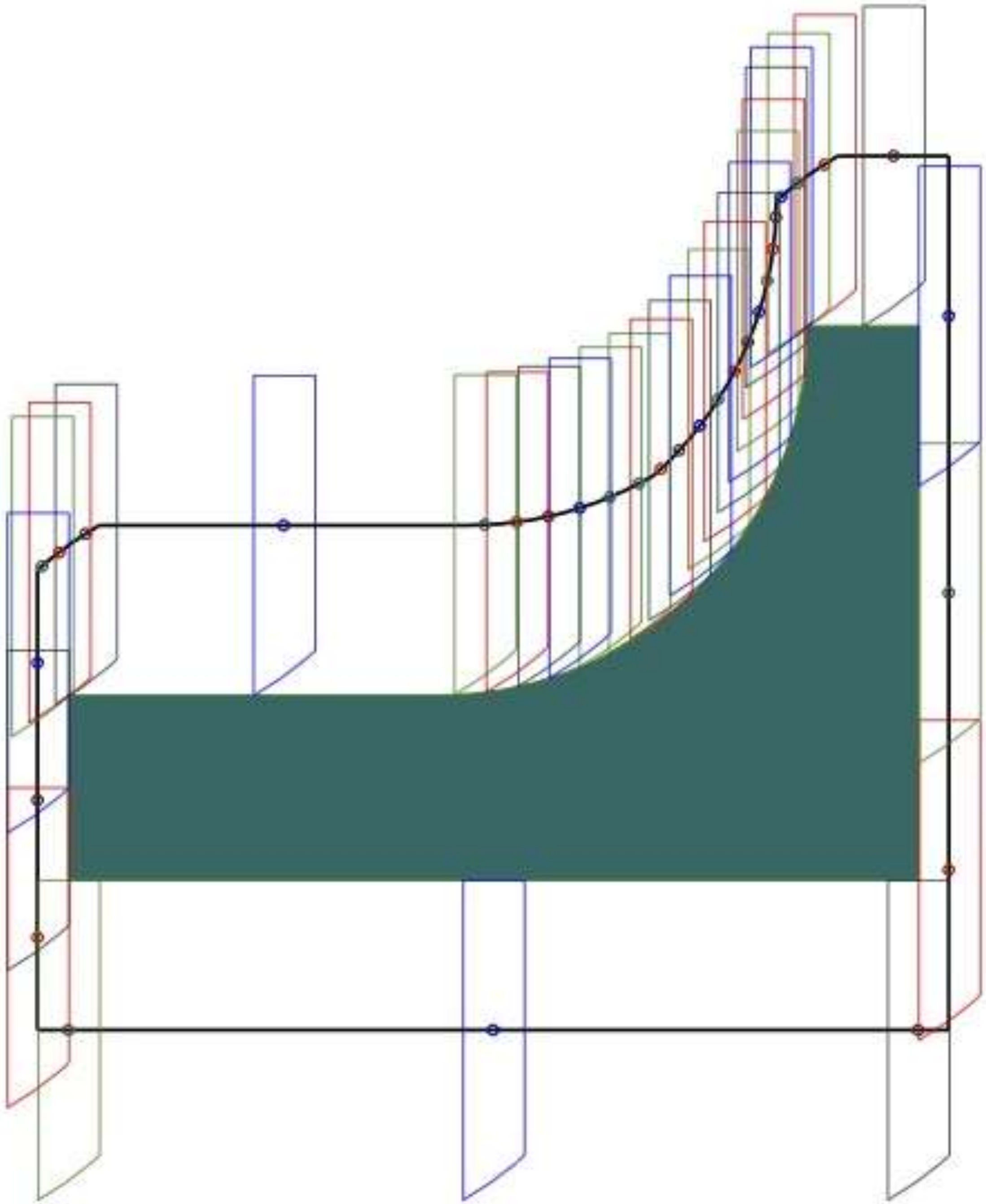


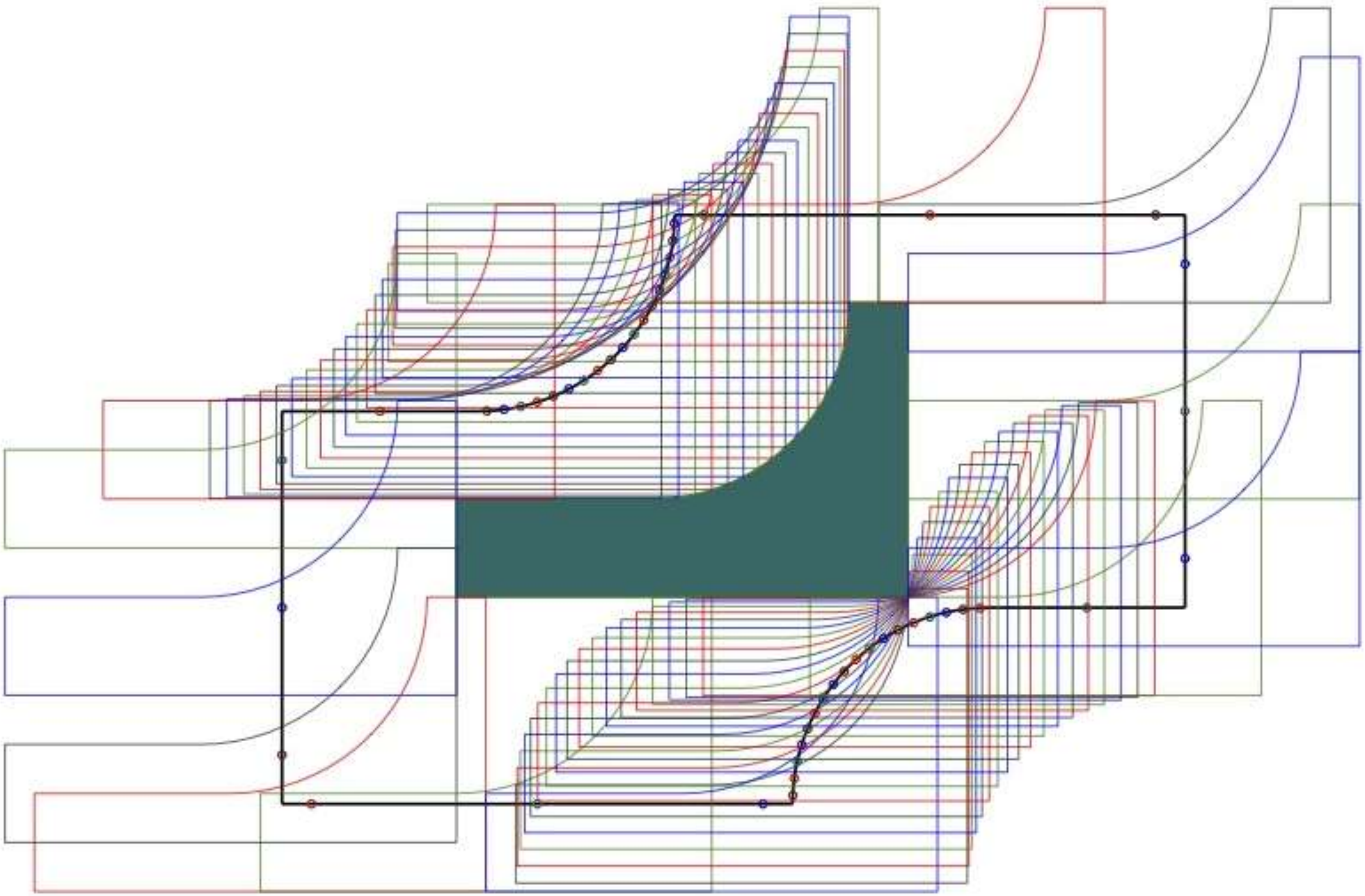












Entrance 3D Solid Angle
of
3D Solid Angles

Theorem of 3D vertex-vertex contact

$$E(\boldsymbol{e}, \boldsymbol{h}) \in \partial E(A, B) \Rightarrow$$

$$int(\measuredangle \vec{e}_1 \vec{e}_2 \cdots \vec{e}_{u-1} \vec{e}_u) \cap$$
$$int\left(\measuredangle \vec{h}_1 \vec{h}_2 \cdots \vec{h}_{v-1} \vec{h}_v\right) = \emptyset$$

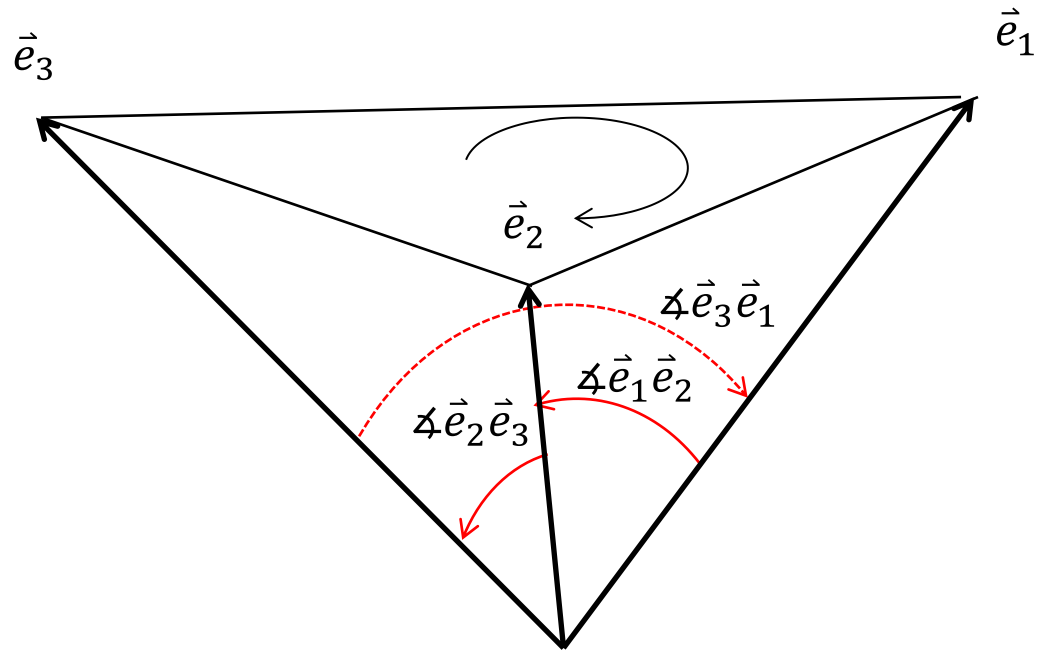
A is 3D block

B is 3D block

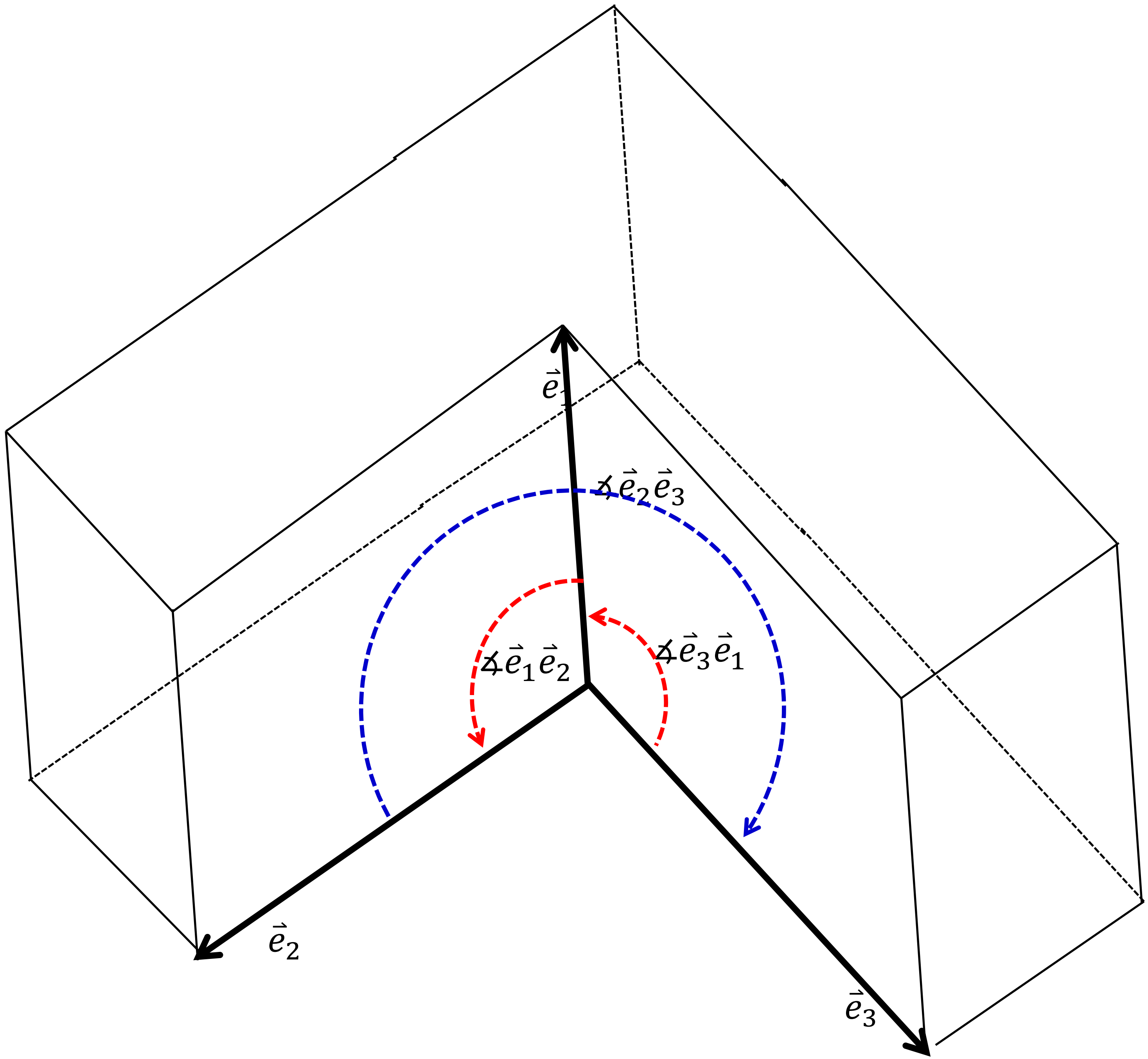
$$E(A, B) =$$

$$\bigcup_{k=0}^{3} E(A(3-k), B(k))$$

$C(0,0)$



$\vec{e}_3$
$\vec{e}_1$
$\vec{e}_2$
$\measuredangle \vec{e}_3\vec{e}_1$
$\measuredangle \vec{e}_1\vec{e}_2$
$\measuredangle \vec{e}_2\vec{e}_3$



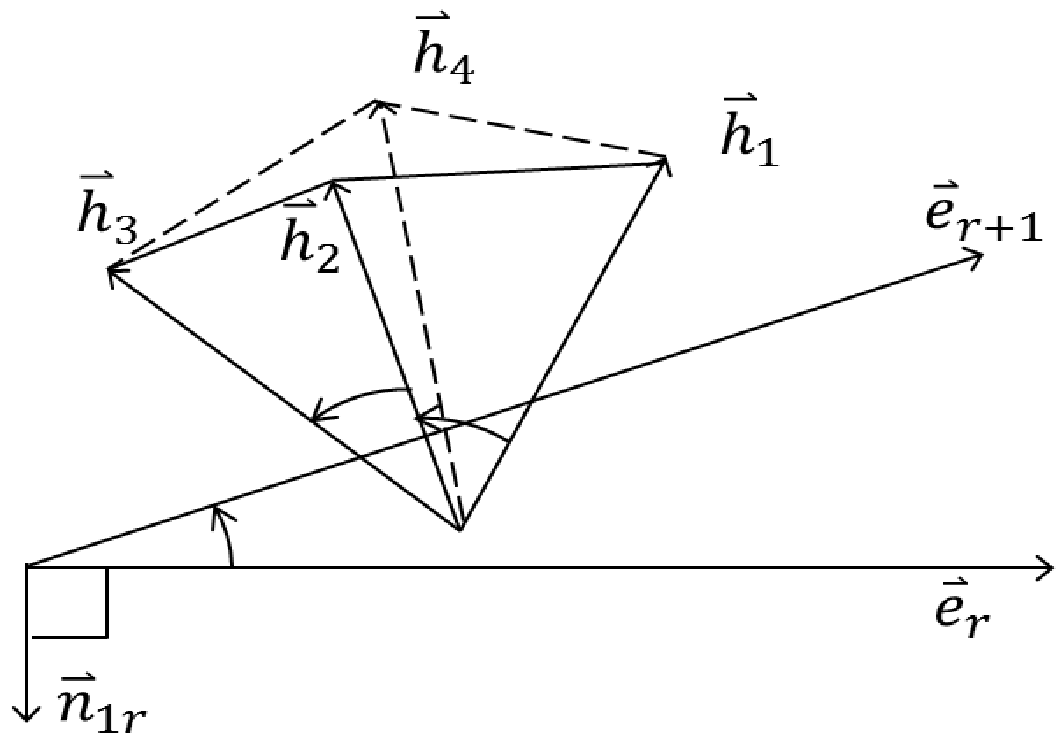
$\vec{e}_1$
$\measuredangle\vec{e}_2\vec{e}_3$
$\measuredangle\vec{e}_1\vec{e}_2$
$\measuredangle\vec{e}_3\vec{e}_1$
$\vec{e}_2$
$\vec{e}_3$

Boundary
of
3D Entrance Angle

$$\partial E(A, B) \subset E(\partial A, \partial B)$$

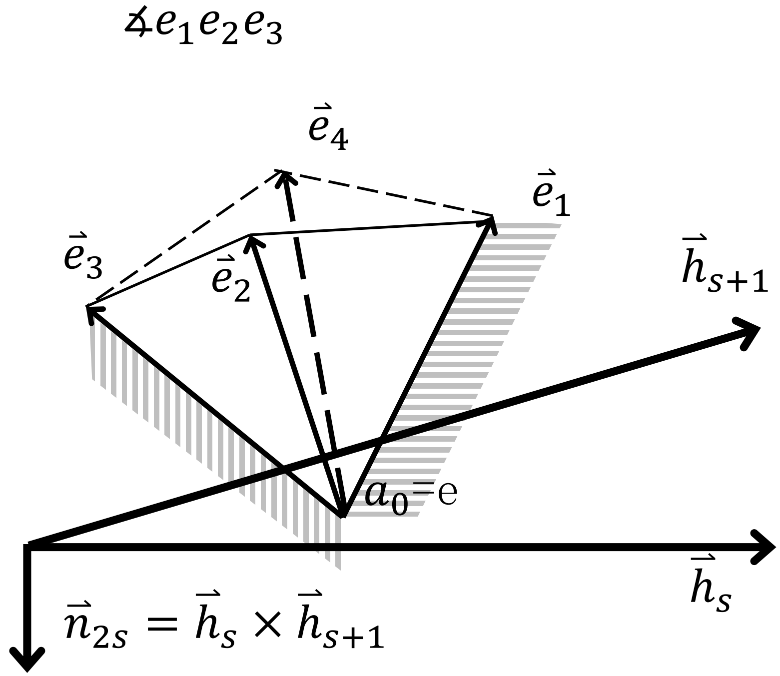
$$\partial E(A, B) \subset E\big(A(0), B(2)\big) \cup E\big(A(2), B(0)\big) \cup E\big(A(1), B(1)\big)$$

$$\partial E(A,B) \subset C(0,2) \cup C(2,0) \cup C(1,1)$$



$C(2,0)$

Contact of Vertex and Boundary2D Angle



$$C(0,2)$$

Contact Angle
of
3D Vector
and Vector

Theorem of 3D vector-vector contact

$$\measuredangle(\measuredangle\vec{e}_r + \boldsymbol{e}) = \uparrow n_{11} \cap \uparrow n_{12}$$

$$\measuredangle\left(\measuredangle\vec{h}_s + \boldsymbol{h}\right) = \uparrow n_{21} \cap \uparrow n_{22}$$

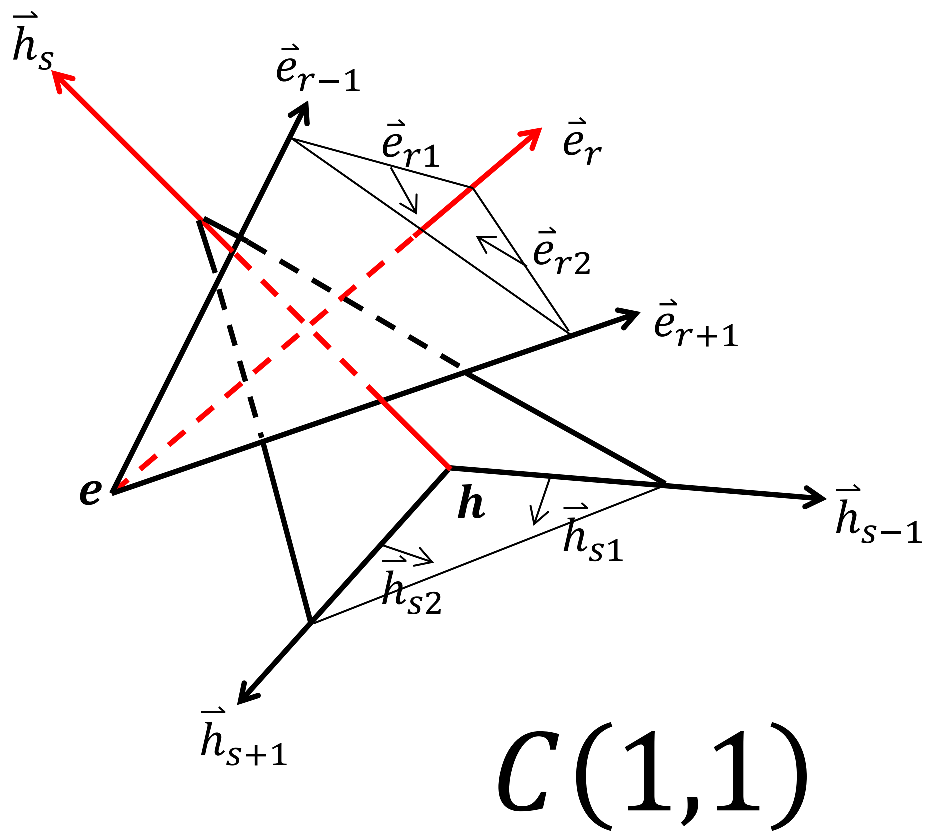
$$\exists \boldsymbol{a} \in int(\measuredangle\vec{e}_r + \boldsymbol{e}), \boldsymbol{b} \in int(\measuredangle\vec{h}_s + \boldsymbol{h})$$

$$E(\boldsymbol{a}, \boldsymbol{b}) \in \partial E(A, B) \Rightarrow$$

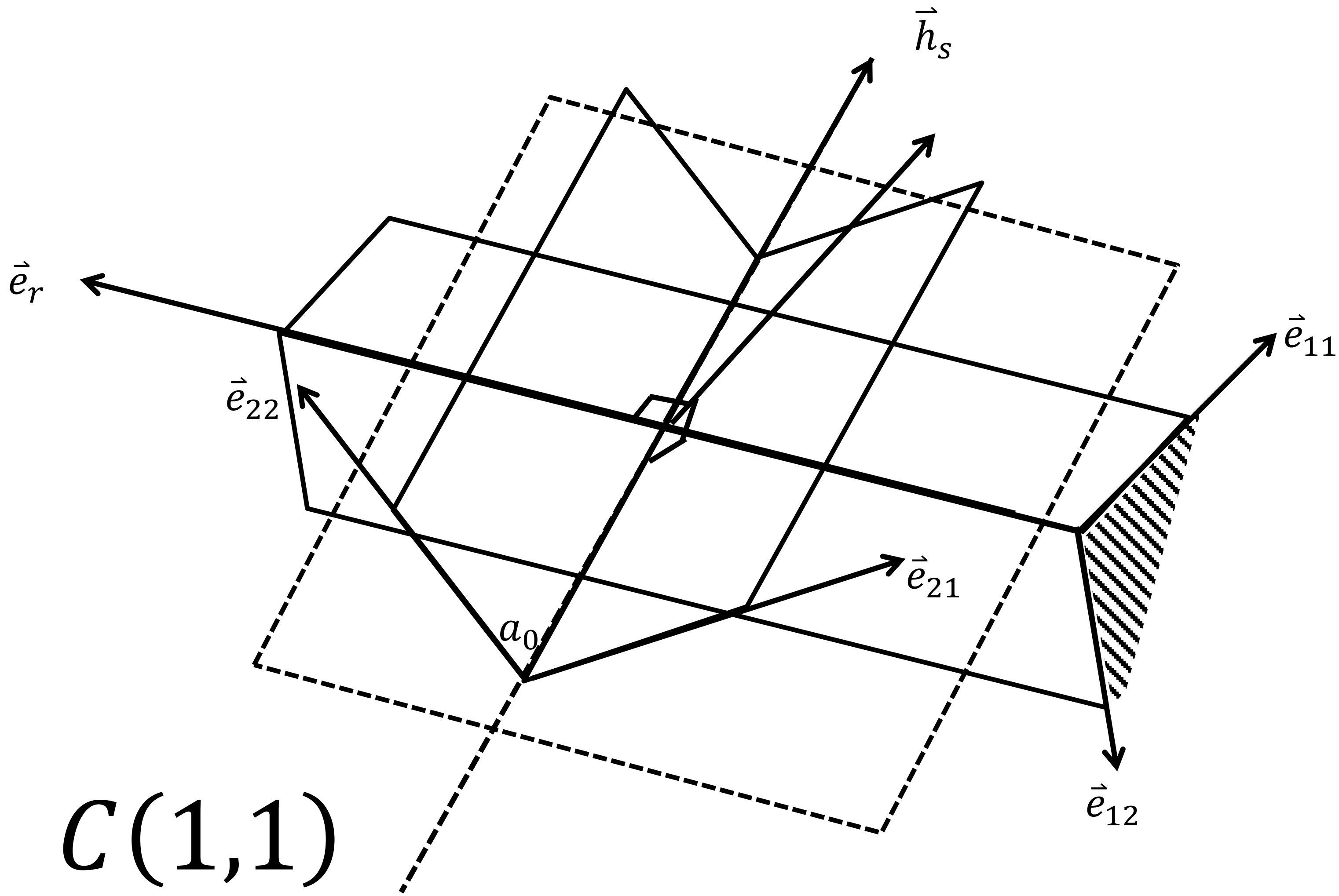
$$int(\uparrow n_{11} \cap \uparrow n_{12}) \cap int(\uparrow n_{21} \cap \uparrow n_{22}) = \emptyset$$

$$\vec{e}_r \upuparrows \vec{n}_{12} \times \vec{n}_{11}$$
$$\vec{h}_s \upuparrows \vec{n}_{22} \times \vec{n}_{21}$$

$$\vec{e}_{r1} = \vec{n}_{11} \times \vec{n}_{12} \times \vec{n}_{11}$$
$$\vec{e}_{r2} = \vec{n}_{12} \times \vec{n}_{11} \times \vec{n}_{12}$$
$$\vec{h}_{s1} = \vec{n}_{21} \times \vec{n}_{22} \times \vec{n}_{21}$$
$$\vec{h}_{s2} = \vec{n}_{22} \times \vec{n}_{21} \times \vec{n}_{22}$$


$\vec{h}_s$
$\vec{e}_{r-1}$
$\vec{e}_{r1}$
$\vec{e}_r$
$\vec{e}_{r2}$
$\vec{e}_{r+1}$
$\boldsymbol{e}$
$\boldsymbol{h}$
$\vec{h}_{s1}$
$\vec{h}_{s-1}$
$\vec{h}_{s2}$
$\vec{h}_{s+1}$
$C(1,1)$

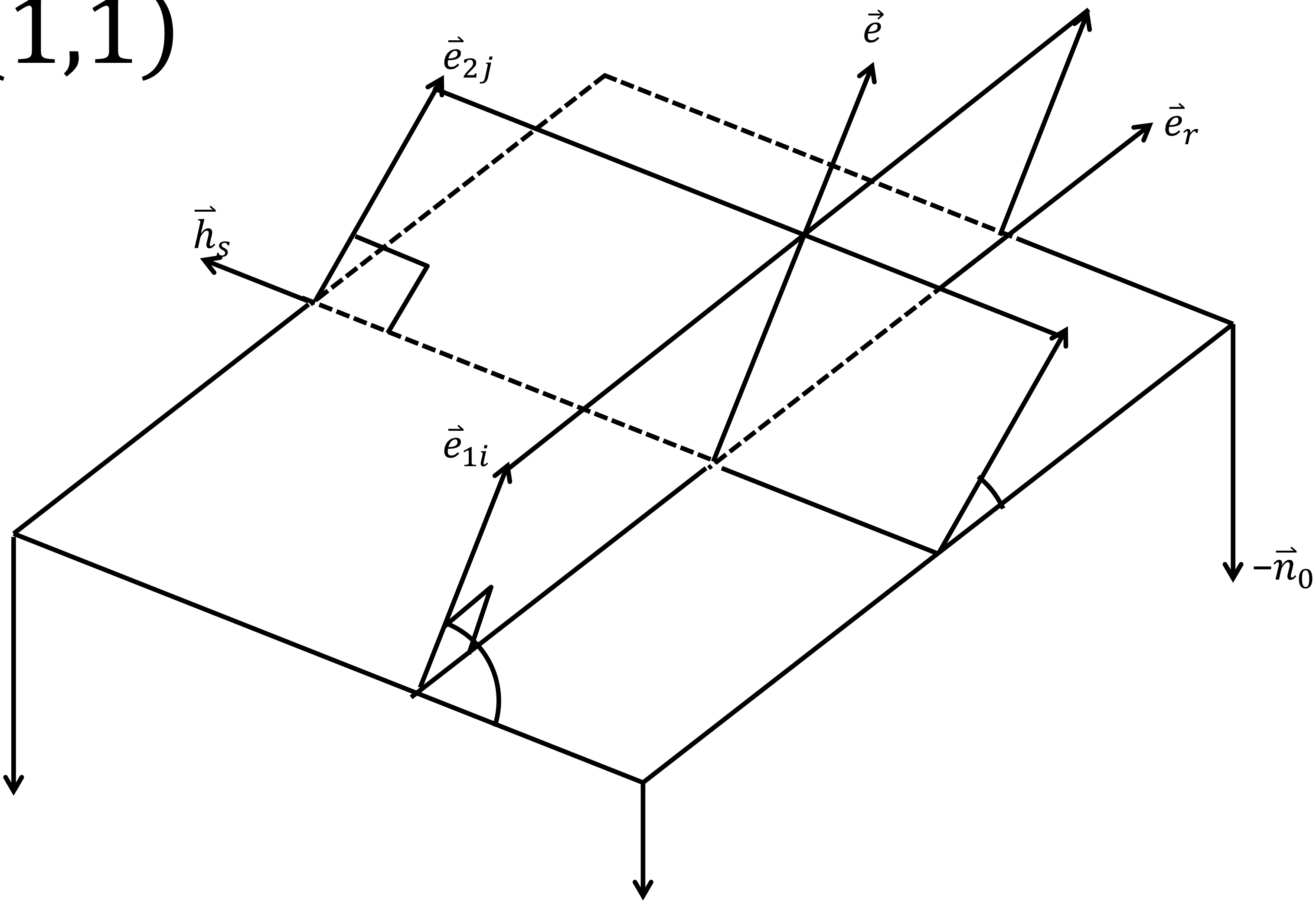
Contact of 3D Vectors



$C(1,1)$

Contact Condition of Two Vectors

$C(1,1)$



Contact Polygons
of
3D Vertex
and Polygon

Theorem of 3D vertex-polygon contact

$$\exists\, \boldsymbol{b} \in int\left(\boldsymbol{b}_1\boldsymbol{b}_2 \cdots\, \boldsymbol{b}_{q-1}\boldsymbol{b}_q\right)$$

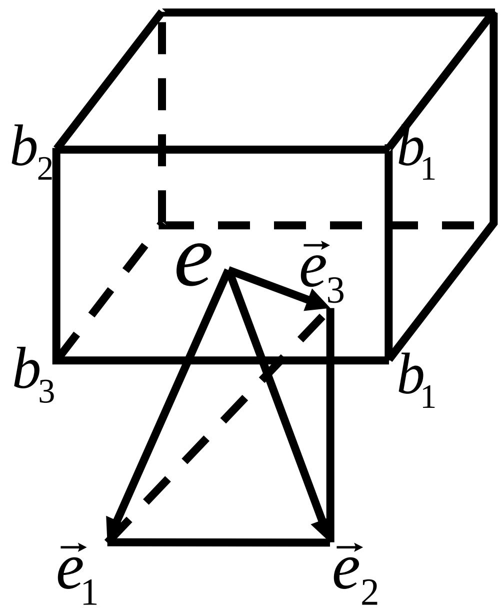
$$E(\boldsymbol{e},\boldsymbol{b}) \in \partial E(A,B)$$

$$\Rightarrow$$

$$int(\measuredangle \vec{e}_1\vec{e}_2 \cdots \vec{e}_{u-1}\vec{e}_u) \cap int(\uparrow m_{2l}) = \emptyset$$

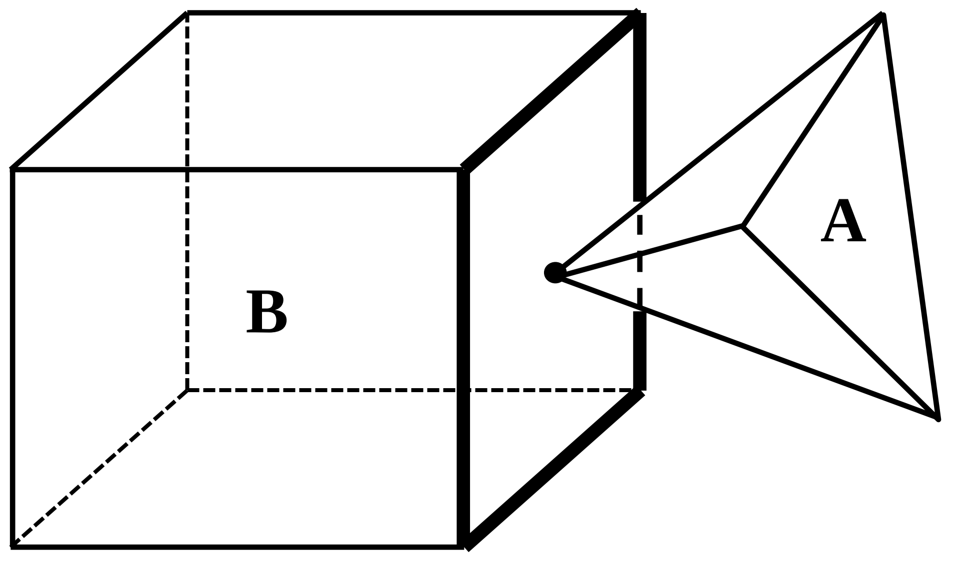
$$E\left(\boldsymbol{e}, \boldsymbol{b}_1\boldsymbol{b}_2\cdots\ \boldsymbol{b}_{q-1}\boldsymbol{b}_q\right)$$
$$= \boldsymbol{b}_1\boldsymbol{b}_2\cdots\ \boldsymbol{b}_{q-1}\boldsymbol{b}_q - \boldsymbol{e} + \boldsymbol{a}_0,$$

Contact of 3D Angle and Face Polygon



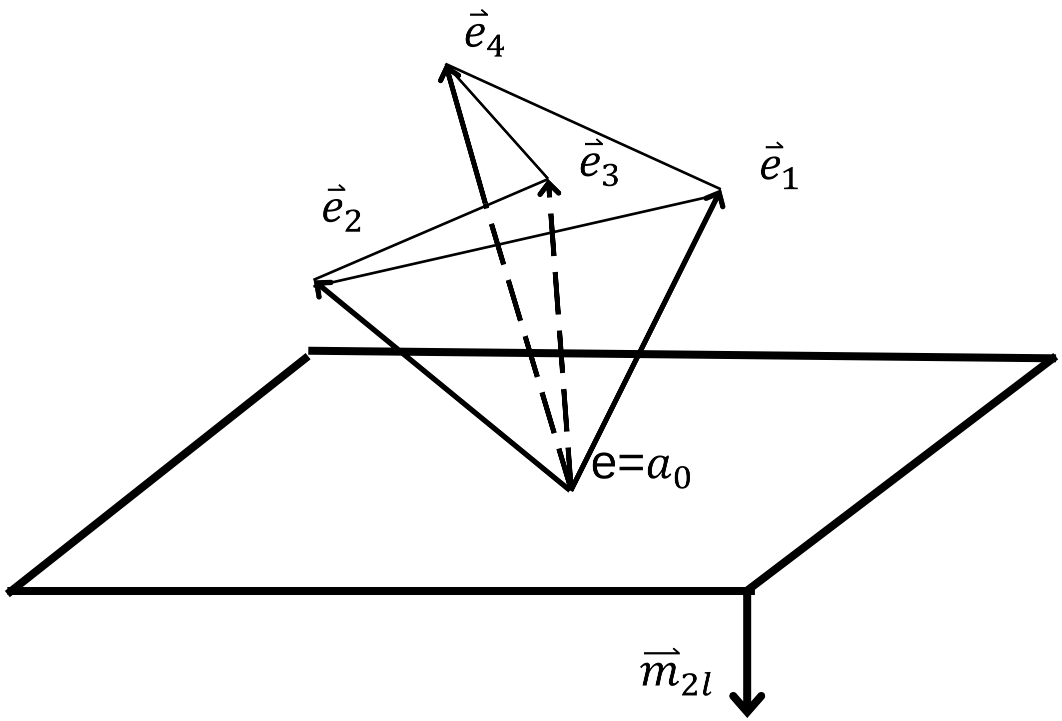
$$C(0,2)$$

3D Vertex-Face Contact

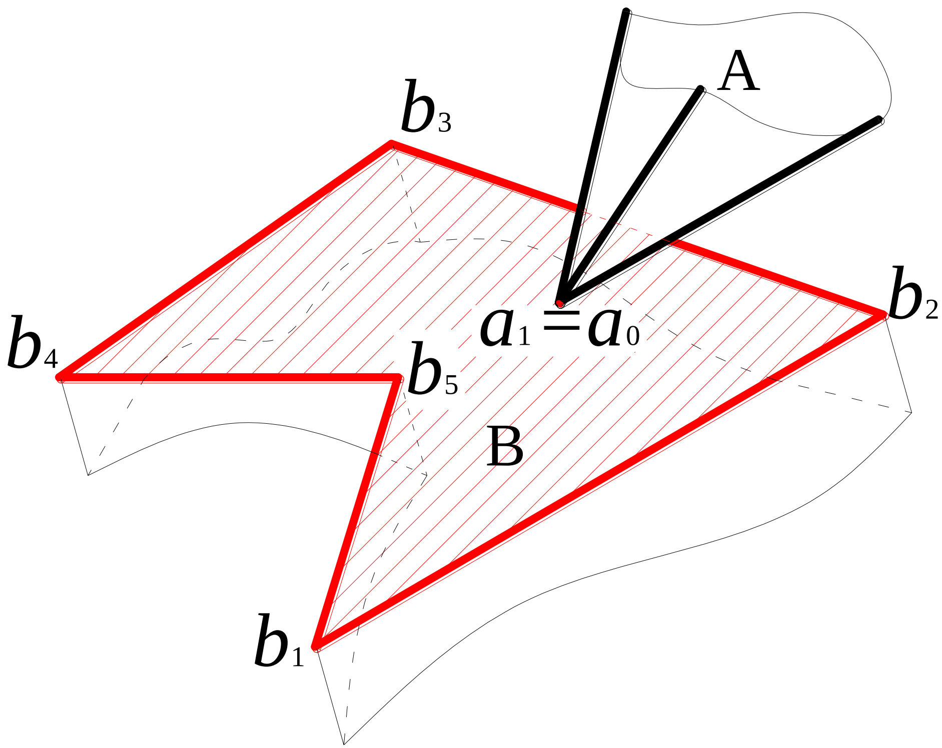


$$C(0,2)$$

Contact Polygon of 3D Vertex and Polygon



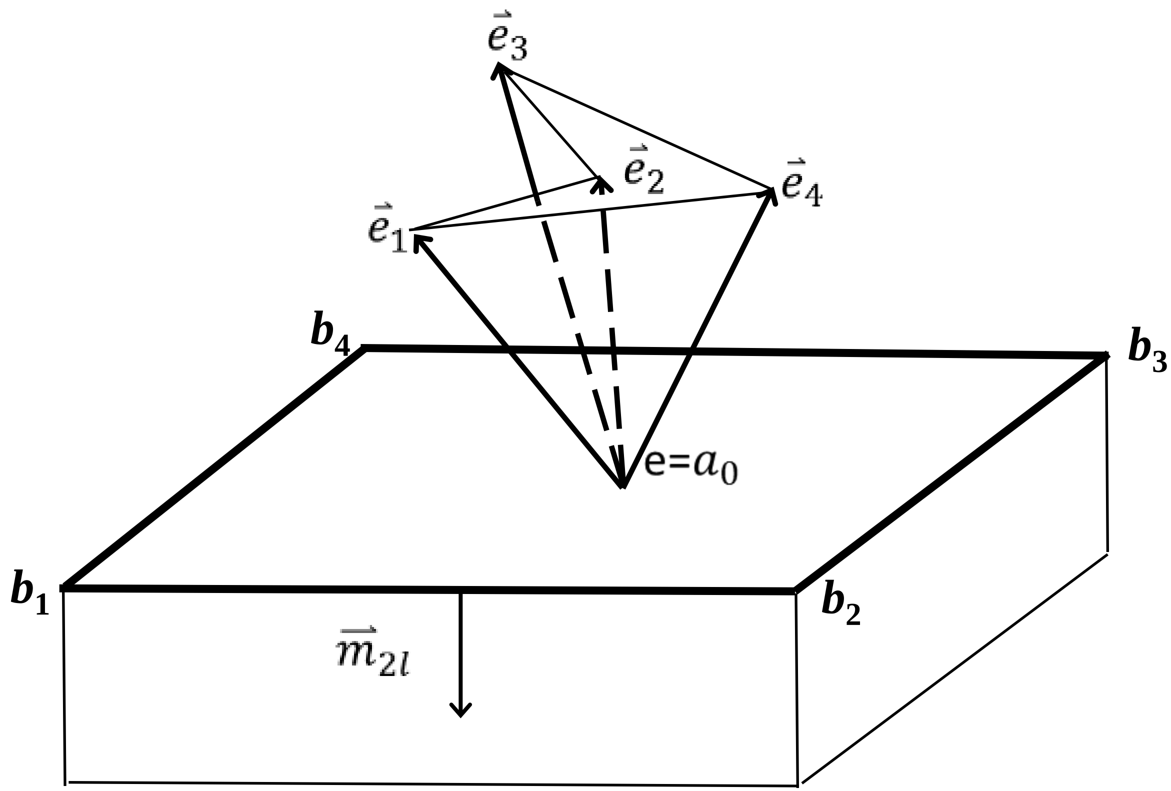
$$C(0,2)$$



$$E(a_1,\ b_1b_2b_3b_4b_5) = b_1b_2b_3b_4b_5$$

$$C(0,2)$$

$C(0,2)$



Contact Polygons
of
3D Edge and Edge

Assume the edges of $\vec{e}_r$ and $\vec{h}_s$ are convex

$$\vec{e}_r \upuparrows \vec{n}_{12} \times \vec{n}_{11}$$

$$\vec{h}_s \upuparrows \vec{n}_{22} \times \vec{n}_{21}$$

$$\vec{e}_{r1} = \vec{n}_{11} \times \vec{n}_{12} \times \vec{n}_{11}$$

$$\vec{e}_{r2} = \vec{n}_{12} \times \vec{n}_{11} \times \vec{n}_{12}$$

$$\vec{h}_{s1} = \vec{n}_{21} \times \vec{n}_{22} \times \vec{n}_{21}$$

$$\vec{h}_{s2} = \vec{n}_{22} \times \vec{n}_{21} \times \vec{n}_{22}$$

$$\measuredangle \boldsymbol{ee}_r = \uparrow n_{11} \cap \uparrow n_{12}$$

$$\measuredangle \boldsymbol{hh}_s = \uparrow n_{21} \cap \uparrow n_{22}$$

Theorem of 3D edge-edge contact

$$\exists \boldsymbol{a} \in int(\boldsymbol{e}\boldsymbol{e}_r), \boldsymbol{b} \in int(\boldsymbol{h}\boldsymbol{h}_s)$$

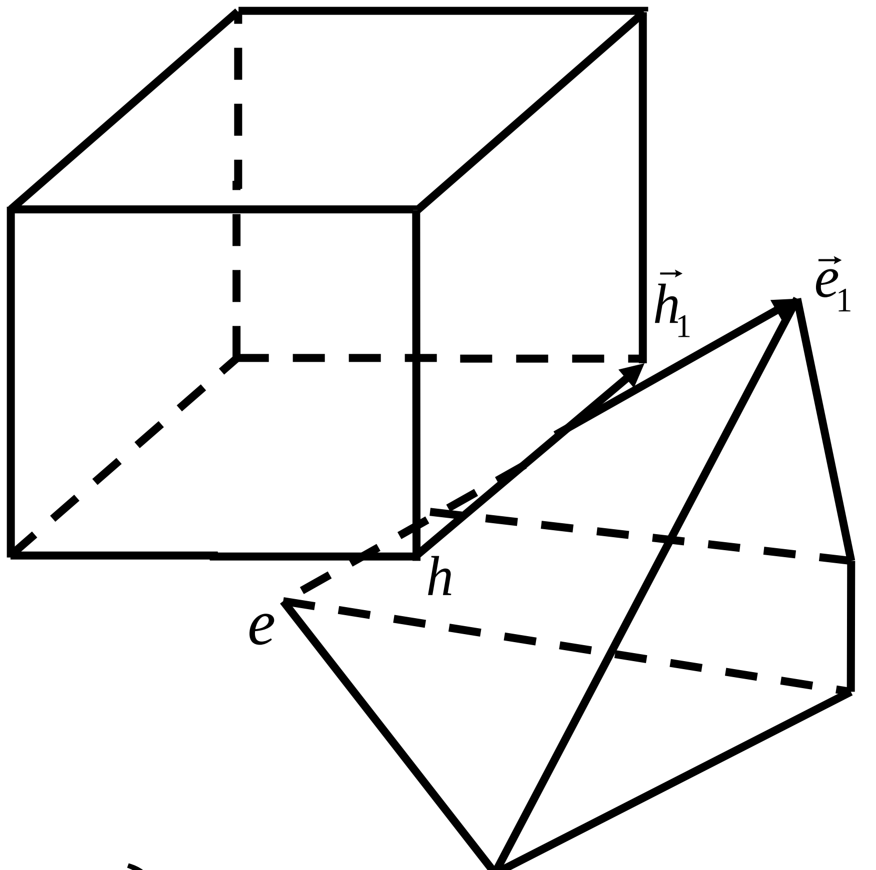
$$E(\boldsymbol{a}, \boldsymbol{b}) \in \partial E(A, B) \Rightarrow$$

$$int(\uparrow n_{11} \cap \uparrow n_{12}) \cap int(\uparrow n_{21} \cap \uparrow n_{22}) = \emptyset$$

$$\Leftrightarrow$$

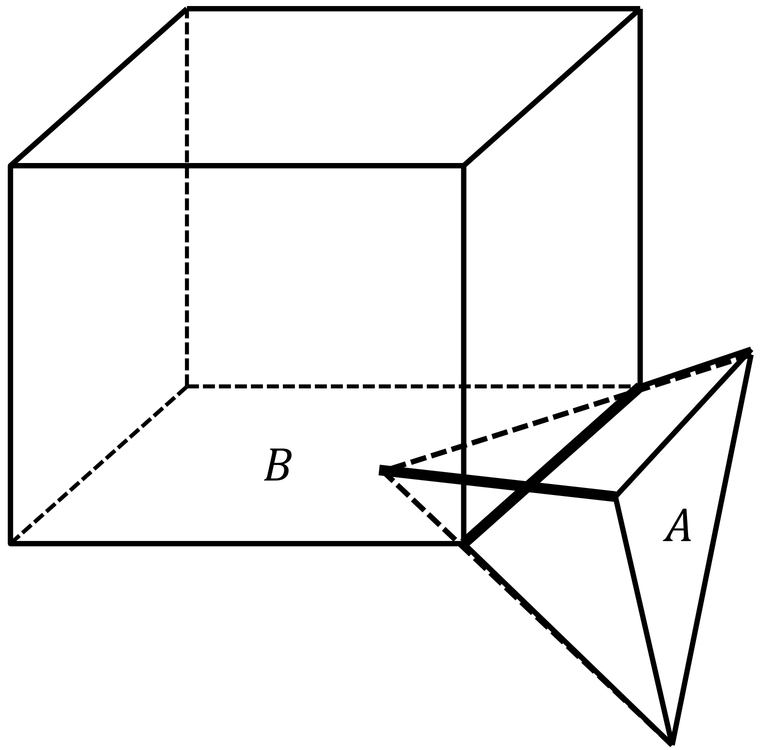
$$int\left((\measuredangle \vec{e}_{r1}\vec{e}_{r2} + \overleftrightarrow{e_r}) \cap \left(\measuredangle \overrightarrow{h}_{s1}\overrightarrow{h}_{s2} + \overleftrightarrow{h_s}\right)\right) = \emptyset$$

Contact of 3D Edge and Edge

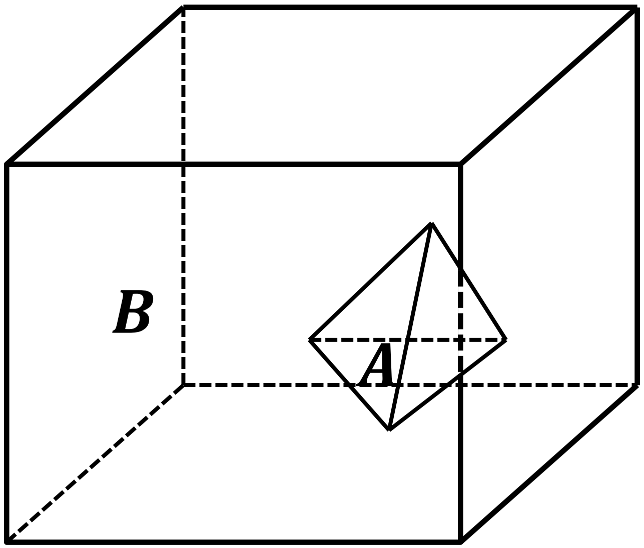


$$C(1,1)$$

3D Edge-Edge Contact

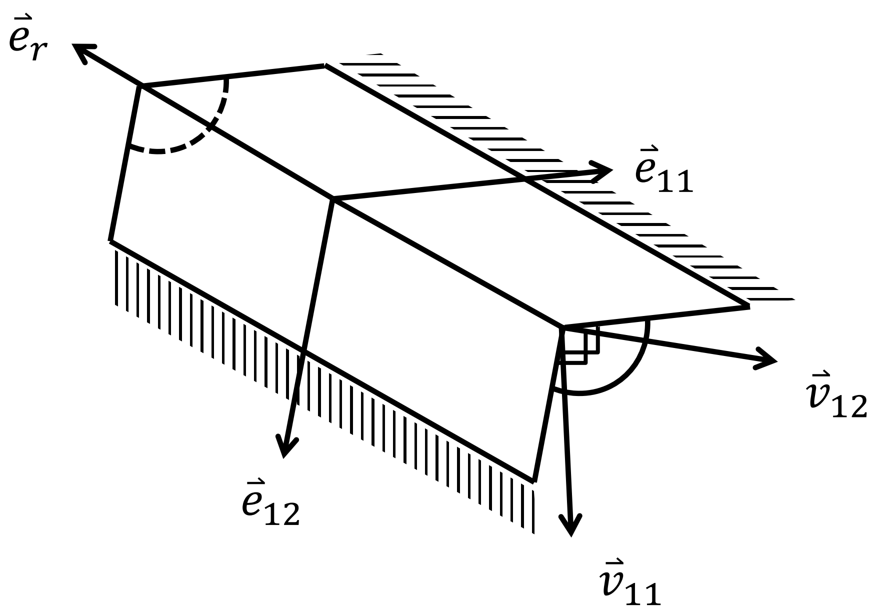


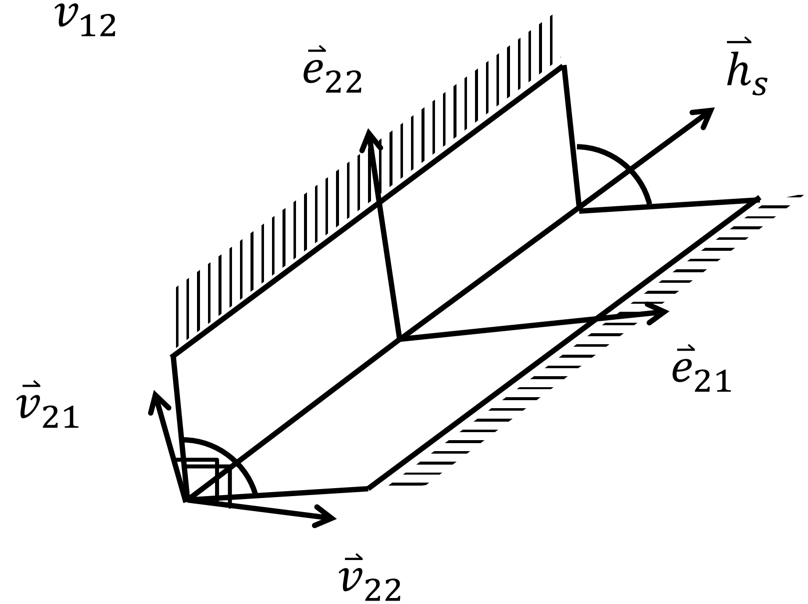
3D Edge-Edge Contact



$$C(1,1)$$

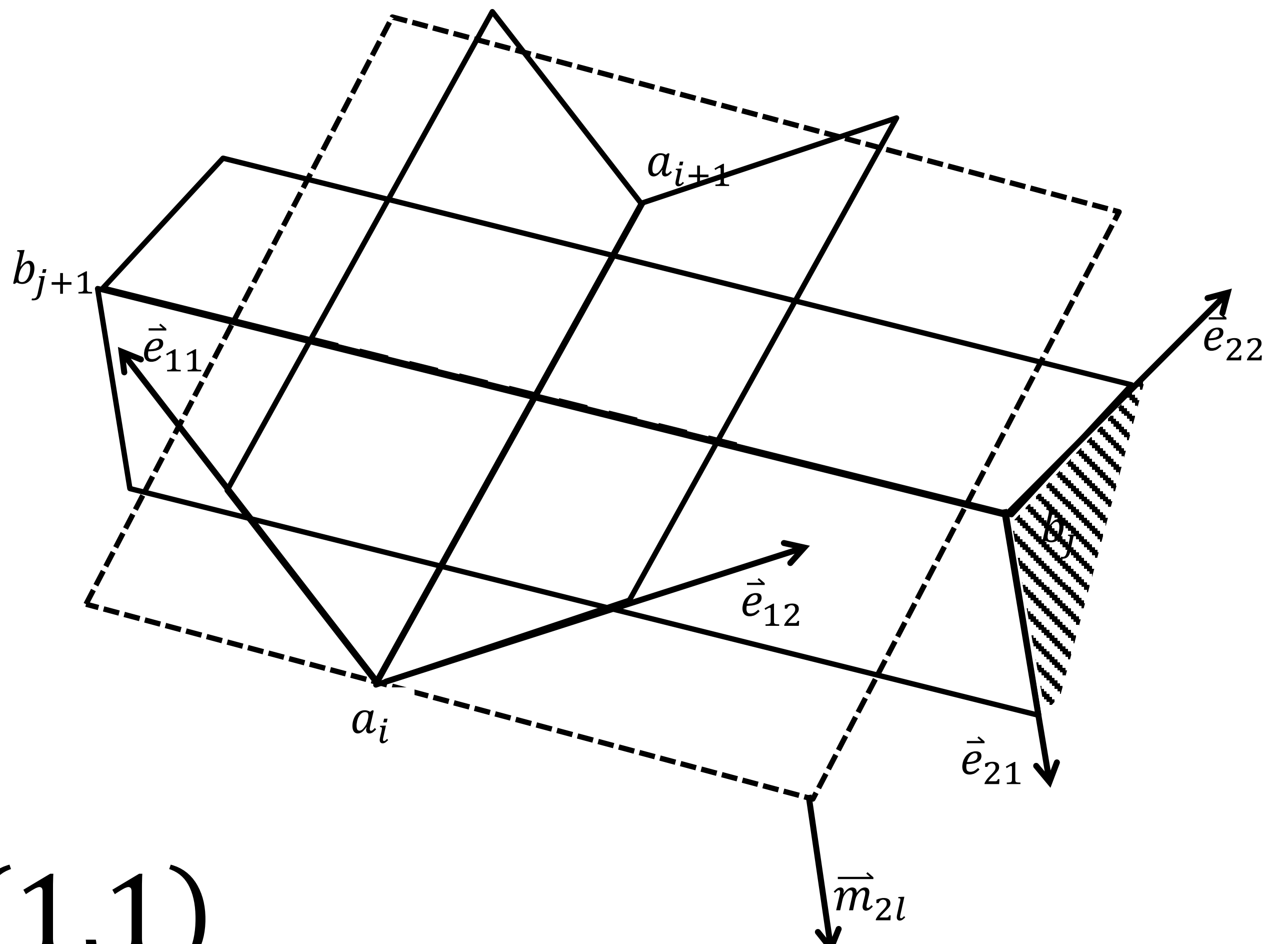
Vector Edges





$C(1,1)$

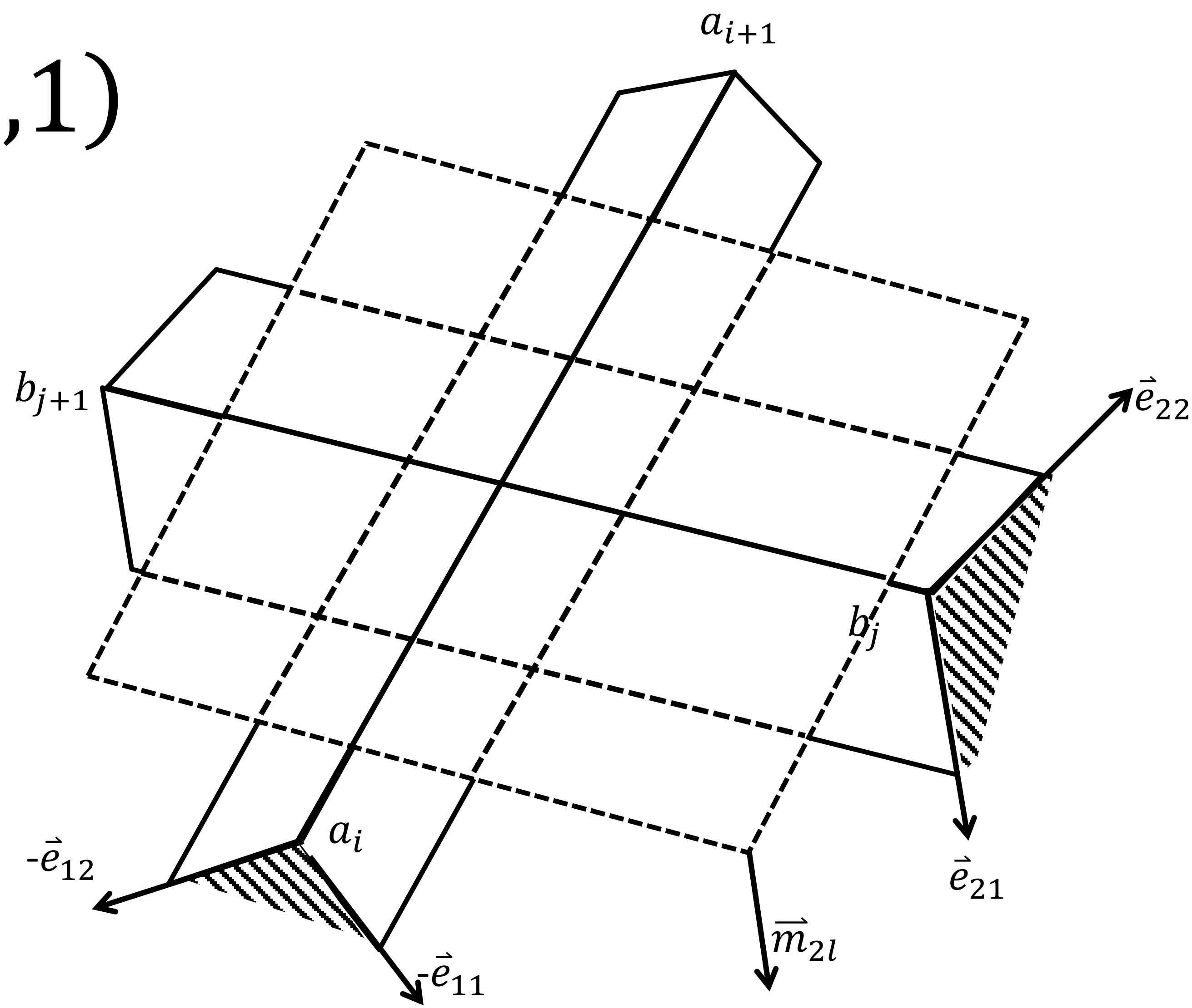
Contact Polygon of Two Edges

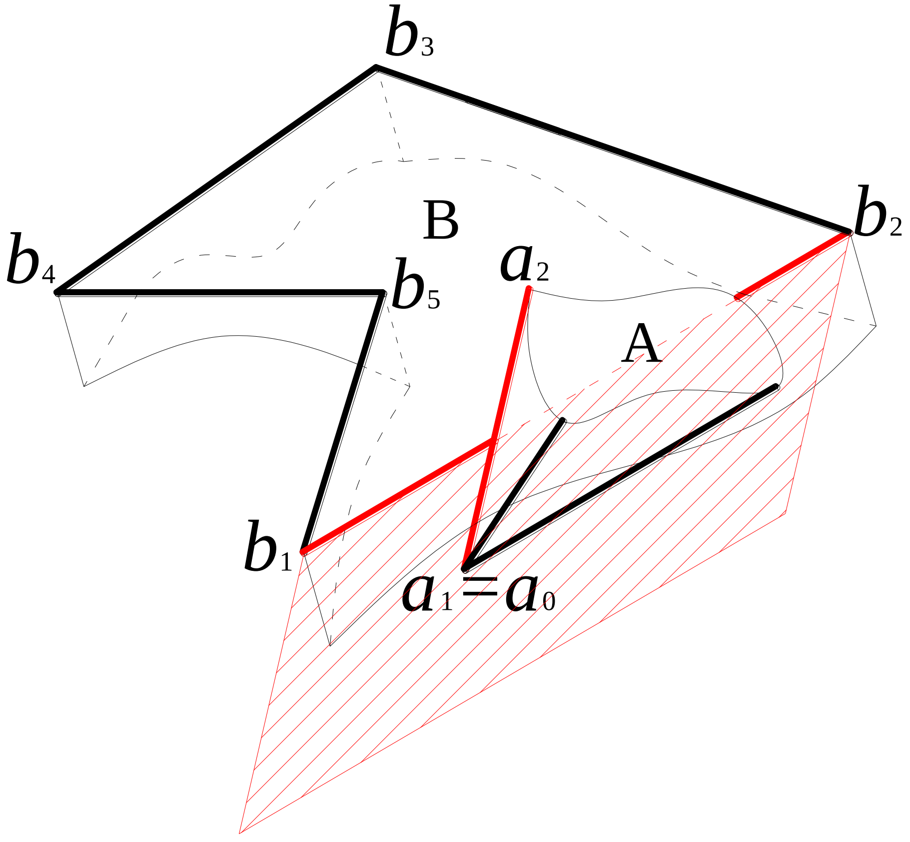


$C(1,1)$

Contact Polygon of Two Edges

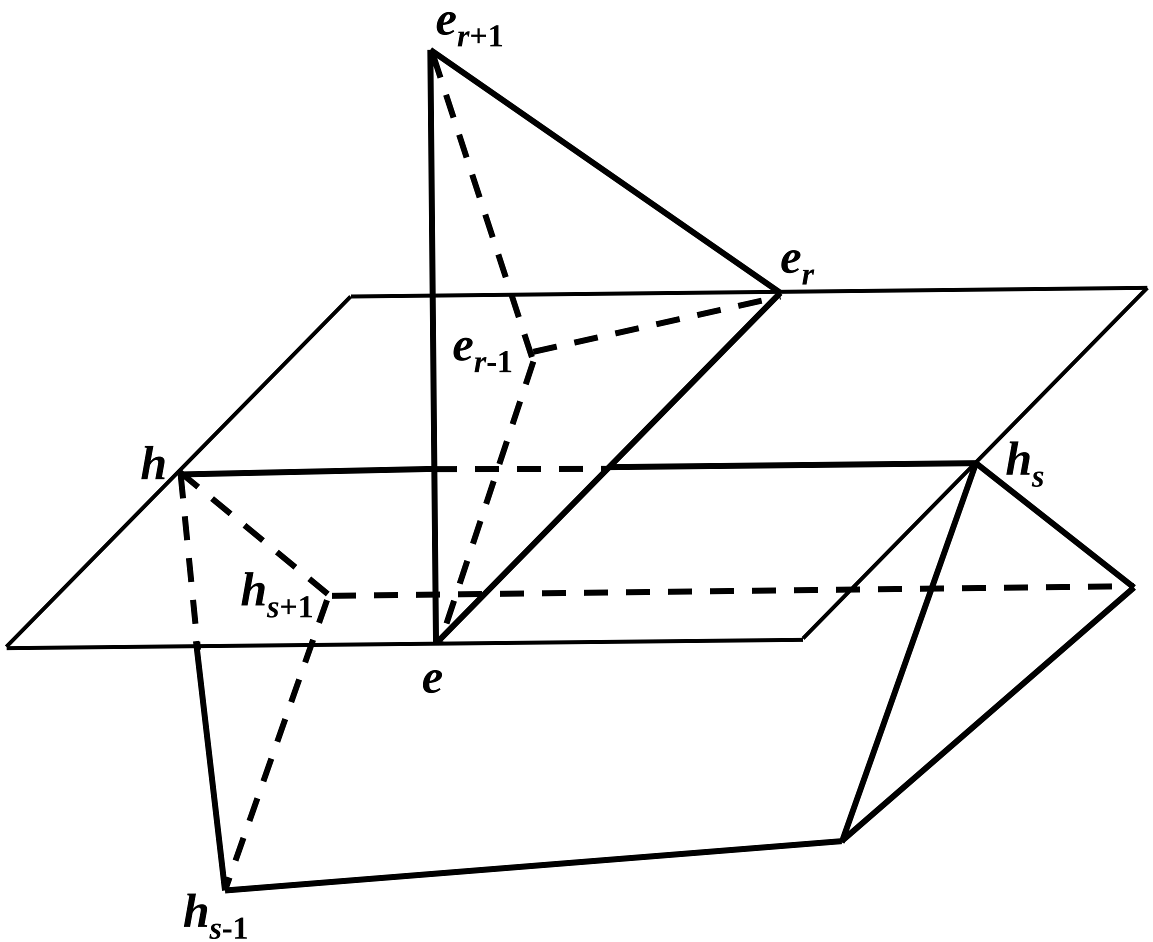
$C(1,1)$





$$E(a_1a_2,\ b_1b_2) = (b_1 - a_1 + a_0,\ b_1 - a_2 + a_0,\ b_2 - a_2 + a_0,\ b_2 - a_1 + a_0)$$

$$C(1,1)$$



$C(1,1)$

$C(1,1)$

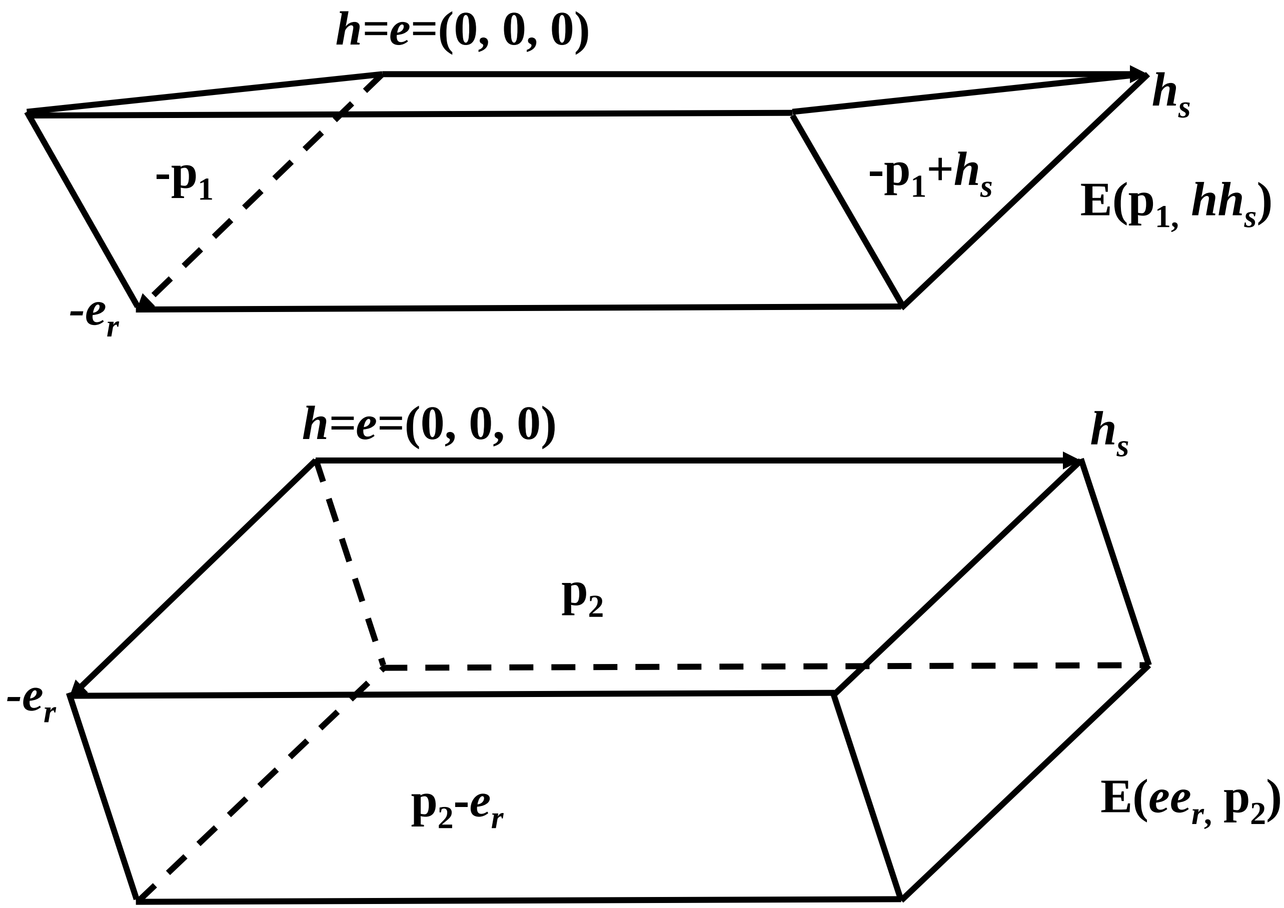
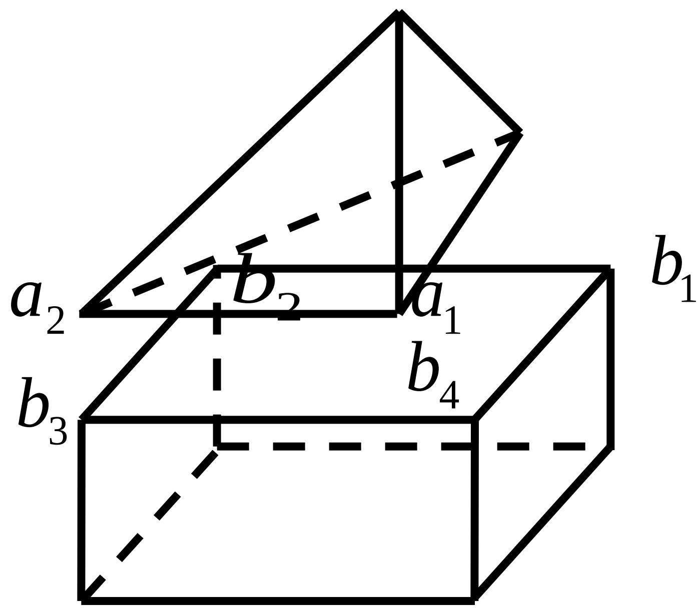


Fig 88b

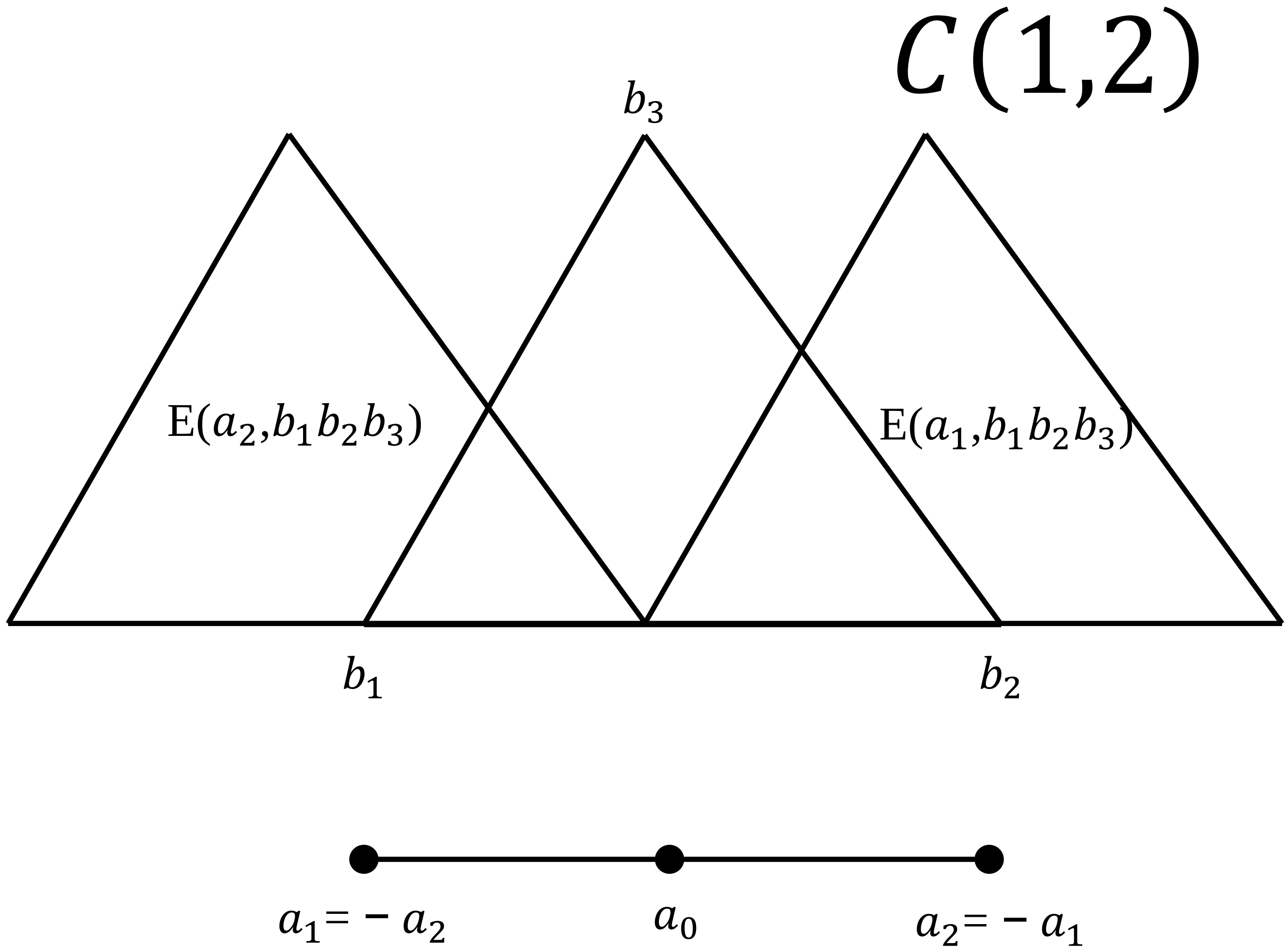
Contact Polygons of 3D Parallel Edge and Polygons

Contact of Parallel 3D Edge and Face Polygon

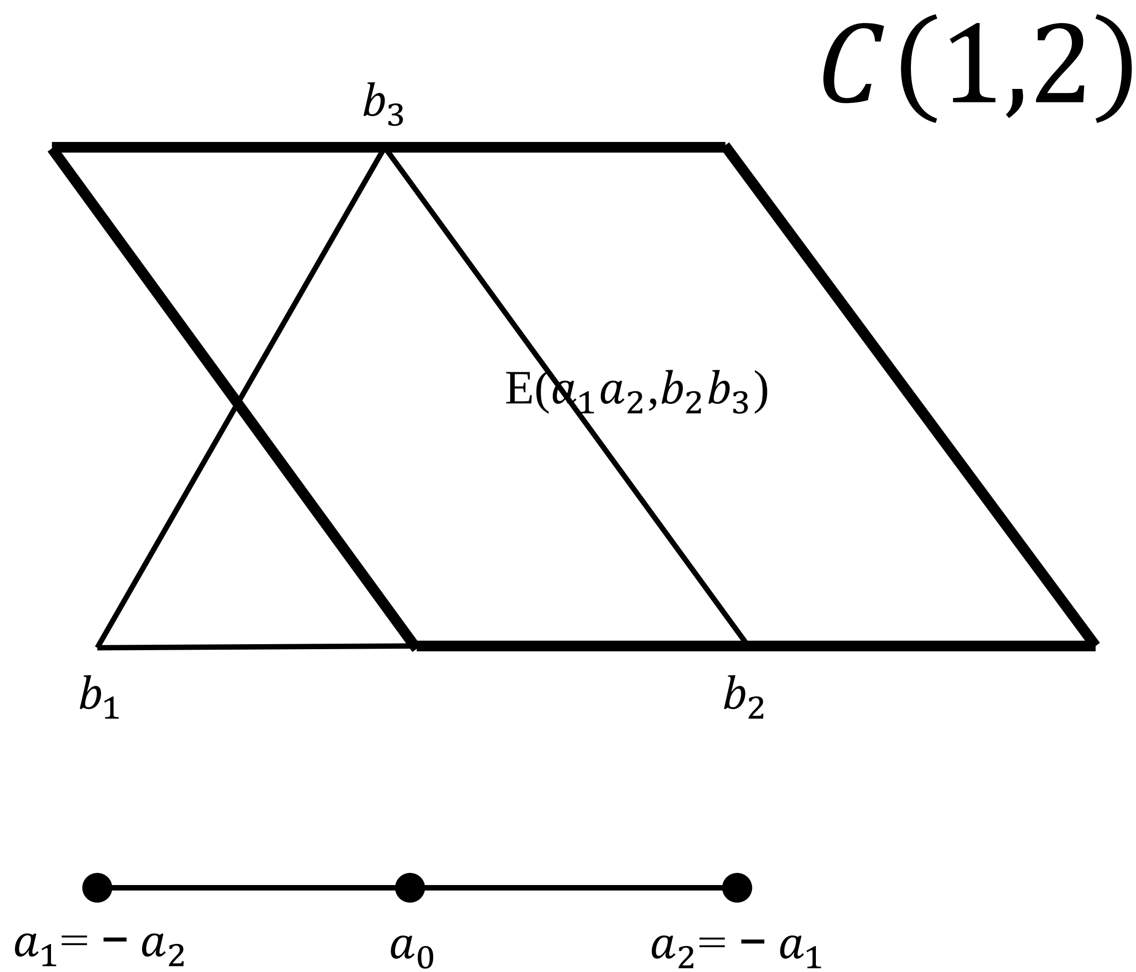


$$C(1,2)$$

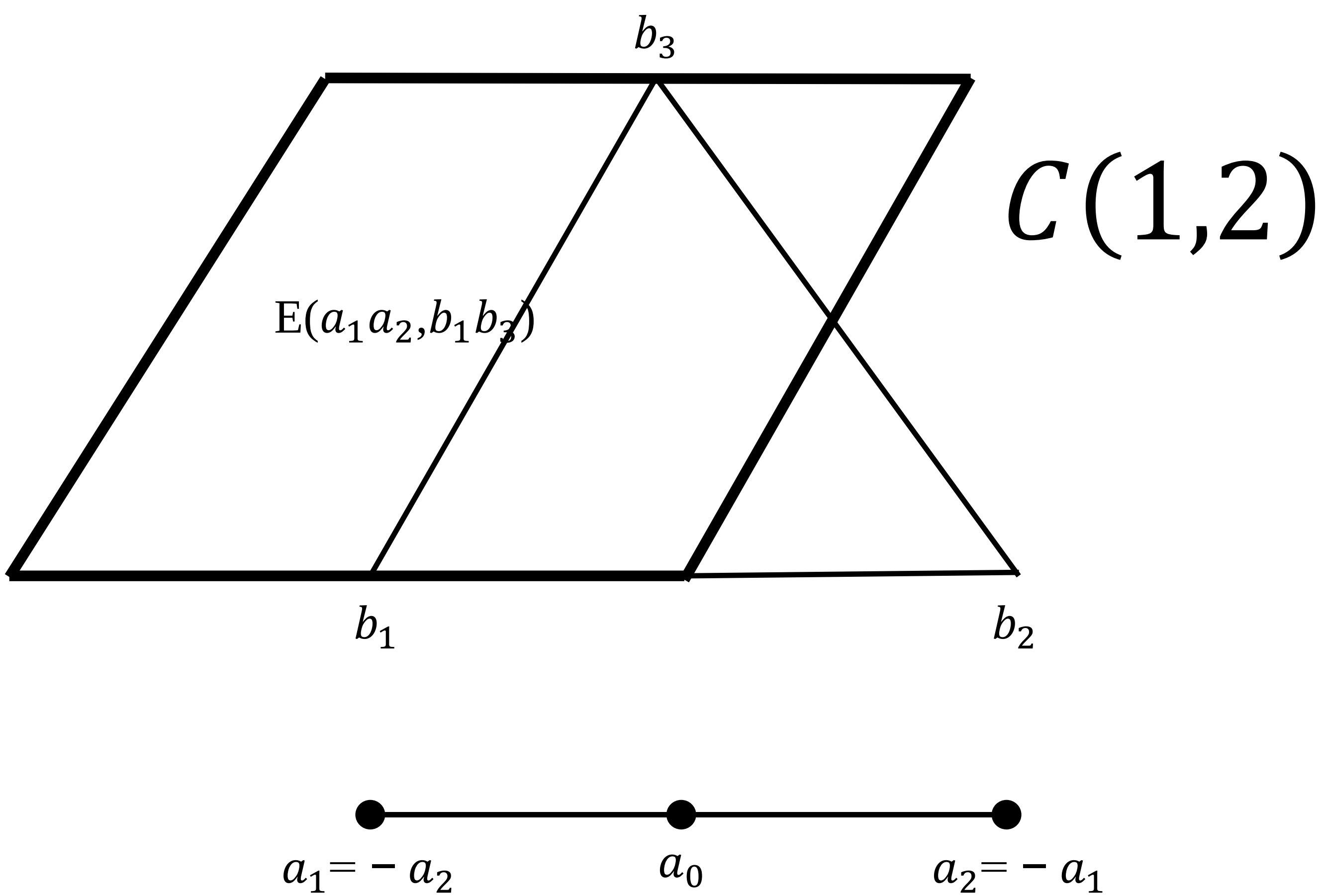
Contact Polygon of Parallel Face Polygon and Edge



Contact Polygon of Parallel Face Polygon and Edge



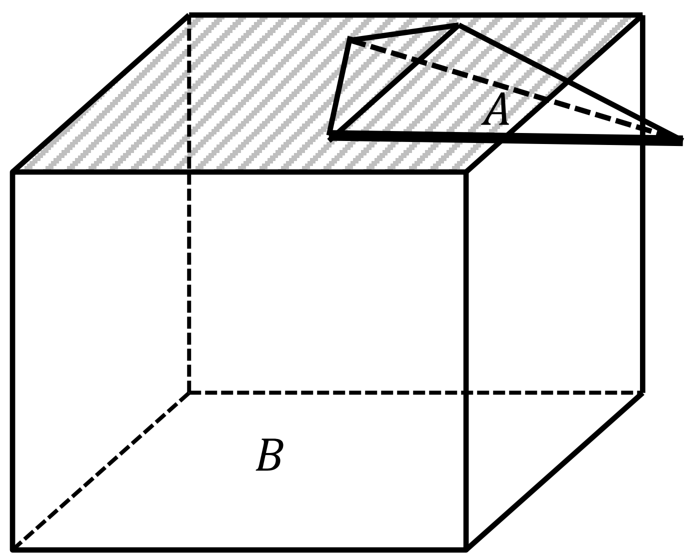
Contact Polygon of Parallel Face Polygon and Edge



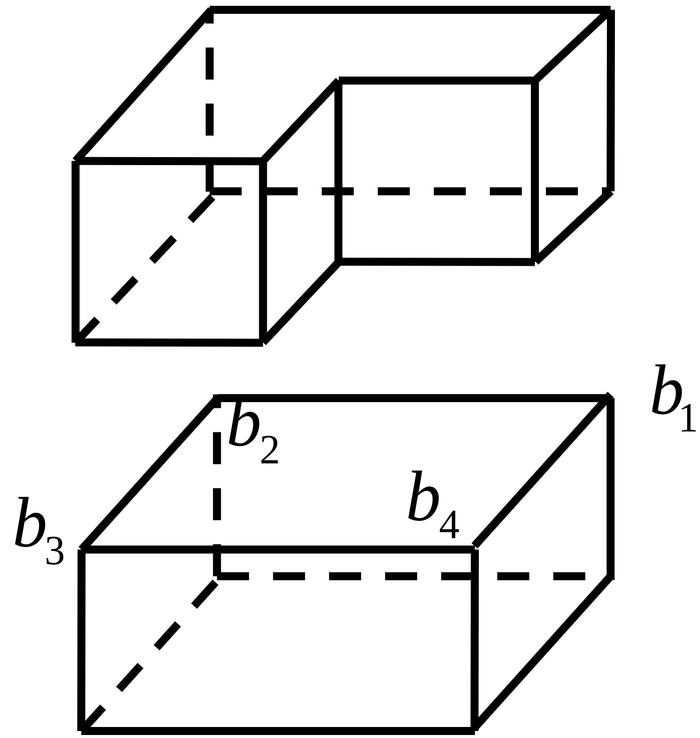
Contact Polygons
of
3D Parallel Polygons

3D Face-Face Parallel Contact

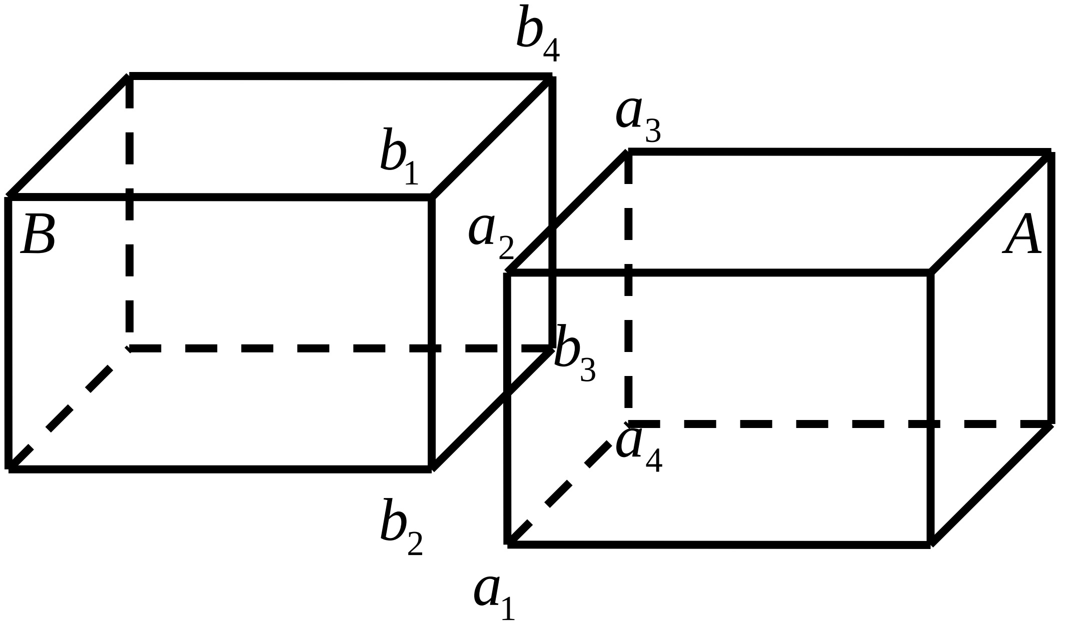
$$C(2,2)$$



Contact of Parallel Face Polygons

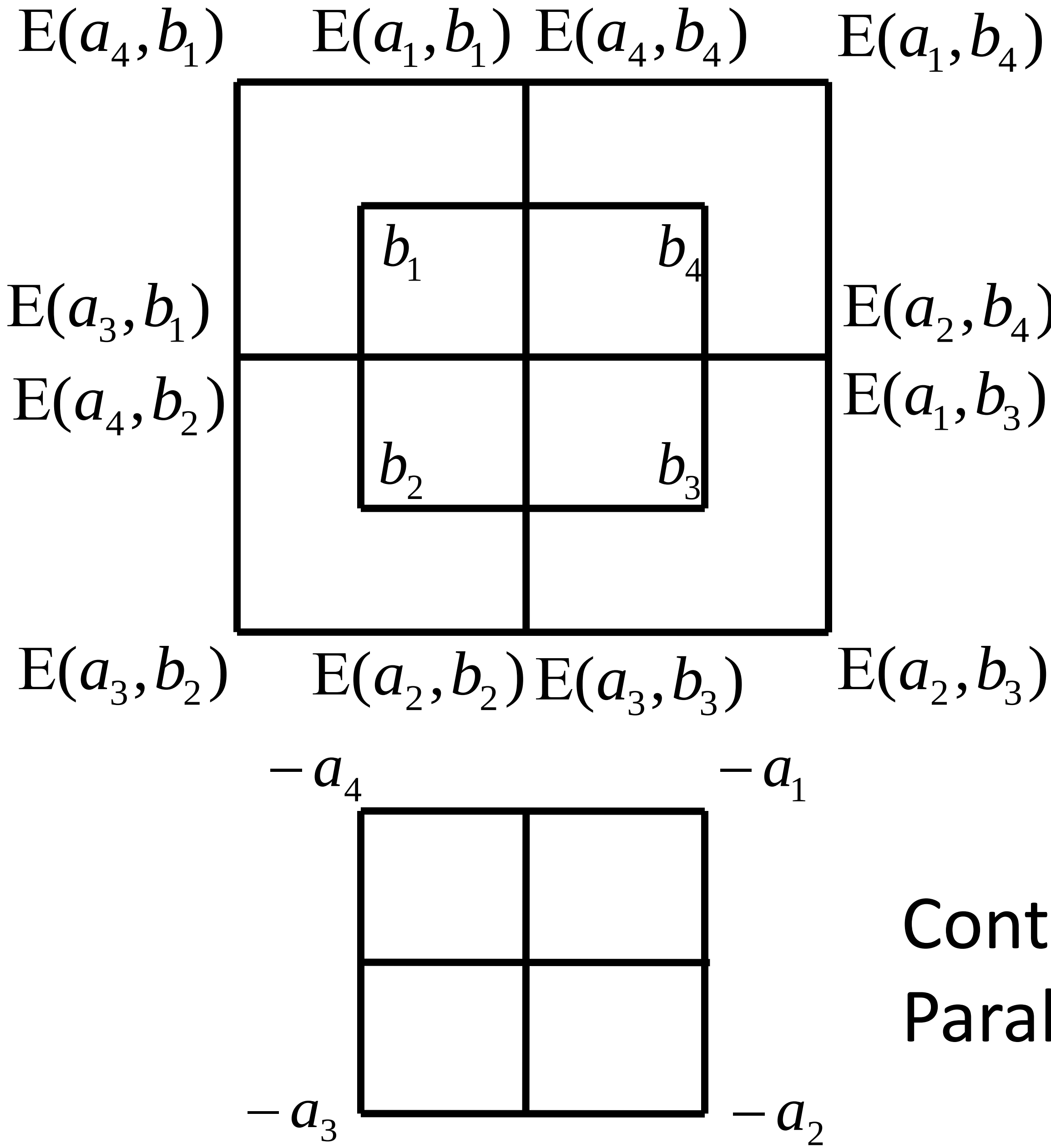


$$C(2,2)$$

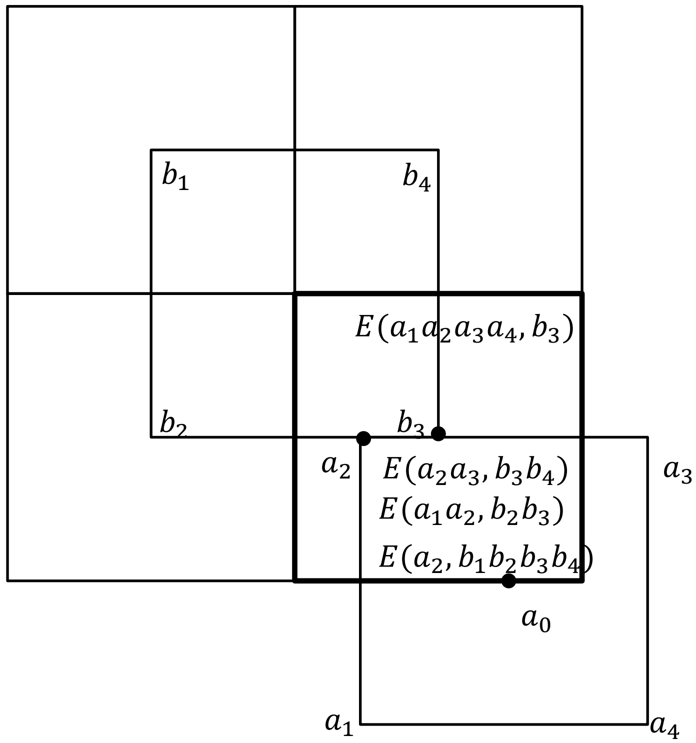


$$C(2,2)$$

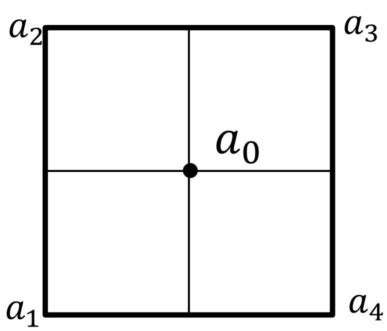
Contact of Parallel Faces



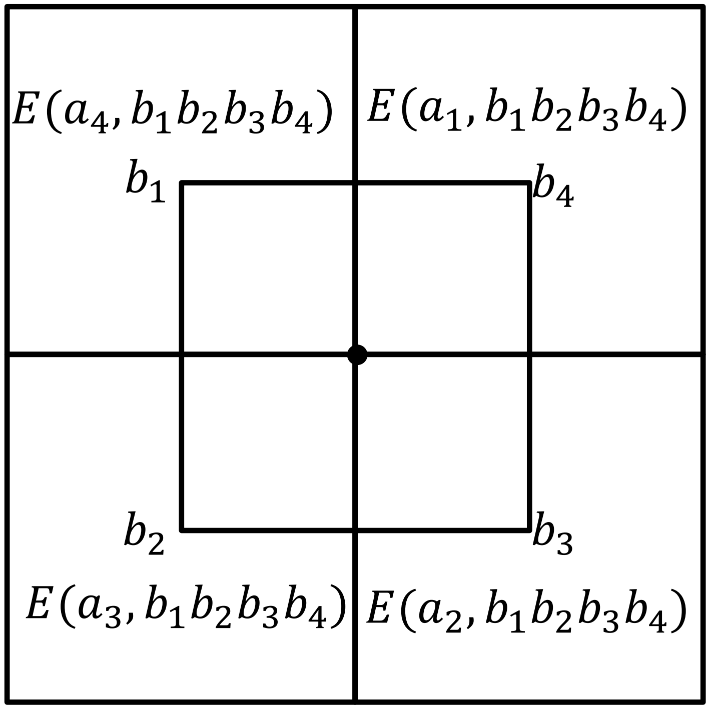
Contact of Parallel Faces

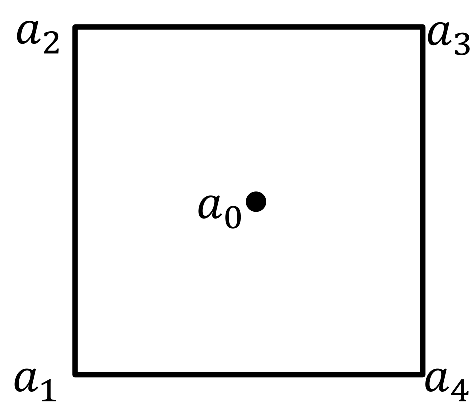


Contact of Parallel Faces

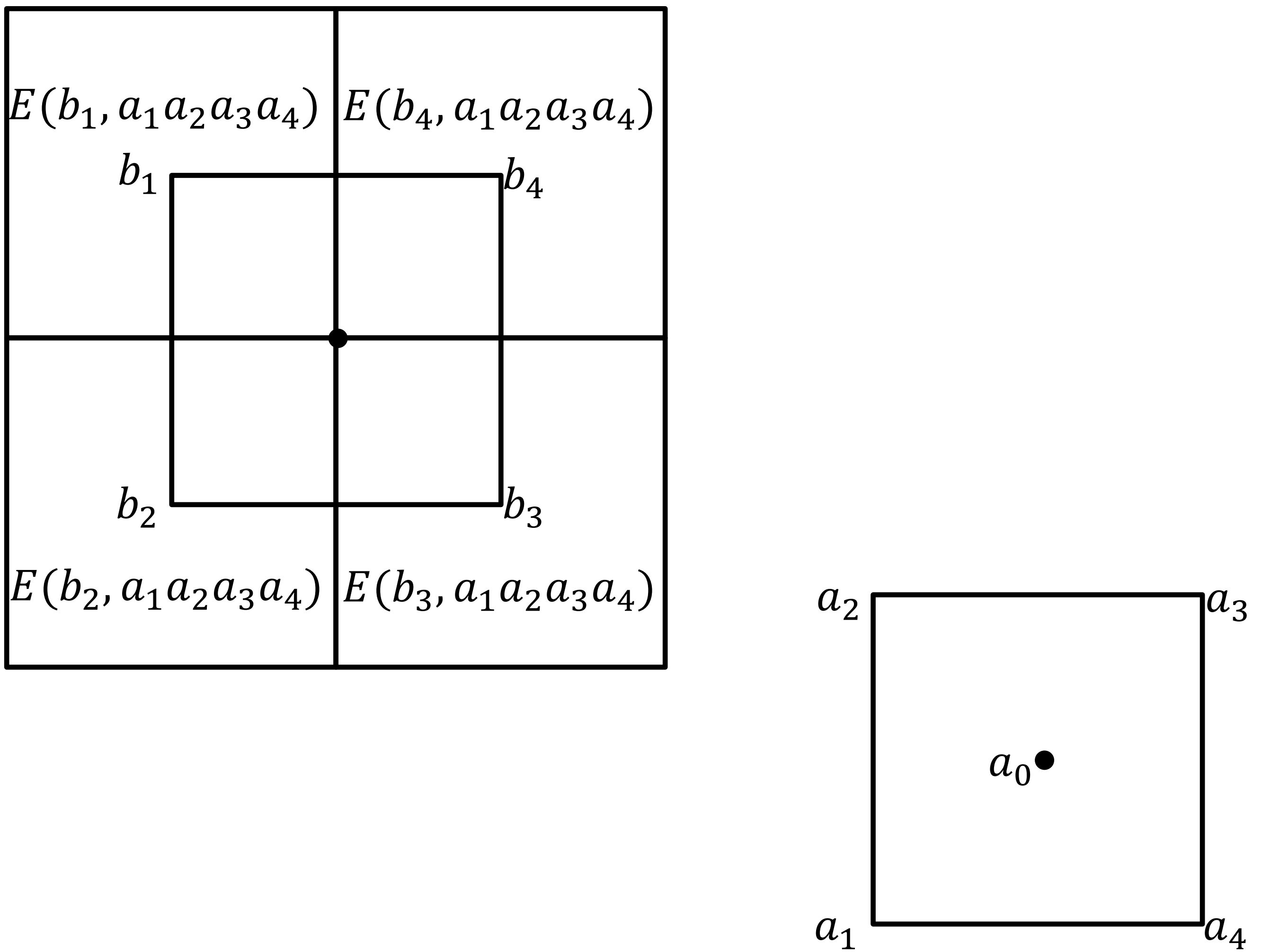


Contact Polygon of Parallel Face Polygons

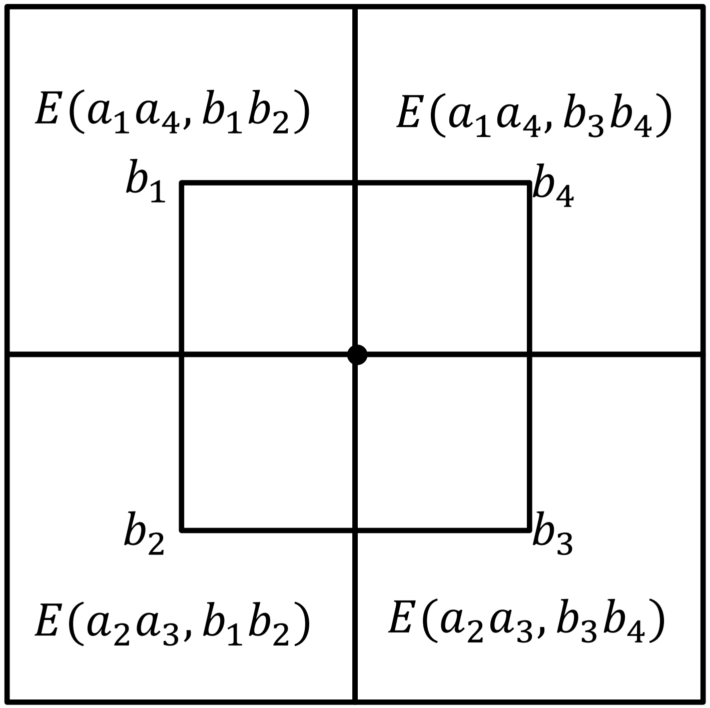


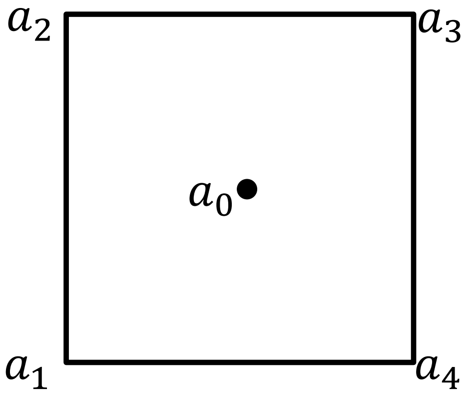


Contact Polygon of Parallel Face Polygons

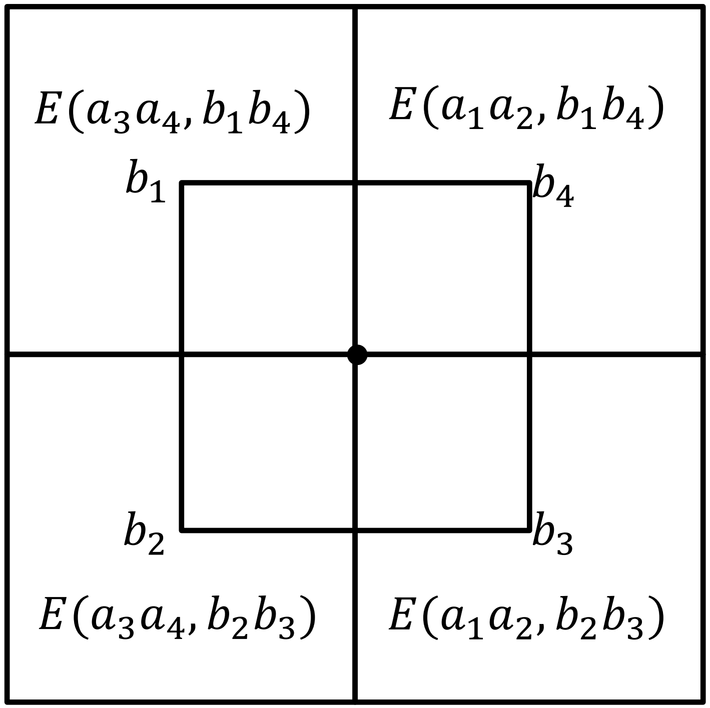


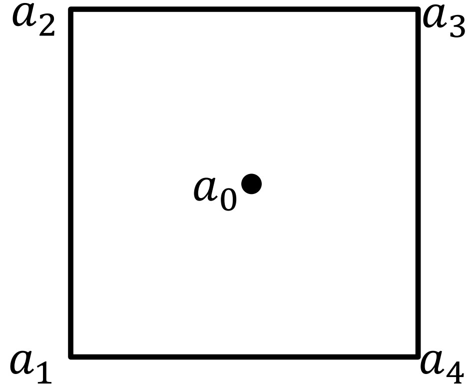
Contact Polygon of Parallel Face Polygons

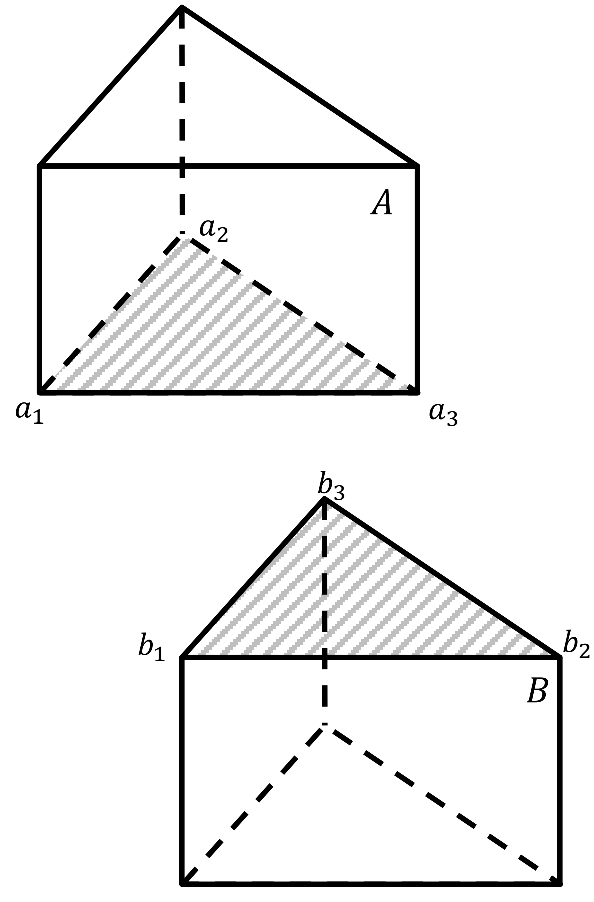




Contact Polygon of Parallel Face Polygons



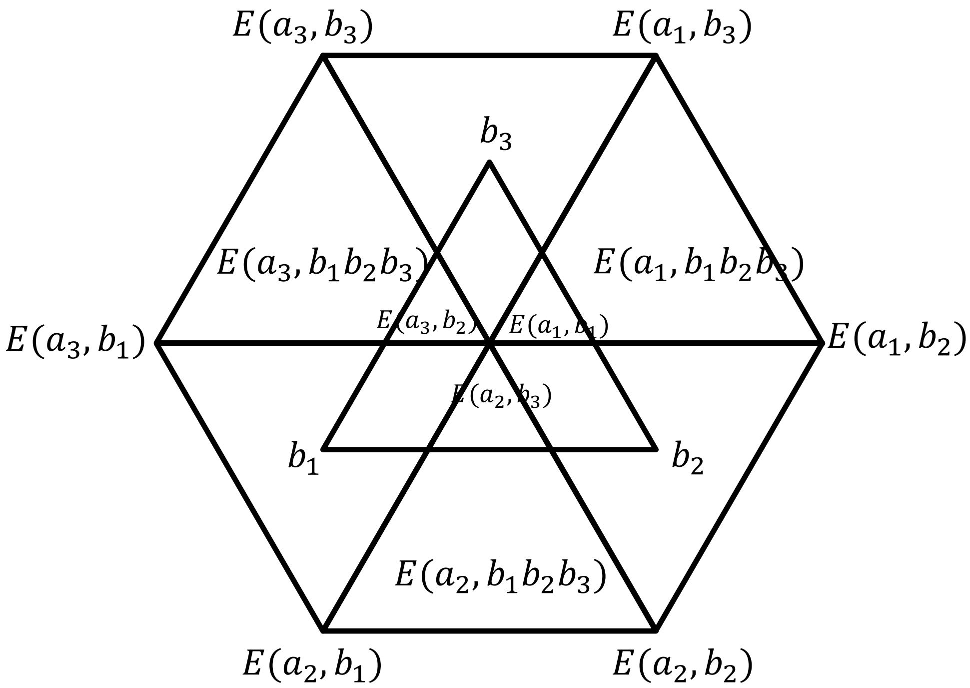


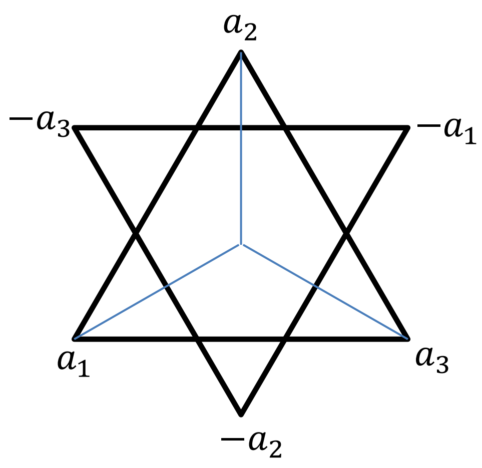


$C(2,2)$

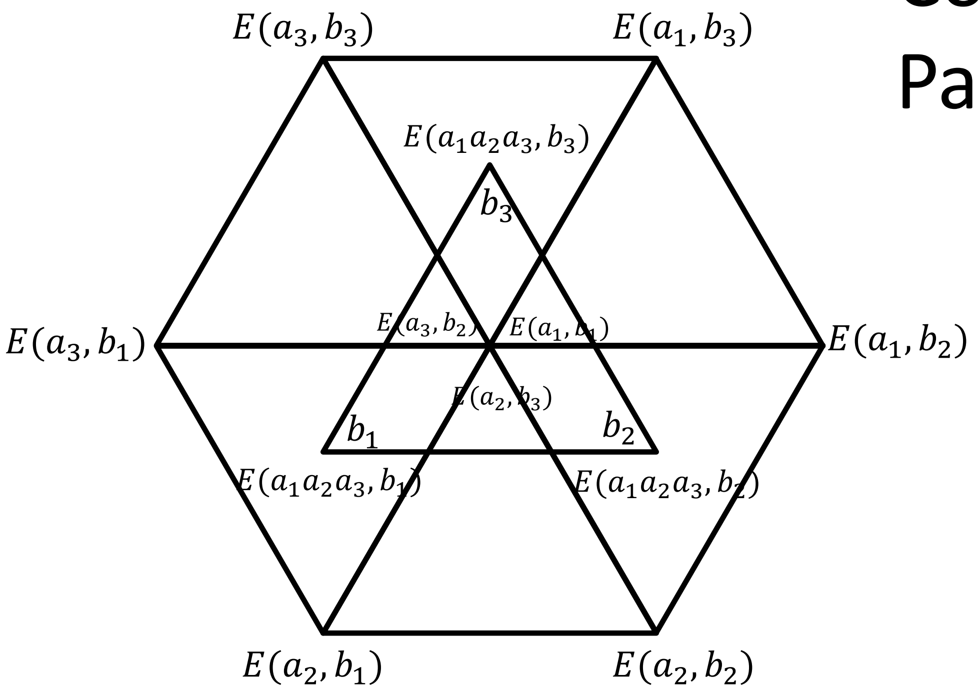
Contact of Parallel Faces

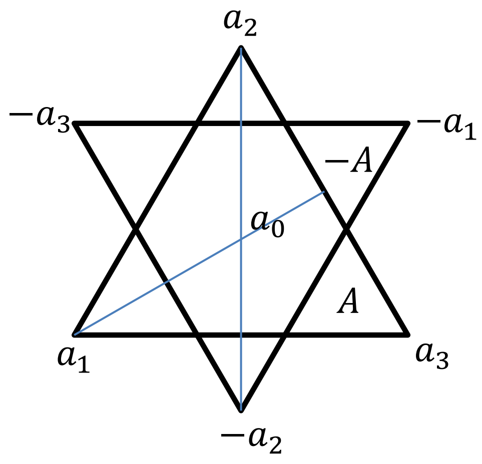
Contact of Parallel Faces



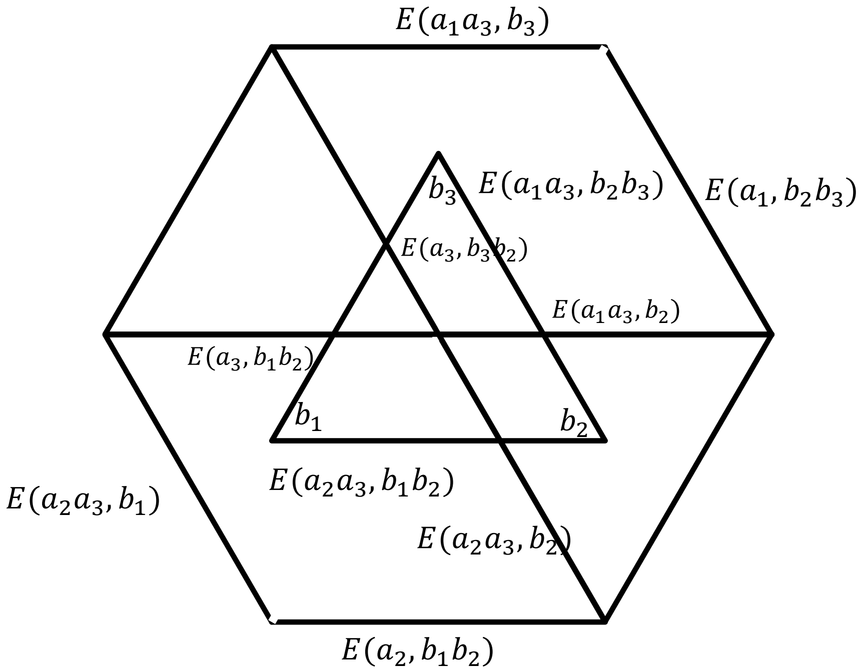


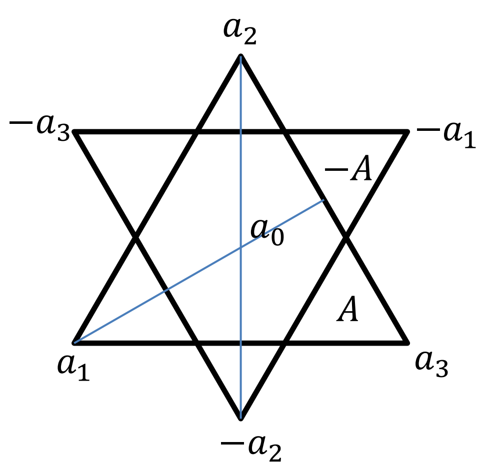
Contact of Parallel Faces



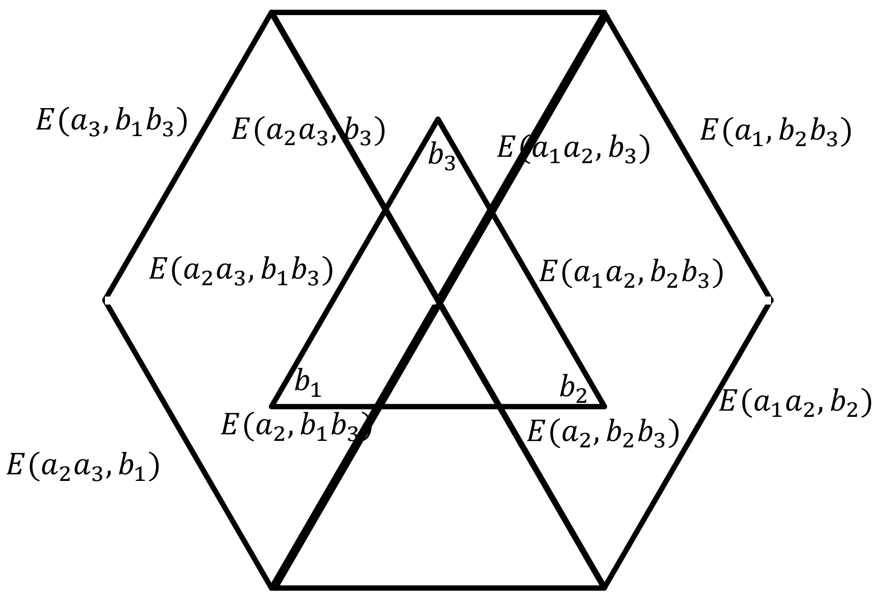


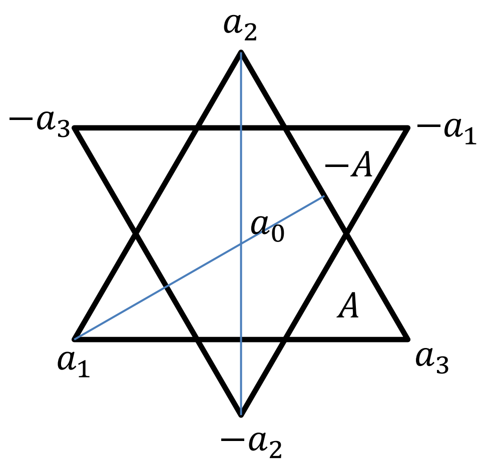
Contact of Parallel Faces





Contact of Parallel Faces





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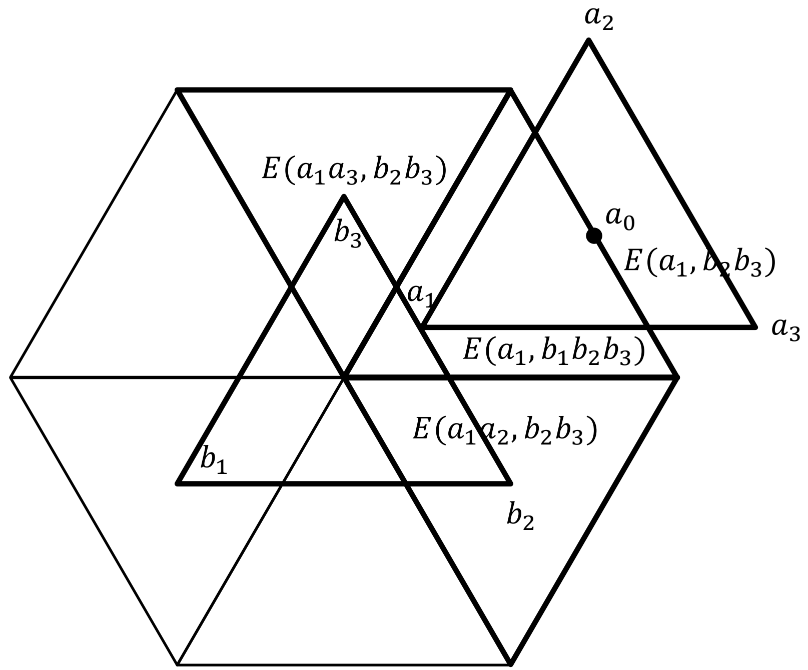
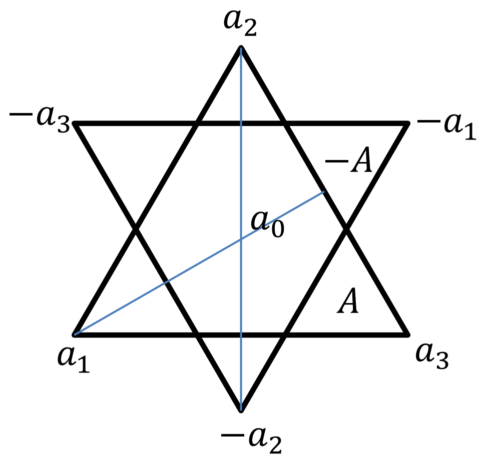
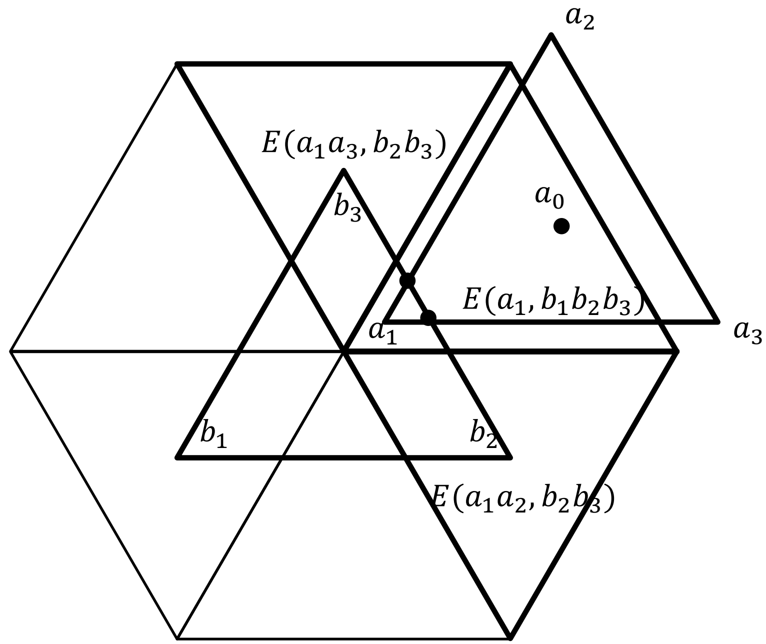
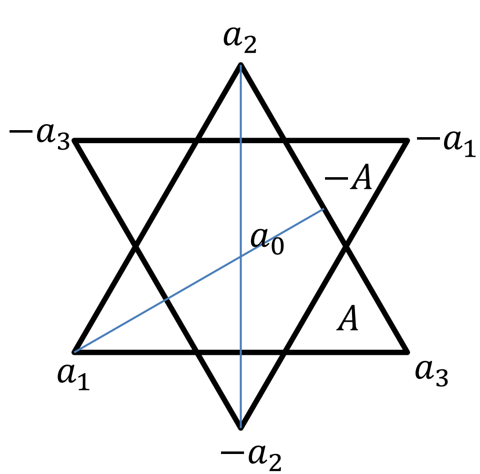


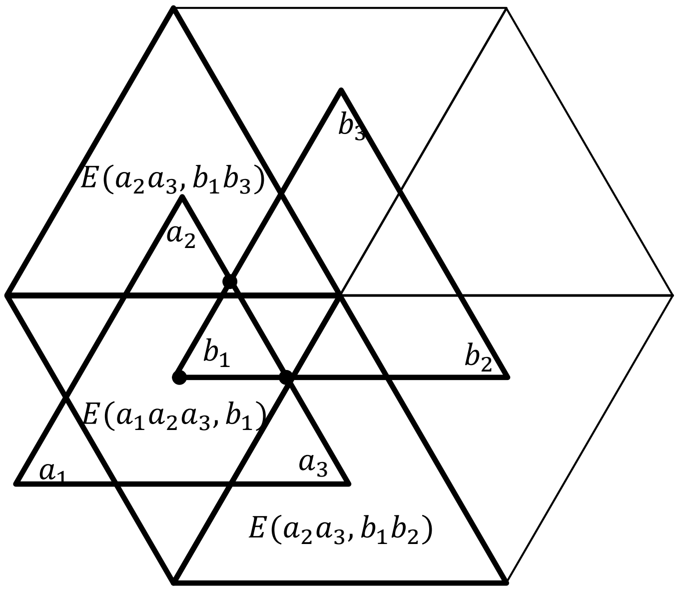


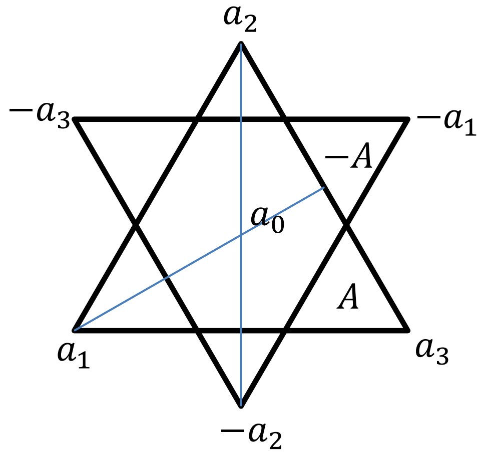
Contact of Parallel Faces





Contact of Parallel Faces



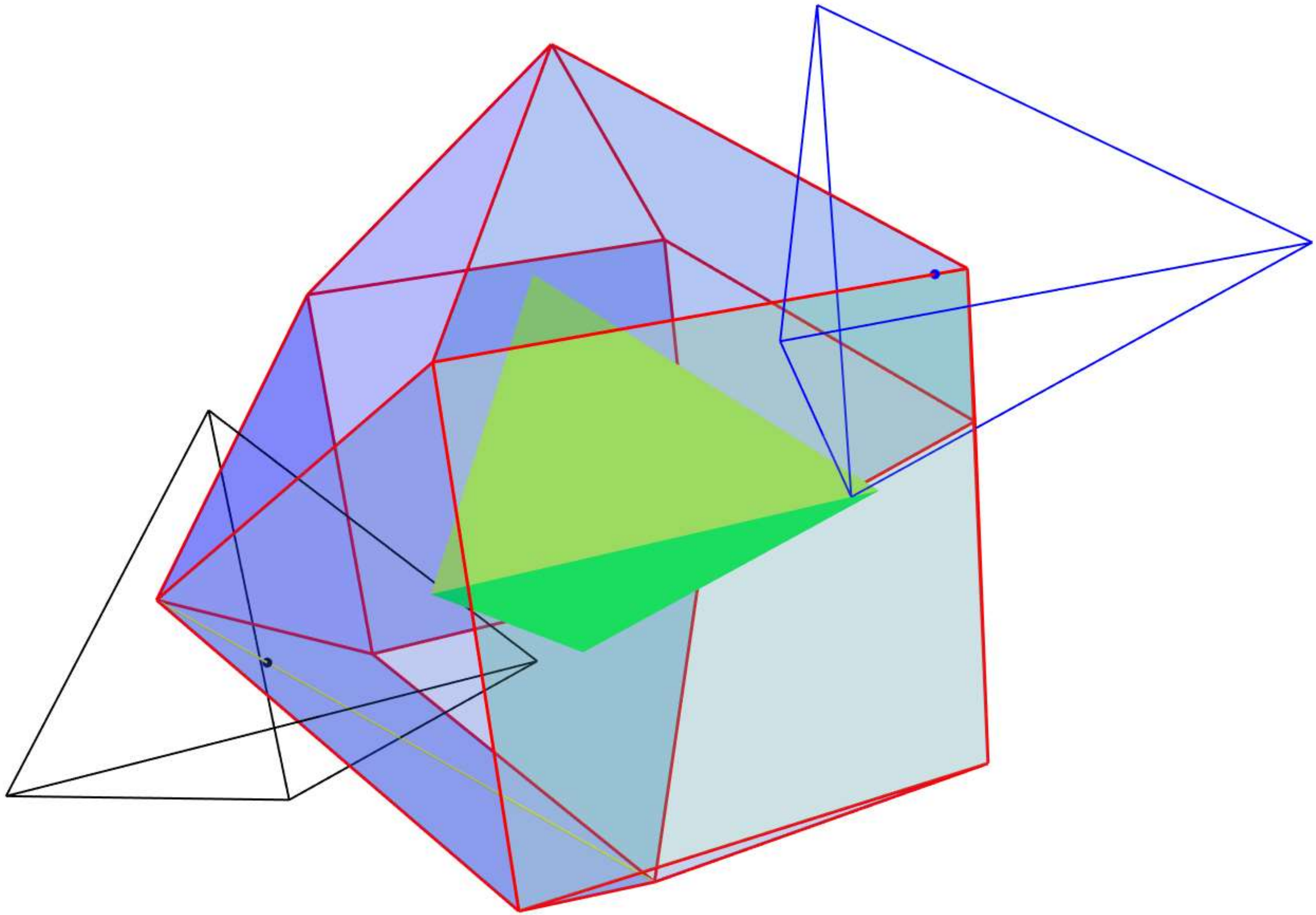


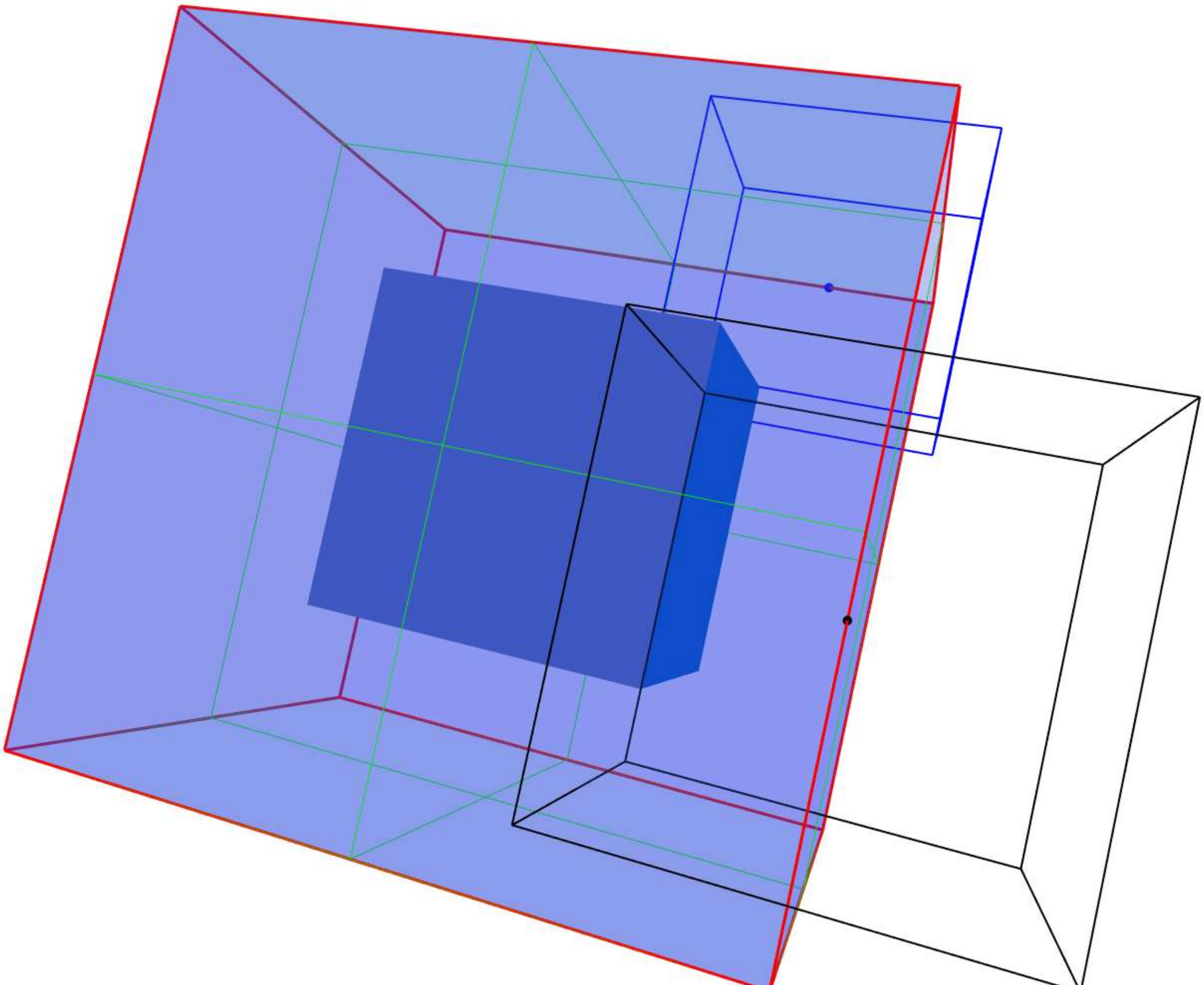
Entrance Block
of
3D Convex Blocks

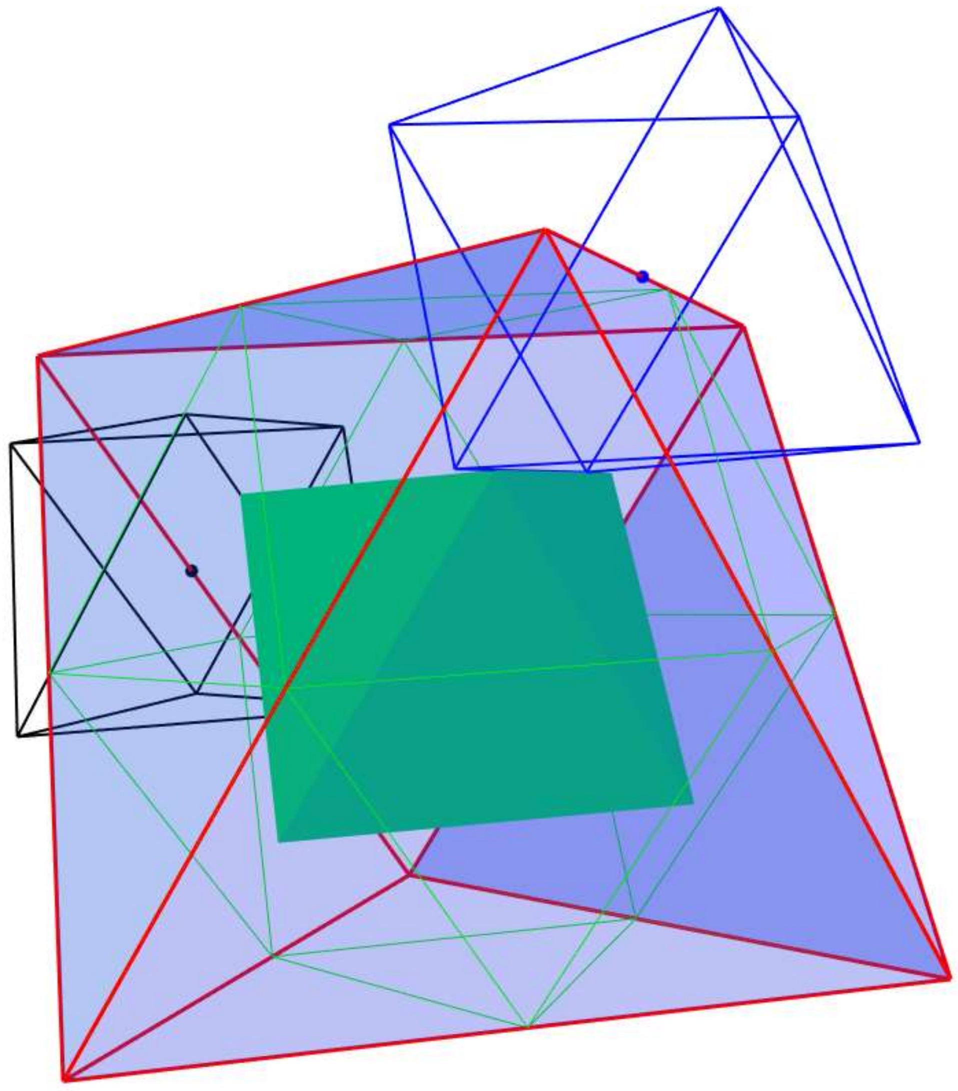
$$\partial E(A,B) \subset C(0,2) \cup C(2,0) \cup C(1,1)$$

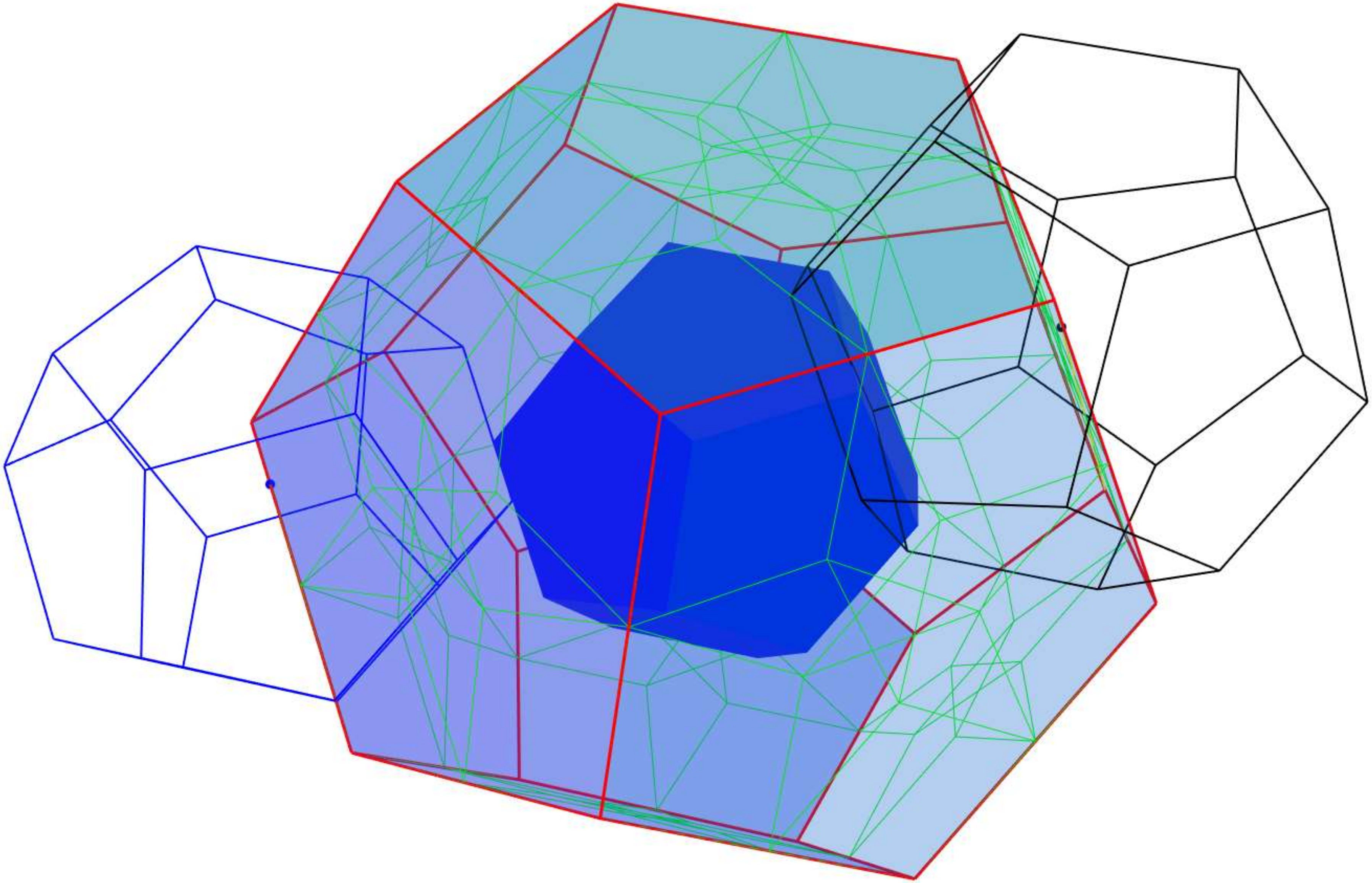
If A and B are convex

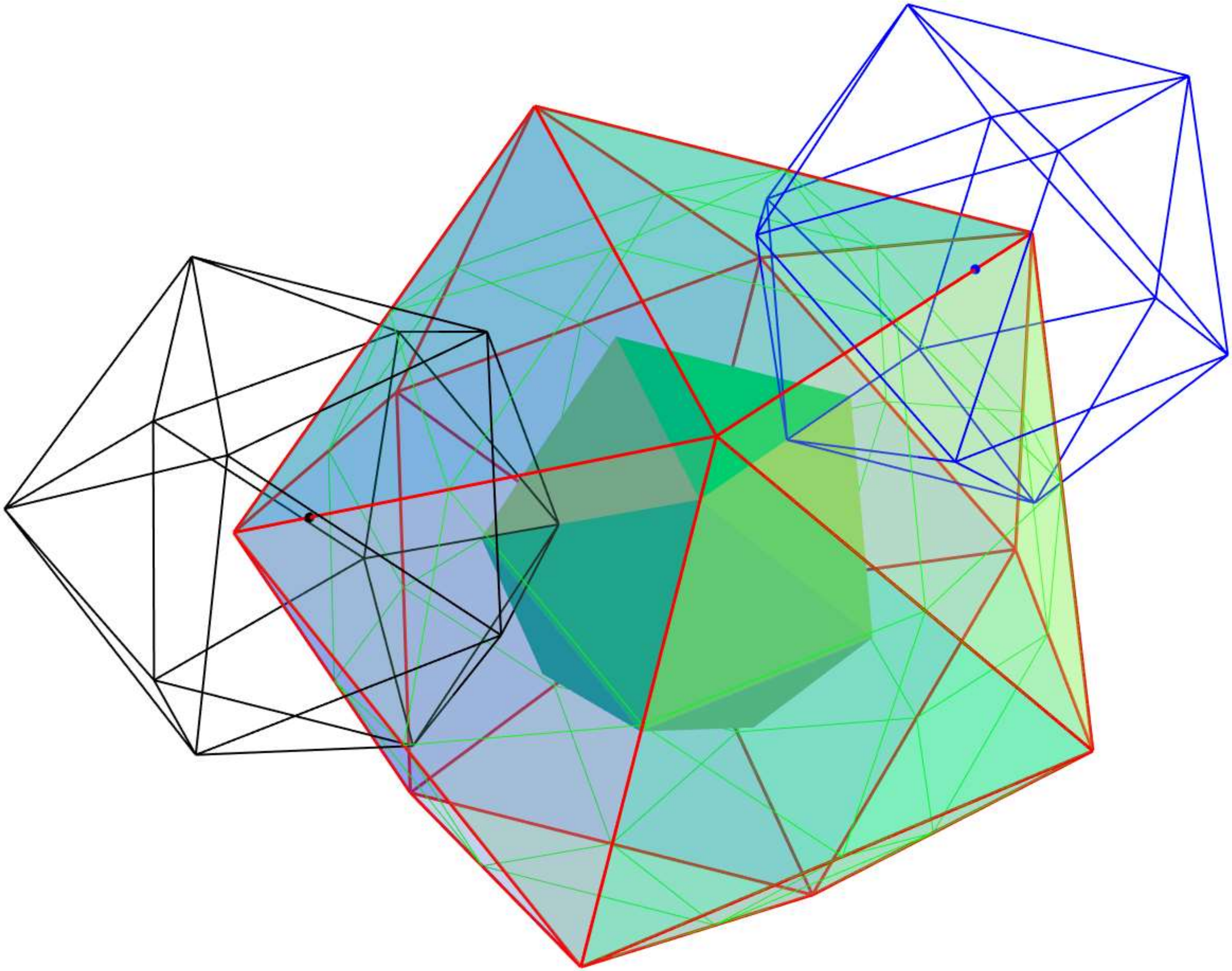
$$\partial E(A,B) \supset C(0,2) \cup C(2,0) \cup C(1,1)$$

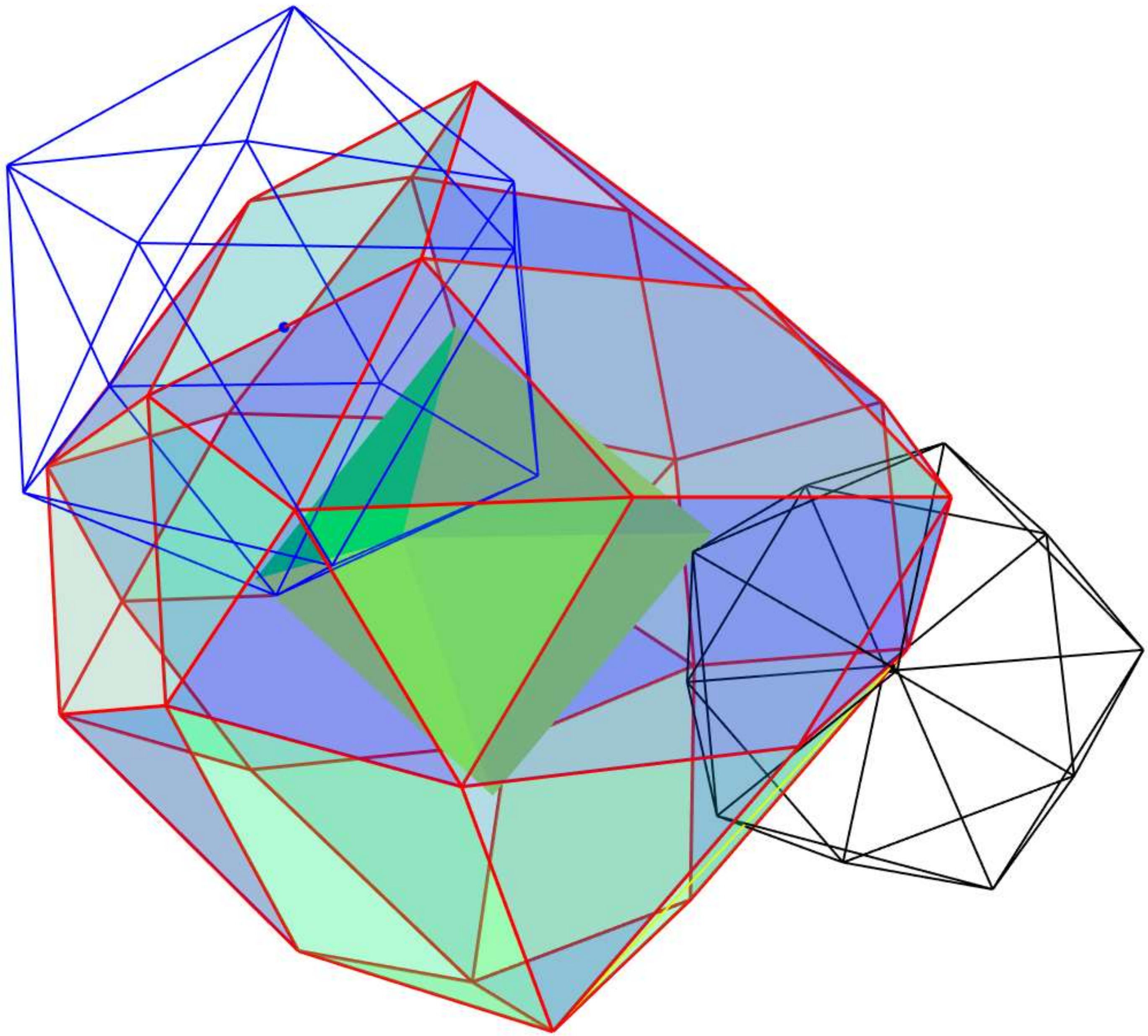


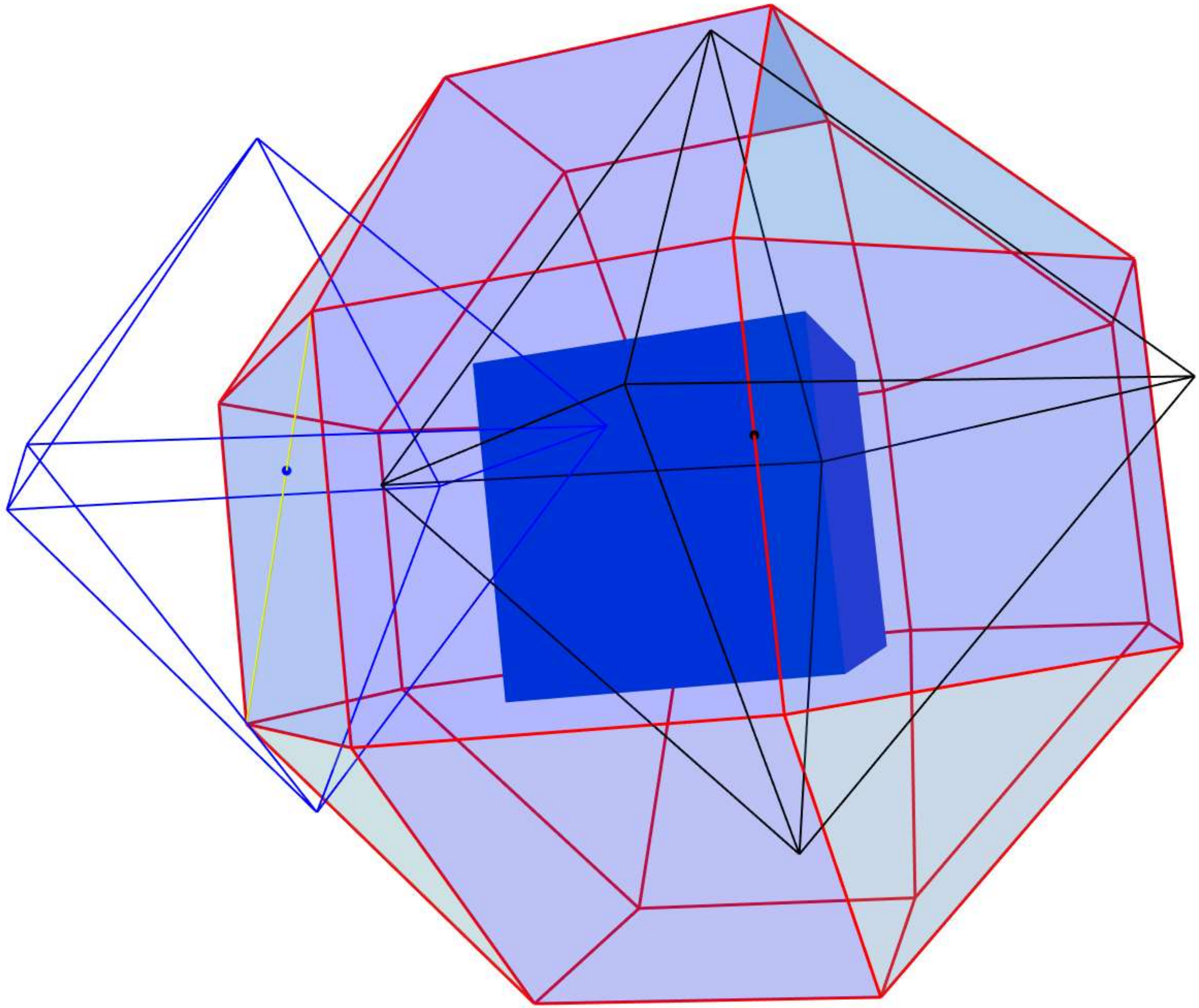


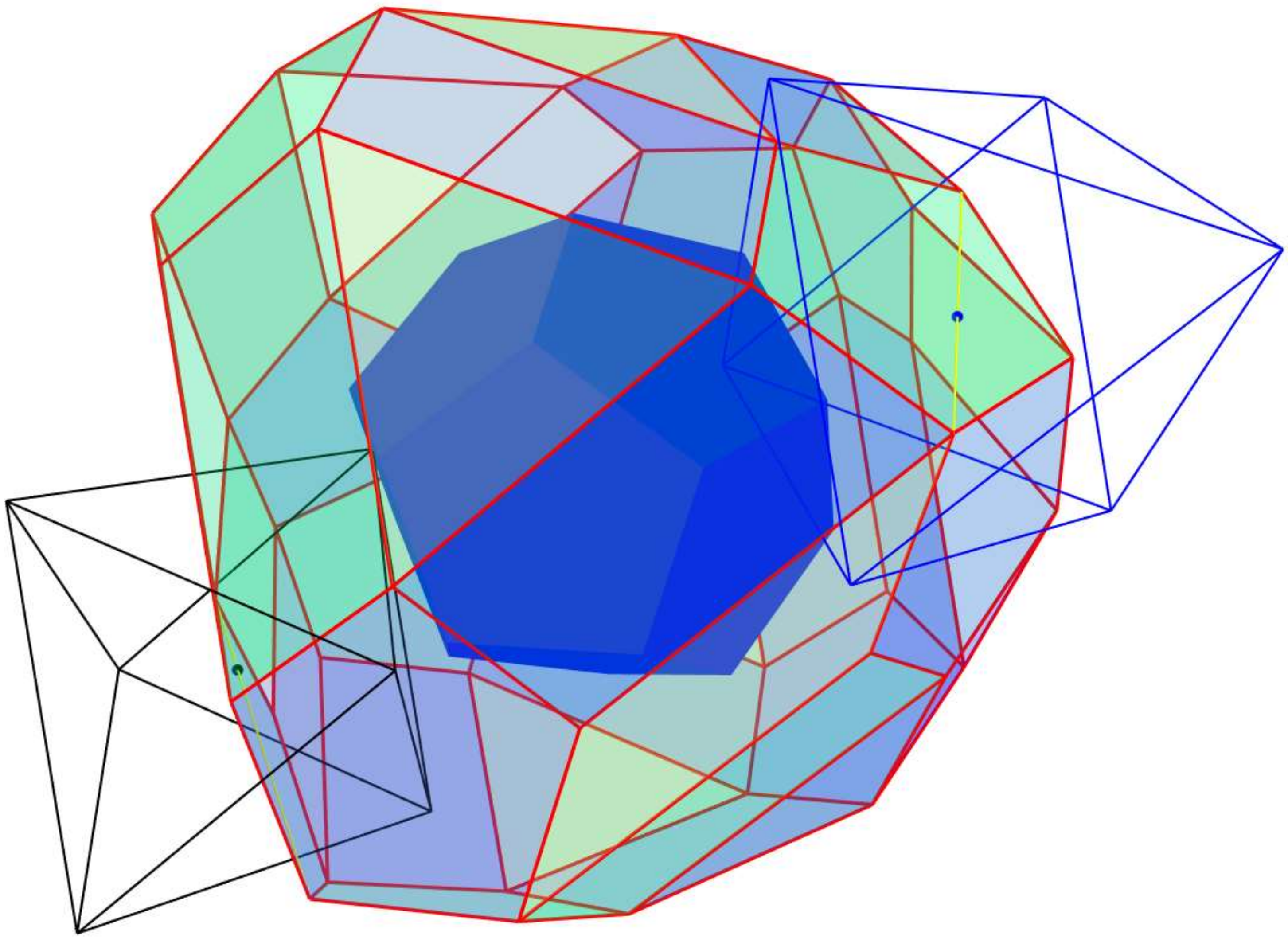


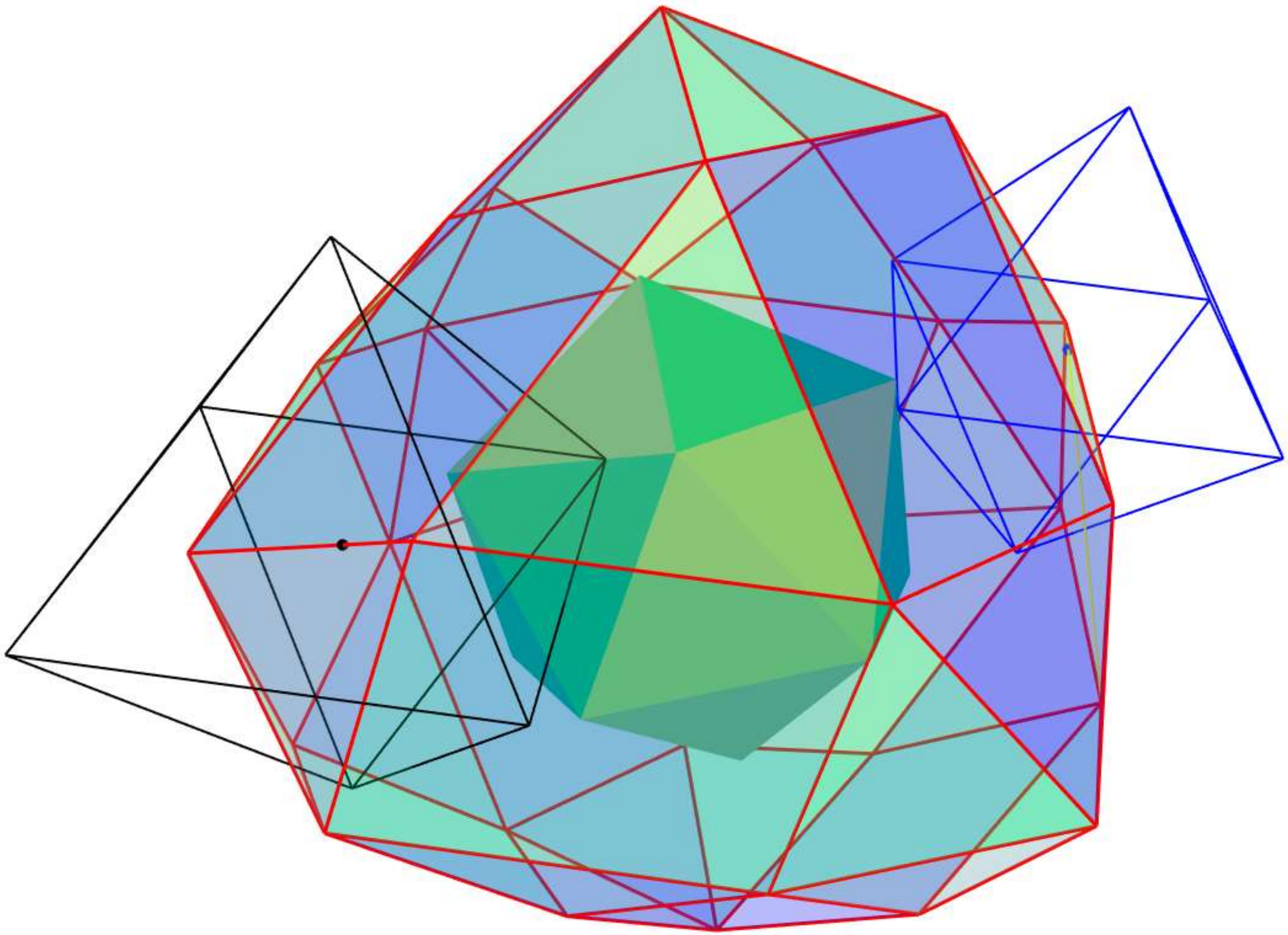


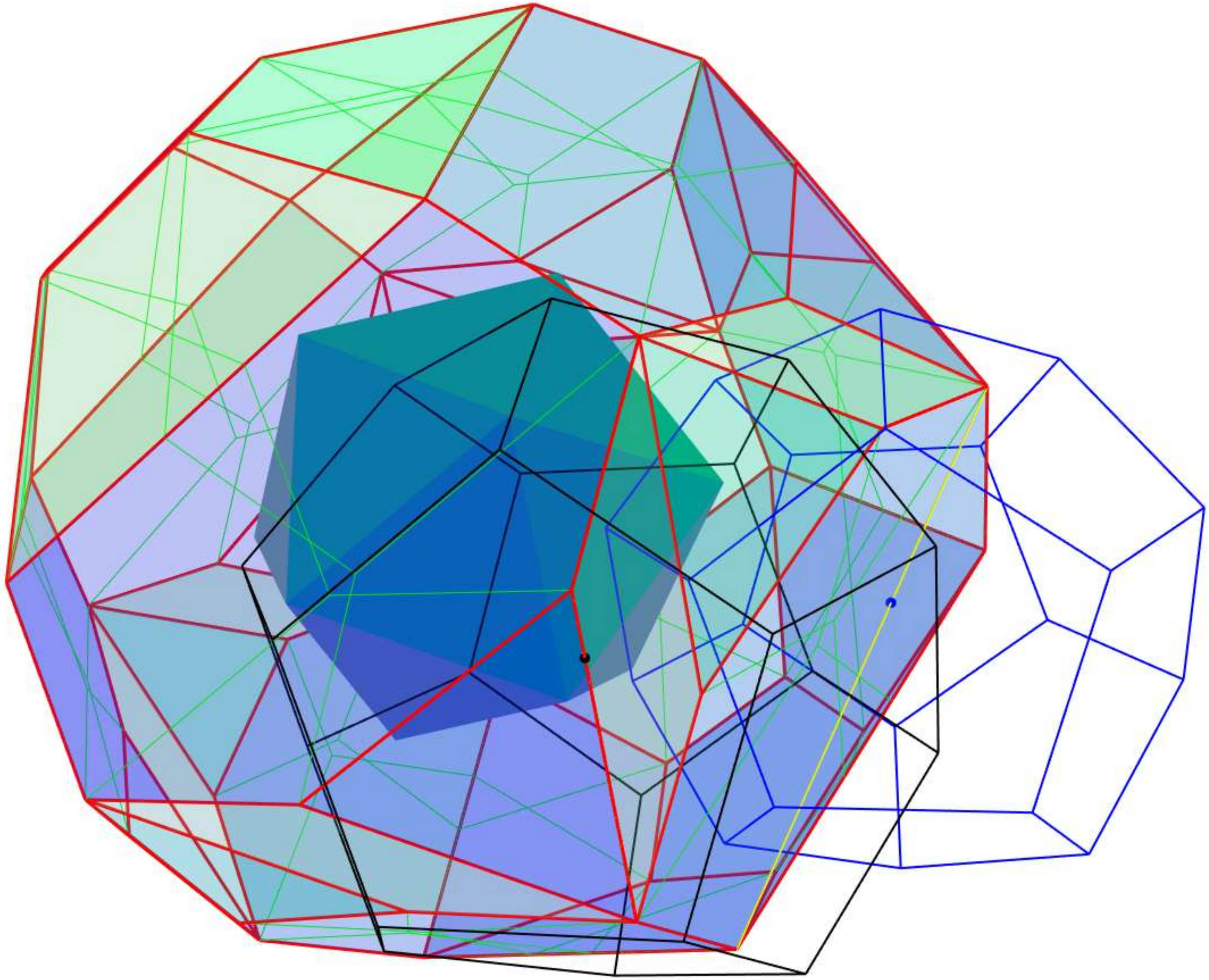












Entrance Block
of
3D General Blocks

If A and B are general blocks

$$\partial E(A,B) \subset C(0,2) \cup C(2,0) \cup C(1,1)$$